A Quest for Understanding Understanding in Mathematics Learning: Examining Theories of Learning

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Abstract
This paper argues that teachers' understanding of understanding in mathematics learning is largely influenced by, and therefore differs according to, the theories of learning they adhere to. This notion is demonstrated through a case study in which three junior secondary mathematics teachers were asked to construct concept maps and write personal essays on their thoughts about understanding in a mathematics classroom. In addition, these teachers were both interviewed and observed while teaching. Implications for teacher education are discussed.

Introduction

Understanding is the most commonly used term both in school and outside school. It can mean an agreement or contract, sympathy or being sympathetic, and comprehension. The word understanding is sweet to hear especially when used in the contexts of emotional appeal such as mutuality and appreciation. In academic contexts, the term is used in relation to intellectual capacity, example of which is a mathematics lesson, where teachers frequently ask learners whether or not they understand mathematical concepts.

The importance of mathematical understanding is construed as crucial by the entire world. To the teacher, students' understanding of mathematics is a sign of achievement—having met the goal of teaching. To the student, it means a furtherance of education, hence a bright future. To a politician, students' understanding of mathematics means economic growth because the subject mathematics is a pre-requisite for the technology-based careers such as engineering. Nonetheless, it seems understanding in a mathematics class is difficult, and some situations, impossible to achieve. To this effect, Hiebert, Carpenter, Fennema, et. al. (1997.) in their book summary have noted that "school systems have always regarded understanding as a critical component of the mathematics classroom, *yet teachers continue to struggle with meaningful ways to teach for mathematical understanding*. Teachers will likely agree that understanding ... includes the ability to reason and make sense of what is being learned, but they may lack ways to design a classroom so that understanding is the central to the students' learning experiences" http://www.toolkitforchange.org/toolkit/documents/659_102_making_sense.pdf.

Several books and articles on this topic exist and are particularly attempting to direct teachers on how they can be able to teach for understanding. Two assumptions are made in this regard: (1) teachers know or 'understand' what understanding entails and (2) that understanding in mathematics is universal—carrying the same meaning to everybody, and throughout the world. Thus, this paper challenges the second view which assumes that understanding has identical meaning to all teachers regardless of their adherence to different theoretical underpinnings to learning, knowledge and knowing.

Background to the study

At least three main theories of learning are known in educational psychology—behaviorism, cognitivism and constructivism. These orientations have different epistemological views about the world, which in turn, shape their practices in the classrooms, particularly techniques for teaching, classroom control, and assessment (Garegae, 2001). The behaviorists, for example, regard learning as change of observable behavior. Thus, the teacher has to aim at producing behavioral change in a desired direction. These assumptions about learning influence teacher's choice of techniques, including drill and practice which may result in rote memorization. Because of the emphasis on behavioral change, understanding is measured by observing change in a learner's behavior through recitation, tests and examinations.

While behaviorism focuses on the external behavior of the learner, cognitivism, on the other hand emphasizes on the internal mental structures of the same—thus lending itself to abstract information processing rather than actual behaviors. According to the cognitivists, mental representations are shaped by learners' beliefs, thoughts, goals and expectations. Thus, a learner is actively engaged in the learning process trying to integrate current information with prior knowledge, so as to make the learning of new information meaningful. The teacher should design "efficient processing strategies [in order] for the learners to acquire knowledge e.g. mnemonic devices to reduce the workload of the short-term memory, rehearsal strategies to maintain information, and the use of metaphors analogies to relate meaning of new information to prior knowledge" http://en.wikipedia.org/wiki/Cognitivism_(psychology). In this regard, understanding is inferred as products of learning measured by the ability to rehearse, retrieve, transform and code the information learnt. That is, the emphasis is on the application of previously learnt knowledge.

In addition to behaviorism and cognitivism there is constructivism. The constructivism paradigm views learning as an active process in which learners are engaged in constructing new concepts based upon current and past own knowledge. In this process, learners are able to go beyond the information given as they incorporate knowledge from their own experiences. Knowledge is regarded as socially constructed, and thus teaching strategies include discovery learning, classroom discourse and discussions. These strategies are geared towards allowing students to voice their ideas to others, and at the same time organizing and refining these ideas. Characteristics of assessment procedures for understanding in this paradigm include testing learners for the ability to apply learned strategies to novice situation, critical thinking as well as the ability to articulate mathematical concept fluently. Journals, impromptu writings, and oral reports are appropriate in this regard (National Council of Teachers of Mathematics (NCTM), 1991). All these theories have different assumptions about the nature of learning; hence people who espouse them are likely to vary in understanding mathematical understanding.

Theories of learning and mathematical understanding

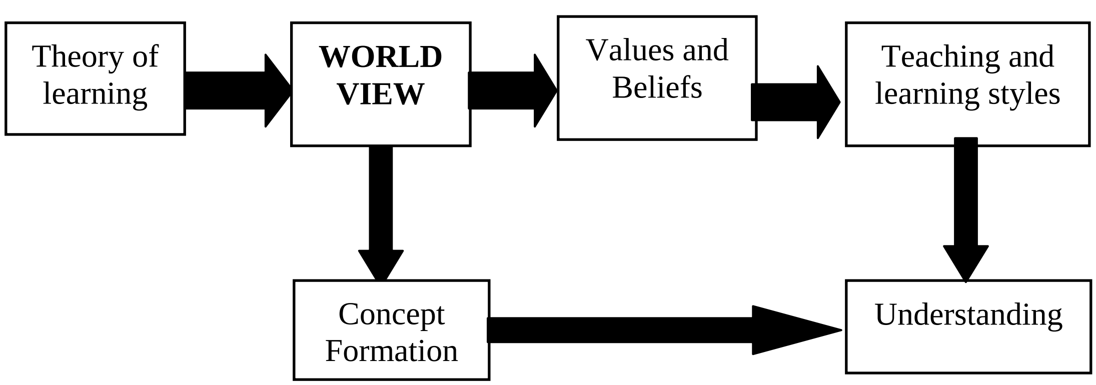
> The design of educational programs is always guided by beliefs about how students learn in an academic discipline. Whether explicit or implicit, these ideas affect what students in a program will be taught, how they will be taught, and how their learning will be assessed. *Thus, educational program designers who believe students learn best through memorization and repeated practice will design their programs differently from those who hold that students learn best through active inquiry and investigation.* http://books.nap.edu/openbook.php?record_id=10129&page=117

Skemp's (1976) paper and Hiebert's (1986) edited book gave the mathematics community ideas of what understanding in a mathematics class could look like. Although the two authors use different words—instrumental and relational for Skemp and procedural and conceptual for Hiebert—their work is one and the same thing. In both cases, relational and conceptual understanding was preferred over instrumental and procedural understanding, respectively. As a result, teaching strategies which are regarded as likely to produce such the favored understanding were suggested in the literature. Some of these include problem solving, investigations, discussions and discourse (Cockcroft, 1982; NCTM, 1991, 1989). The snag is that assumption that every one has the conception of what is being discussed is implied in the heavily emphasized importance of teachers' role in creating classroom atmosphere appropriate for such teaching. To the contrary, not all people may have the same mental representation of conceptual or relational understanding as indicated in the discussion above. Skemp (1976/1987:153) recognizes this by saying

> Instrumental understanding I would until recently not have regarded as understanding at all. It is what I have in the past described as 'rules without reason', *without realising that for many pupils and their teachers the possession of such a rule, and ability to use it, was what they meant by understanding."*

By so saying, Skemp acknowledges that the teacher's view of the nature of mathematical thinking may influence his/her views of mathematical understanding. The position this paper takes is that, teachers' points of view about the nature of mathematical understanding (and of mathematical thinking) is largely influenced by their affiliation to theories of learning. The concept of understanding to a behaviorist is not that same as that of a cognitivist or a constructivist. These theories form lenses through which one views the world, hence impacting on his or her beliefs about teaching, learning and understanding. Figure 1 below summary of how theories can shape one's practice. Thus, we can conclude by saying that there are several versions of understanding that teachers can posses. These understandings are conceived, born and bred by different ways of knowing and knowledge.

Figure 1: Teachers' view of understanding as influenced by learning theories



The study

Sampling and selection

In this case study, a qualitative research approach was used to explore lived experiences of three junior secondary school mathematics teachers. These teachers were selected through purposive sampling (Maxwell, 1996) from schools within the radius of 100 kilometers from the capital city of Gaborone. The criteria used for selection were years of teaching experience, qualification, and the institution of their initial training. Those holding the Diploma in Secondary Education (DSE), had trained from either of the two colleges of education, as well as having been in the field for more than two years, were selected. This excluded DSE holders who have trained at the University of Botswana. The participants were males with pseudonyms Thamo, Letsomane, and Kgosing.

Data collection and analysis procedures

Although the study employed several methods to collect data, this paper confines itself to data collected through personal essays, concept maps, classroom observations and interviews. The qualitative data gathered was subjected to Tesch's (1990) approach to data organization and analysis and Bogdan and Biklen's (1992) folder style of separating themes and categories was used. This paper reports themes on the category 'learning and understanding mathematics'.

Findings and Discussions

Mr. Thamo, who was perceived to be a behaviorist, believes that mathematics understanding is achieved through doing several problems on a certain topic. His teaching is characterized by sporadic explanations to individual or group of students and seat-work where students perform repeated calculation. He instead of letting students discover pi ratio, he told students about it by saying "we are told that somebody has discovered that. If we take any circumference and divide by its diameter, the result is 22/7".

And yet Mr Thamo construes this as teaching for understanding. In his essay and interview proceedings, he claims that

> The most important way of learning mathematics is being able to make sense of what is being taught, being able to relate what you are taught with previous knowledge. ... Students should be able to know formulas. ... The students should continuously work out questions. ... As long as pupils work out problems, they can understand mathematics".
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> The teaching of mathematics should be towards preparing students for an exam as the learning is supposed to be followed by testing. However, the teaching should also prepare students for life outside classroom it should emphasize critical thinking where learners are taught problem solving techniques. … As students will be writing exams … teaching should also be focused on the process of getting the answer. Without focusing on the answer, the desired results would not be achieved. (Essay)

During interviews, Thamo repeatedly said that the teacher should make sure that students do the right thing, and that understanding is important. It seems, to him, doing a lot of problems through drill and practice is a process of understanding. When examining his students' exercise books, it was found that he marks the answer only. He never considered the method—a practice that is contrary to his claim.

Mr. Letsomane emphasized mental processes when teaching. He was fond of employing a Socratic dialogue with the whole class, trying to diagnose students' prior knowledge, which always formed a basis on which current information was built on. His preferred style of teaching is discovery, and he provided a discovery-oriented environment in which students were engaged in 'experiments'—for instance, the use of soda cans to discover pi ratio. In his essay, he noted that "learners must learn mathematics through the combination of conceptual and procedural knowledge". He further stated that

> The objective of teaching mathematics should be for relational understanding. This will help them to relate the mathematics which they are taught in the classroom to every day life situation and also in the world of work. They should also see mathematics as a tool that can be used in many activities.

In an interview, Letsomane said that mathematics topics are related. "So, when I am teaching, I bring in those topics that I have done previously. … I then ask them questions, some would have forgotten all I have taught but others would remember and then I give them a chance to discuss in class". Letsomane emphasized retrieval of previously learnt knowledge to be applied in the current situation. He acknowledged that learners can commit some errors without necessarily indicating lack of understanding. His practice (as observed from learners' exercise books) on assessing students' understanding matches with his professed beliefs about assessment. He marks the method as well as the answer.

Mr. Kgosing's class was characterized by sense of humor. He is talented in creating a comfortable environment for students' participation, and yet firm enough to control or guide interaction. He opens his essay by saying that "learning mathematics with understanding can be promoted through the use of progressive methods such as problem solving, investigations and practical work". He echoed the same sentiments during interviewing episodes. In every class, Kgosing made sure that students engaged in discussions of some kind. He was regarded as espousing a constructivist view of learning because he emphasized practical work and investigations, giving students an opportunity to elaborate on their thought processes. Whenever a student is called to work a problem on the chalkboard, he would encourage such a student to 'voice' his/her thought processes. He said that what should be emphasized in mathematics teaching is students' understanding. His methods of assessing understanding include oral presentation, tests, quizzes and exercise books.

Conclusion

Understanding concepts in a mathematics lesson is essential. It gives students some experience such that the learning of subsequent concepts becomes easier. However, it is important that educators and other practitioners have a knowledge that an individuals' impression of understanding depends on one's adherence to a particular theory of learning. And this adherence goes beyond classrooms in that it influences one's perceptions about the meaning of schooling and the purpose therein. Usually, this is ignored or taken for granted especially when curricular innovations are made. This study has indicated that besides their mathematical beliefs, teachers' classroom decision making, particularly about their instructional practices, is essentially rooted on their views about the rationale for learning, which in turn is strongly influenced by their theoretical underpinnings on what understanding is, and how it is assessed.

Implications for teacher education

Often times, student teachers are tailored into robots or machines in that instead of developing their thinking capacity, it is being stripped off from them. As such, they are denied to explore their cognitive nature. Teacher educators usually teach them theories of learning in an abstract manner. They are never made to comprehend that each theory of learning has its own assumptions about understanding and its assessment. This error of omission has to stop given that decisions made by teachers about nature and quality of mathematical understanding have important consequences both for classroom practices and the meanings attached to such practices. Student teachers should be taught explicitly that, the assumptions about the nature of understanding differ according to the theories of learning, and that these assumptions influence the teacher's methods of explaining concepts as well as his or her expectations on students. The instruction should be aimed at challenging student teachers' internalized or inner core beliefs about understanding and how it relates to schooling. This teaching approach will help them become aware of their internalized theory of learning, thus giving them the opportunity to reflect on the expectations and demands they are likely to make on learners at secondary schools.

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