Developing Calculus Concepts through Applications Related to NASA's Space Exploration Program

Chris Giersch, NASA; Langley Research Center
Josephine Letts, Virginia Beach Public Schools
David K. Pugalee, University of North Carolina, Charlotte

Abstract

The United States' National Aeronautics and Space Administration [NASA] has a new focus for manned space travel to go beyond Earth's orbit for purposes of human exploration and scientific discovery. This emphasis on the Earth, Moon, Mars and beyond opens new opportunities for mathematical applications. It is essential that students in schools are exposed to these ideas so that they may develop an appreciation for how mathematics is connected to space travel. The problems that are the focus of this workshop provide authentic situations developed to provide conceptual and intuitive meaning for applications of calculus concepts and skills. These tasks support calculus reform efforts by emphasizing real-world and contextualized problems, providing focus on multiple representations of topics, and the development of formal concepts and procedures from work on real world applications. Problems encourage students to use technology as a tool for problem solving. As such the problems provide investigative experiences to stimulate an increased depth of understanding.

Background

Calculus is an essential course at the secondary level to adequately prepare students for mathematics courses at the university level. Interest in increasing students' performance in calculus while positively impacting the number of students taking advanced mathematics courses resulted in calculus reform efforts. Tall and Ramos (2004) assert that calculus is the culmination of several areas of mathematical development depending on numerical calculations, symbolic manipulations and graphical representations. They further argue that teaching of calculus should involve interactions with the outside world and representative thought experiments. In arguing that some approaches do little to help students understand better what calculus is about or what it is used for, David Mumford (1997) posed this question, "Are we teaching calculus in the hope that a small percentage of our students will catch our love of rigor, or so that most of our students will emerge with the ability to use calculus in their specialities?" (pg. 563)

Helfgott (2004) advocated that students should see substantive applications throughout their study of calculus and that such experiences should not be relegated as a culminating activity. Rich tasks provide occasions to develop critical thinking. In addition, tasks provide teachers with performance tasks which assess students' conceptual and procedural understanding. As performance tasks, the items emphasize two complementary perspectives necessary for understanding: mathematical content (including topics, themes, and mathematical and cognitive aspects) and connections with mathematical issues (role of contexts, access to mathematics, communication, collaborative experiences, career explorations, and symbol manipulation) (Shoenfield, 1997). The NASA related problems also require students to interpret real situations and represent those situations mathematically. This experience addresses weaknesses in variable manipulation that have been identified in research as a common difficulty in students' errors with calculus related tasks particularly when those tasks involved modeling a situation (White & Mitchelmore, 1996). These difficulties emerge when variables are presented as symbols to be manipulated. Problem situations which emphasize representation using variables and the manipulation of those symbols fosters the development of an abstract-general concept of variable. These problems connect representation, problem solving, algebra and calculus concepts so that students understand how mathematical ideas interconnect as they relate and apply mathematics in real-world contexts (NCTM, 200).

Problem Development

Each problem was designed to emphasize a mathematical connection to NASA's space exploration program. Problem development began with research into the activities of NASA where potential mathematics applications were likely. The conceptualization of the problem situation was followed by contextualizing the problem within the skills and concepts relevant for students study calculus in secondary school settings, particularly AP courses. All problems encourage the effective use of technology. The problem was developed and reviewed by multiple experts to assure accuracy, relevancy, and pedagogical appropriateness. Reviewers included scientists and engineers at NASA particularly those at Langley Research Center in Hampton, VA. Problems were further reviewed by university faculty and field-tested with high school students. Calculus problems related to NASA's work will initially be available through the Texas Instrument Activities Exchange website (search by topic – calculus- or NASA as keywords) [http://education.ti.com/educationportal/sites/US/sectionHome/classroomactivities.html]. All problems include an educator's section and student worksheets.

Example Activity: Next Generation Spacecraft

Background Information for Problem

NASA's new spacecraft, the Crew Exploration Vehicle (CEV), is the key to making the Vision for Space Exploration a reality. The CEV will use an improved, larger, blunt-body capsule, much like the shape of the Apollo spacecraft. With an outside diameter of approximately five meters, the spacecraft will have more than three times the volume of the Apollo capsules. This design will shorten development time, reduce reentry loads, increase landing stability and permit safe travel for up to six crewmembers.

The CEV will be able to support landings anywhere on the moon's surface and sustain itself for six months in lunar orbit. Reusability is another important feature since the number of flights per vehicle is a key cost driver.

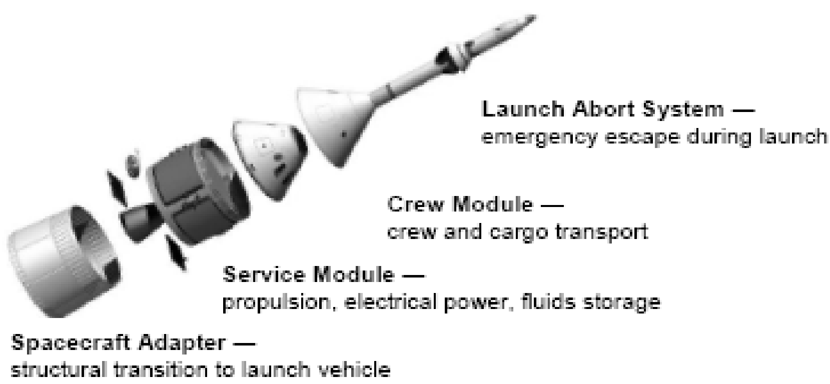

Figure 1: Components of the Orion Spacecraft (NASA Concept)

During the CEV planning, NASA studied several different kinds of entry vehicles and rockets. NASA didn't set out to make this vehicle identical to Apollo, though several Apollo era researchers were consulted. Ultimately, this design was found to meet the requirements while being the most effective within the safety goals. NASA will launch as early as possible, but no later than 2014. This date is budget driven. Figure 1 shows the components of the CEV. For more information about Orion and the Vision for Space Exploration, visit www.nasa.gov. Figure 1 shows components of the Orion spacecraft and Figure 2 shows the crew module


NASA concept of the Orion Crew Module


Apollo Capsule

Figure 2: Comparison of the Orion Crew Module and the Apollo Capsule

Instructional Objectives
Students will:

- use integration to find the volume of a solid generated by a region, R.
- determine the equation of a circle using the standard form and the general form.
- determine the equation of a line using the point-slope form.
- solve a system of equations with three equations and three unknowns.

NCTM Standards
Algebra standards

- Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.
- Write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency—mentally or with paper and pencil in simple cases and using technology in all cases.
- Judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.
- Draw reasonable conclusions about a situation being modeled.

Geometry standards

- Visualize three-dimensional objects and spaces from different perspectives and analyze their cross sections.

Problem Solving standards

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

Problem
Use the sample Computer Aided Design (CAD) drawing provided on the next page to answer the following questions.

1. Find the volume, in cubic feet, of the Crew Module (CM). Make sure you use the correct significant digits.

2. Inside the outer shell of the CM will be a crew cabin that is pressurized. Suppose the pressurized volume (crew cabin) is approximately 55% of the total volume. What is the volume of the crew cabin? Round your answer to the nearest cubic foot.

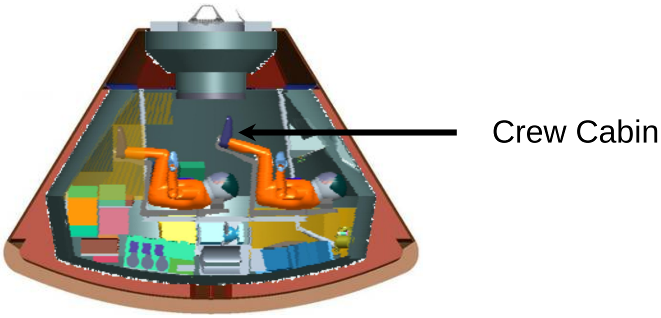

Figure 3: Cross-section of the Crew Module (NASA Concept)

3. What room in your home has about the same volume as the crew cabin of the CM?

4. Using the information provided in “Background Information”, estimate the total volume of the Apollo capsule? Round your answer to the nearest cubic foot.

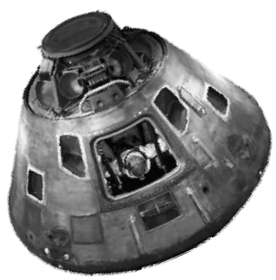

Figure 4: Apollo Capsule

The following figure shows a page from the Student Edition of the problem depicting the NASA Concept of the Orion Crew Module CAD Drawing.

NASA Concept of the Orion Crew Module CAD Drawing:
(Reference Student Edition CAD Drawing for Full Page Diagram)

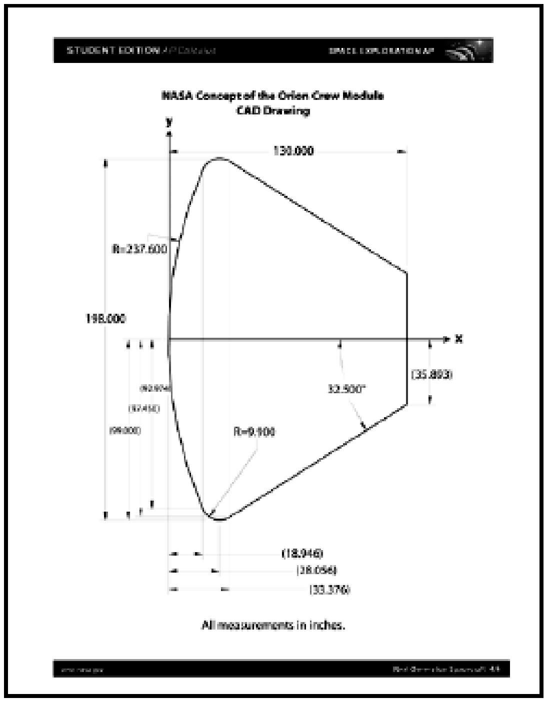

Figure 4: Student Edition - NASA Concept of the Orion Crew Module CAD Drawing

<u>*Solutions*</u>

The Educator's guide presents a detailed solution, including alternate approaches to solving the problem. All materials are available via the Texas Instruments website.

References

Helfgott, M. (2004). Can we do something to improve the teaching of first year calculus? *Proceedings of the International Congress on Mathematical Education.* Denmark, Coppenhagen. [Available at http://www.icme-organisers.dk/tsg12/papers/helfgott-tsg12.pdf].

Mumford, D. (1997). Calculus reform for the millions. *Notices of the AMS*, May 1997, 559-563.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics.* Reston, VA: author.

Schoenfeld, A. (Ed.) (1997). <u>Student Assessment in Calculus: A Report of the NSF Working Group on Assessment in Calculus</u>. Washington, DC: The Mathematical Association of America.

Tall, D. & Ramos, J.P.M. (2004). Reflecting on post-calculus reform. *Proceedings of the International Congress on Mathematical Education.* Denmark, Coppenhagen. [Available at http://www.icme-organisers.dk/tsg12/papers/tall-mejia-tsg12.pdf].

White, P. & Mitchelmore, M. Conceptual knowledge in introductory calculus. *Journal for Research in Mathematics Education*, 27(1), 79-95.