Mathematics in the Middle: Enhancing Teachers' Understanding of the Interplay Between "School Math" and Professional Uses of Mathematics

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Abstract
As American society becomes more technologically reliant, applications of mathematics are less obvious and less understood. Teachers tie mathematics to the world of their students while assuming that mathematics is fixed body of knowledge, rather than a developmental tool. In this study, 27 teachers were asked to identify the explicit and implicit mathematics they used during a 7-day period. 621 individual "mathematical events" were reported of which over 74% included measurement and algorithms. 25% of the reported mathematics supported planning and estimation. Only 1% of the mathematical events tied mathematics to the technological underpinnings of society. This suggests that teachers have difficulty recognizing the role that mathematics plays in the use and development of technology. Both teachers and students are likely to devalue the role of mathematics and are unlikely to recognize the importance of mathematics as a support to and in the continued technological development of our modern world.

Introduction

We face a dilemma in elementary school mathematics education: many teachers teach a mathematics they do not fully understand to students who see, recognize and use less mathematics in their lives than ever before (Hastings, 2007, February 2). As American society becomes more technologically reliant, (Noss, 2001; Skovsmose, 2005), the application of mathematics is less obvious and less understood (Friedman, 2005; Schiesel, 2005) by K-8 classroom teachers and their students.

In practice, mathematics curricula stress the importance of linking school mathematics to students' daily lives and realities as a way of connecting academic mathematics to contextual and/or practical realties and understandings (Desimone, Smith, Baker, & Ueno, 2005; National Council of Teachers of Mathematics, 2000; 2006). K-8 teachers try to explicitly tie mathematics to the world of their students (Delpit, 2006; National Council of Teachers of Mathematics, 2000) and look to make realistic connections between mathematical rules and algorithms and the events children participate in on a daily basis. Additionally, many teachers assume that mathematics is fixed body of knowledge that offers clear-cut answers to numerically-based problems; they do not recognize the flexibility and experimental nature of mathematics nor do they appreciate how mathematics informs planning, organizing, and ethical decision making (Bakalar, 2006; Delpit, 2006; National Council of Teachers of Mathematics, 2000). Formal mathematics is associated with scientific, technological, and engineering practices and there is little recognition that important mathematics is embedded in many professions not usually associated with mathematical understanding (Lesser & Nordenhaug, 2004; Masingila, 1996; Nicol, 2002; Zlotnik & Galambos, 2004).

Certainly, the role of mathematics in society is changing. The more that technology impacts and influences our daily lives, the less mathematics is visible (Noss, 2001; Skovsmose, 2005). While mathematicians, scientists, and engineers recognize that these technological advances require a deeper understanding of mathematics (Tate & Malancharuvil-Berkes, 2006), societally, we do not explicitly "see" much of the mathematics that is used on a daily basis

(Empson, 2002). Implicit uses of mathematics are ubiquitous in the United States (e.g., bar codes that monitor inventory, fast food restaurant cashier counters that display pictures of food items instead of numerals), yet these implicit uses of mathematics obscure explicit mathematics. If teachers do not recognize the many ways that mathematics is embedded into our daily lives, then, regardless of the depth of their mathematical content knowledge, they may be unable to help students make connections between school mathematics and the reasons for studying the mathematics.

The goal of mathematics education for preservice teachers focuses on ensuring that they understand the basic mathematics concepts they will teach (Hill, Rowan, & Ball, 2005) and have access to developmentally appropriate pedagogy and practices (Dahl, 2005; Donnell & Harper, 2005). Additionally, it is hoped that they recognize connections between "school math" and daily practices (e.g., calculating unit cost or interpreting a graph in the newspaper). Yet little attention is paid to ensure that educators acknowledge implicit mathematical practices that are part of daily life beyond the connections made in textbooks (Reys, Lindquist, Lambdin, & Smith, 2007; Sheffield & Cruikshank, 2005). In the United States, some curricula have been developed that tie classroom mathematics to explicit concerns that reflect the lives of students, their families, and their communities (Gutstein, 2006). These curricula and lesson plans illustrate how mathematics is embedded into the political and economic fabric of our society. However, they do not routinely explore the implicit, "hidden" mathematics included in, for example, computer design and architecture, product standardization, advertising graphics, scheduling the seasonal game schedule for a sports leagues, and health policy decision-making. Thus, neither students nor most teachers are able to articulate how the "school mathematics" taught in elementary and middle school translates into important knowledge that is used in professional practice in technical and non-technical fields. Yet, when teachers are able to make these connections, there is evidence that students begin to both recognize of the role of mathematics in technology, innovation, planning, and decision-making and the understanding that mathematics is more than just "right answer"(Gerofsky, 2004; Gutstein, 2006).

In this study, preservice and practicing teachers (collectively referred to as "teachers") recorded their recognition of mathematics usage in their daily lives in a typical week. This qualitative analysis of the mathematics they reported addresses the main areas of their mathematical recognition and acknowledgment: how they define mathematics, how well their definitions reflect the mathematics they recognize and report, and how much and what types of "implicit" (less visible) mathematics do they acknowledge.

Methods

Participants

Participants were preservice and practicing teachers (n=28) enrolled in one of two Introduction to Research courses as part of a graduate-level Masters of Education program at a regional university in the northeast during the Spring of 2006. Eleven were licensed and certified teachers and had been or currently were elementary school teachers; seventeen were completing initial licensure for elementary (K-8) or adolescent (6-12) teaching certification. Fifteen completed college level calculus courses (n=9) or algebra/statistics courses (n=6). One calculus student and one algebra student also completed a mathematics methods course designed for prospective K-6 mathematics specialists. Four others completed at least one of two mathematics content courses designed for prospective teachers during which they developed an understanding of the NCTM mathematics curriculum.

Method

We began with a discussion of overt, explicit, covert, and implicit mathematics that we use on a daily basis. Teachers shared examples of mathematics they used and recognized, such as balancing checkbooks and measuring recipe quantities. They also discussed how mathematics is embedded in much of today's technology. The teachers then agreed on a definition of mathematics based on their own recollection of previous mathematics classes, their own practices of mathematics, K-8 classroom expectations, and our classroom discussion. Using this definition, the teachers spent seven days monitoring their recognition, practice, and use of mathematics. This study was a qualitative analysis of the mathematics that the practicing and preservice teachers recognized and recorded in their journals.

Results

Definition of Mathematics:

Teachers in each of the two classes ($N_1 = 16$, $N_2 = 12$) spent a full class session defining mathematics. This definition became the working definition they used to identify mathematics they recognized in their world.

Both classes initially agreed that mathematics included two unifying ideas: 1) mathematics involved numbers (e.g., content); and 2) mathematics was an applied tool, used to solve problems (e.g., process). As classroom discussion continued, they questioned whether mathematics always included numerical understanding. They began exploring issues of pattern recognition, relationships between ideas numbers, algorithms, and specific calculations, and the universality of mathematical ideas, better reflecting the mathematical definitions suggested by NCTM and TIMMS (Mulls et al., 2004; National Council of Teachers of Mathematics, 2000). Several teachers questioned whether mathematics had depth beyond numbers (e.g., "Math is just formulas and things, it really doesn't explain much" [Female, Class 1]; "Math is the universal language we talk with numbers" [Female, Class 2]) but most recognized that mathematics involved relationships between ideas, numbers, and concepts.

By the end of the afternoon, each class had defined mathematics in similar ways that somewhat mirrored the content/process linkage that, in fact, underlies school mathematics:

> Mathematics is a way of representing or explaining relationships through a system of numbers and symbols. Mathematics is a universal language. [Class 1]
>
> Mathematics is the systematic application of methods and techniques in relationships to achieve a useful end. Mathematics is operations involving number systems and/or variables in relationships. [Class 2]

These working definitions of mathematics were very similar: teachers recognized that mathematics involved relationships between numeric ideas, although those ideas might be expressed symbolically. Numbers were not a necessary precursor to the identification of mathematics itself; a logical structure to these relationships was implied.

Identification of Mathematics:

In the 7-day period during which teachers recorded their recognition of mathematics-related phenomenon, 27 teachers identified 695 interactions of which 621 (89.3%) were defined as explicitly mathematically related. (One student did not complete the mathematics diary.) Less than 11% of reported mathematical encounters are defined as "non-mathematics," or "number recognition only."

Nearly 90% of all mathematical encounters were classified as "explicit mathematics," defined as an activity that required use of mathematical strategizing beyond the simple recognition of numbers. Explicit mathematical relationships often involved numbers (e.g., budget planning, bill paying, calculating sports statistics) but were not limited to the use of numbers (e.g., reading maps, choreographing a dance).

Most of the reported mathematics included explicit use of numbers or formulas, although many of the journal entries reflected uses of mathematics as a tool for logic and decision-making that was not rely on explicit calculations. Some journal entries reflected more than one mathematical interaction; these were placed into the "highest" mathematics use category.

Measurement, Calculations and Algorithms, which account for over 70% of the mathematical encounters, represent the most straightforward uses of mathematics. Teachers recognized this type of mathematics both at home and at work, for recreational, administrative, and professional purposes. This type of mathematical enterprise most closely mirrored school uses of mathematics to solve problems that were easily described.

> Considered if I could drive to work (number of miles) on the amount of gas (% of tank, fraction of gas in tank). Considered cost of gas vs. cost of running out of gas. Decided to get gas later. [Female 4]

Just over 25% of the reported uses of mathematics recognized the mathematics as a tool for Estimation and Planning. Within this category, formulae and algorithms were not explicitly discussed. Logical understandings described nearly half of the reported entries in this category (n=73, 46.5%) and were invoked to make purchases ("Mentally calculated how much wood we'd need to make a bookshelf at home depot" [Female 6]), plan a project ("Create a portfolio at a glance. Must estimate how much information I will need to fill a tri-fold brochure" [Female 4]).

Mathematics as a decision making tool accounted for the other half of journal entries in the Estimation and Planning category (n=84, 53.5%). This was described in terms of approximation, comparing/contrasting, and probabilistic estimation. Teachers described using mathematical ideas to interpret charts, identify best value for money, and make gaming decisions.

> Playing games: I have a group of friends that I play some obscure games with, but all of them involve some math. First is Bonanza, which involves a lot of probability. Knowing the number of each type of card that is left and playing the odds is an important part of the game. It is math that is done in my head, but can be difficult to track because there are different amounts of each type of card and the more rare they are the more they are worth. [Male 25]

During their week of data collection, teachers were especially encouraged to identify embedded mathematics, such as implicit uses of technology and hidden mathematics, in their daily mathematics encounters. However, less than 2% of the responses identified such embedded mathematics. Of the six responses in this category, half discussed how a computer translates keypad instructions to electronic impulses:

> I use a computer- When I use a computer, I press a symbol on the keypad. The computer uses binary math (ones and zeros) to perform a specific operation and display an output (mathematics is being used here because the computer does not have the ability to speak English, rather each symbol on the keypad has its own mathematical formula understood by the computer [Male 18].

The other three responses focused on issues of pattern recognition and encryption (e.g., "Open door with code" [Female 9]) although there was limited discussion of the connection between the mathematics involved in the technological enterprise.

Discussion

The mathematics definitions developed by the teachers was less clear than the more formal understandings of mathematics (Garii, 2004; Gerofsky, 2006; Mulls, Martin, Gonzalez, & Chrostowski, 2004; National Council of Teachers of Mathematics, 2000). The teachers' definitions acknowledged that mathematics represented and/or explained relationships between numbers and/or symbols and suggested that mathematics is a tool to solve problems, yet they harbor an elusive understanding of what mathematics entails and their definition reflects this.

While the teachers overtly acknowledge that mathematical ideas underlie much of the technology that they encounter, they did not recognize this mathematics. Their concrete definitions required that the mathematics be visible and solve an explicit problem. Making change and scheduling and organizing events were seen as mathematical because the teachers recognized that mathematics embodies both algorithmic understanding and logical planning. However, less tangible uses of mathematics and that are not easily visible – such as the mathematics that underlies technology – was rarely mentioned. The mathematics identified reflected the teachers' ambivalence about mathematics and suggested a lack of confidence in their knowledge of what is mathematics and what they should label as mathematics.

It is possible that the teachers' definition of mathematics, as developed in this study, affected the mathematics that they recognized. While 16 of the teachers had completed advanced mathematics in high school and/or college level mathematics, and thus had been exposed to more abstract understandings of mathematical thought, they did not internalize this understanding as inherently "mathematical." and they continued to recognize only explicit manifestations of mathematics. This raises questions about what should we expect teachers (and students) to recognize, understand, and value in terms of mathematics and mathematics education. Formal definitions of mathematics (Mulls et al., 2004; National Council of Teachers of Mathematics, 2000; 2006) strive to help teachers create a classroom environment that allows students to explore mathematics itself. What is missing, however, is the link that helps teachers and students connect the important mathematics that is part of the K-12 curriculum to the less visible mathematics that undergirds the technological supports of our society. If we are teaching mathematics as an arcane set of skills that helps students hone their abilities to think, organize, and solve straightforward problems, then the mathematics curriculum we are teaching today is appropriate. If teachers do not recognize the many uses of mathematics in our lives, then they cannot be expected to prepare students for using mathematics to build a viable tomorrow.

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