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Investing with a Broker [1]

The Problem. A risk neutral, subjective expected utility maximizer has money to invest. She has the following alternatives:

- deposit it into a bank account and earn zero for sure
- buy a risky asset, which yields 1 or -1 if it goes well or bad —events resp. h and l (for high and low)
- go and see a broker, pay a commission c, listen to his advice, which can be either "buy" —event β— or "don't buy" —event δ—, and decide whether to follow it or not.

A suitable state space for the problem is $S = \{h\beta, h\delta, l\beta, l\delta\}$; and abusing notation we may let $h = \{h\beta, h\delta\}$, $\beta = \{h\beta, l\beta\}$, and so on. Suppose the decision maker's subjective probability P on S is determined by the following judgments on the quality of asset and broker:

$$\mathsf{P}(h) = 0.6,\ \mathsf{P}(l) = 0.4; \quad \mathsf{P}(\beta \mid h) = \xi_1,\ \mathsf{P}(\delta \mid l) = \xi_2.$$

Note that ξ_1 (resp. ξ_2) measures the ability of the borker to recognize a good (resp. bad) asset. And **assume** that $\xi_i > 0.5,\ i = 1, 2$. This says that the broker has at least some professional skill (in general it is plausible that $\xi_1 > \xi_2$, since good news circulate more easily that bad ones; but we will not need such an assumption).

Determine the optimal decision for $(\xi_1, \xi_2) \in (0.5, 1]^2$.

[1] Based on a problem in D.V. Lindley, *Making Decisions*, Wiley 1985.

Solution. Given SEU and risk neutrality, the decision maker's preferences are represented by the expected value function $f \mapsto \mathbb{E} f \equiv \sum_s f(s)\,\mathsf{P}(s),\ f\colon S \to \mathbb{R}$.

For $A \subseteq S$ let $\mathbf{1}_A\colon S \to \{0,1\}$ be the indicator of A (defined by $\mathbf{1}_A(s) = 1$ if $s \in A$ and zero otherwise); and denote by D, I, F, R respectively the following acts (functions from S to $\mathbb{R}$): deposit, invest without advice, go see the broker and follow his advice, go see him and reject his advice (i.e. do the opposite of what he recommends). So $D = \mathbf{1}_\emptyset,\ I = \mathbf{1}_h - \mathbf{1}_l,\ F = -c + \mathbf{1}_{h\beta} - \mathbf{1}_{l\beta},\ R = -c + \mathbf{1}_{h\delta} - \mathbf{1}_{l\delta}$.

Obviously $\mathbb{E}\, I = 0.2 > 0 = \mathbb{E}\, D$, so D is dominated. Problem is whether to invest with or without the help —and commission— of the broker. Also, in principle we must consider, besides I, F and R, the acts of going to consult the broker and follow his advice only *partially*, that is only if he says 'buy' or only if he says 'don't buy'. On the other hand it is easy to see that also these acts are dominated: e.g. following advice only if it is 'buy' means going to se the broker and buying anyway —dominated by buying without the broker.

To go see the broker and do the opposite of what he says —act R— is dominated by F *because* $\xi_i > 0.5$. To see this observe that $\mathbb{E}\, F = 0.6[\xi_1(1-c) - (1-\xi_1)c] - 0.4[(1-\xi_2)(1+c) + \xi_2 c] = -c + 0.6\xi_1 - 0.4(1-\xi_2)$ and $\mathbb{E}\, R = -c + 0.6(1-\xi_1) - 0.4\xi_2$; from this it is easy to see that for any $\xi_1, \xi_2 > 0.5$ it is $\mathbb{E}\, F > \mathbb{E}\, R$.

Thus the question is reduced to comparing F and I. For this, it is again elementary to verify that $\mathbb{E}\, F > \mathbb{E}\, I$ iff $3\xi_1 + 2\xi_2 > 3 + 5c$.

Conclusion: the optimal decision is to invest without the broker if $3\xi_1 + 2\xi_2 < 3 + 5c$, and go see the broker and follow his advice under reverse inequality. Of course on the segment $3\xi_1 + 2\xi_2 = 3 + 5c$ the acts I and F are indifferent. If you like pictures, you can draw one for e.g. $c = 0.1$.