Affective Pathways and Structures in Urban Student's Mathematical Learning

Gerald A. Goldin, Yakov M. Epstein, Roberta Y. Schorr
Rutgers University geraldgoldin@dimacs.rutgers.edu, yakov.epstein@gmail.com, schorr@rci.rutgers.edu

Abstract
We address the complex affect that can occur when students in the social context of an urban, inner-city classroom engage in conceptually challenging mathematics. Building on some earlier work describing idealized affective pathways and their interaction with mathematical problem solving heuristics, we formulate and discuss the concept of an "archetypal affective structure". This refers to a recurring, idealized pattern, inferred from observations of classroom and interview tapes, that includes characteristic patterns of individual and social behavior, sequences of emotional states, "self-talk" and associated affective responses, meta-affect, and interactions with goals and strategic decision-making. We describe several examples of such affective structures that appear to have important consequences – positive and negative – for mathematics education.

Affect and Conceptually Challenging Mathematics

Skilled teachers often seek to engage students in *conceptually challenging* mathematical activity, where existing understandings are changed, new understandings gained, or new representations constructed. Conceptually challenging problems are typically nonroutine for the students, who at some point are likely to reach a point of impasse or blockage. Classroom social interactions around such mathematics may include students presenting ideas that are challenged by publicly by their peers. Disagreements are likely, with some students' conjectures turning out to be incorrect while others are accepted by the teacher or the class.

Such situations can evoke strong emotional expressions, and may have lasting emotional effects. Large numbers of adults describe themselves as experiencing negative feelings in connection with mathematics – e.g., "math anxiety" – and many people recall painful or humiliating experiences they had as children in connection with school mathematics. At the same time, some describe important emotional breakthroughs – realizations of their own mathematical capabilities, or moments of profound satisfaction in mathematics.

The authors are presently leading a major, exploratory study of affect as it occurs and develops in three middle school urban classrooms in low-income, predominantly minority communities in the United States (1). We are particularly interested in the development of *powerful mathematical affect* in students (McLeod, 1992, 1994; Goldin, 2000, 2007). By this we do not mean exclusively positive emotional feelings, such as curiosity, interest, and satisfaction – rather we mean affect that may include feelings such as impasse, frustration, and disappointment, but that contributes to mathematical engagement, persistence, problem-solving success, and achievement.

The affective domain is beginning to receive long overdue attention from mathematics education researchers and cognitive scientists (Evans, 2000; Malmivuori, 2001; Leder, Pehkonen, & Törner, 2002; Dai & Sternberg, 2004; Hannula, 2002, 2004; Lesh, Hamilton, & Kaput, 2007). Much recent research has focused on individual expressions of affect during mathematical problem solving (e.g., Zan, Brown, Evans, & Hannula, 2006). This article, motivated by our desire to understand affect at the classroom level, discusses a theoretical construct that has emerged from a study of students' affect during interactions with their peers in urban, inner-city classrooms – a construct we term an *archetypal affective structure.* Preliminary descriptions of some of these structures as inferred from our observations, as well as further details regarding our overall study, have been reported elsewhere (Epstein, Schorr, Goldin, Warner, Arias, Sanchez, Dunn, & Cain, in press; Schorr, Epstein, Warner, & Arias, in press; Alston, Goldin, Jones, McCulloch, Rossman, & Schmeelk, in press).

Here we shall focus on the (idealized) psychological components that in interaction with each other, make up archetypal affective structures as we presently understand them. We remark that our analysis is still evolving, so that this report describes a "work in progress" rather than a definitive statement of our theoretical stance.

Description and Components of Archetypal Affective Structures

We have earlier characterized an archetypal affective structure as a recurrent, idealized pattern, situated within individuals, that involves: a situational component (particular behaviors, or a salient feature of the stimulus constellation the individual encounters), emotional feelings, and self-talk. Here we would like to describe it somewhat more comprehensively, as including the following (simultaneous and mutually interacting) components:

(1) a characteristic pattern of behavior, beginning in response to particular circumstances in the social environment, and culminating in a characteristic behavioral outcome,

(2) a characteristic sequence of emotional feelings, or affective pathway,

(3) information or meanings that may be encoded by the emotional feelings,

(4) characteristic self-talk or inner speech, in response to and evocative of the person's emotional feelings,

(5) characteristic problem-solving strategies and heuristics for decision-making,

(6) interactions with the individual's systems of beliefs and values,

(7) interactions with the individual's structures of self-identity, integrity, and intimacy,

(9) meta-affect, that includes feelings about feelings, feelings about cognition about feelings, and self-monitoring of affect,

(10) expressions from which affect may be inferred that are socioculturally-dependent as well as idiosyncratic, which can serve some communicative function; including eye contact and facial expressions, posture and "body language," hand and body movements, interjections and exclamations, tears and laughter, blushing, etc.

Thus an archetypal affective structure is, roughly speaking, a *behavioral/affective/social constellation* within the individual. Examples we have described elsewhere that occur in mathematical contexts include the following:

"Don't Disrespect Me": This archetype involves the person's experience of a perceived challenge or threat to his or her well-being, status, dignity, or safety. Resistance to the challenge raises the conflict to a level above that of the original mathematical task. The need to maintain "face" supersedes the mathematical issues.

"Check This Out": This archetype entails the individual's realization that solving the mathematical problem can have a payoff – either immediately, or at some future point. The resulting motivation to engage mathematically can lead to (intrinsic) interest in the task itself, or heighten (extrinsic) interest in an external payoff.

"Stay Out Of Trouble": This archetype involves the person's need to avoid commitments or interactions that may lead to trouble – either with peers, or with authority. Aversion to risk then supersedes mathematical aspects of the task.

"It's Not Fair": This archetype involves the person's experiencing a sense of unfairness in the work distribution across a group, or the level of participation by others in a group, leading to disinvestment in the mathematical ideas in the task and a desire just to "get it done."

These are just four of a larger set of archetypal affective structures that we have thus far identified, in the course of our observations in urban mathematics classrooms. In a subsequent section we shall discuss one of these, "Check This Out," in greater detail. But before proceeding, let us discuss and clarify some important, general theoretical aspects of this approach.

Discussion and Comments

The term "archetypal" that we use to describe these structures is intended to convey two ideas. First, our descriptions are idealized. Real-life responses to actual social situations may approximate these patterns to some degree, and to the extent that they do, the analysis of the archetype helps us to understand the individual's affect and cognition. Second, the structures that we describe occur widely and frequently in most human social interactions. They are not specific to mathematical thinking, or to urban classrooms. Nevertheless, understanding these structures may be essential to understanding the psychology of classroom mathematical learning, and how to enhance that learning.

We also remark that *archetypes are not stereotypes* – they are not specific to any one social context, or to any particular cultural, racial, or ethnic group. For example, some of the face-saving issues central to the "Don't Disrespect Me" archetype have been described in important studies of inner-city street life (Anderson, 1999; Dance, 2002). However, we understand that the same archetypal affective structure can be inferred from behavior in college faculty meetings, in situations involving difficult negotiations, in formal social gatherings and in many everyday situations (as well as in school classrooms engaged in mathematical discussions). The particular *expressions* of the affect – the final component in our list above – can differ substantially according to different sociocultural norms in different contexts, as well as across individuals; but the underlying affective structure remains essentially invariant.

Some archetypal affective structures may at times contribute directly to mathematical engagement and persistence, while others may at times impede such engagement. However, we do not see some of these structures as "good" and others as "bad". Rather, we see most or all of them as present within individuals. The challenge is to learn how to create an *emotionally safe environment* for conceptually challenging mathematics, so that (for example) serious mathematical engagement with integrity contributes to (rather than jeopardizes) safety, status and "face", and is experienced as leading away from "trouble" rather than toward it.

In observing classrooms where students are working in groups on mathematical problems, we sometimes notice what seem to be critical "choice points" or "branch points." As we refer to them here, branch points occur in archetypal affective structures when someone can act (consciously or otherwise) in one way rather than another, thus experiencing one set of feelings rather than another and evoking one structure rather than another.

For example, in our work we have seen many instances where a peer challenges a student's work. This can elicit a series of actions typical of the "Don't Disrespect Me" structure, with the student immediately becoming defensive of her position – often to the point of unwillingness to actually consider the argument of the other student, with all comments thereafter interpreted as "attacks" on her mathematical identity. On the other hand, the very same type of challenge can evoke responses more consistent with the "Check This Out" archetype, where the student seeks to justify and defend her mathematical ideas while simultaneously considering the comments or arguments of the student who challenged her. She seeks to maintain her mathematical integrity either by strengthening her justification, or by revising her solution accordingly.

The concept of self-statements, important to the notion of an archetypal affective structure, comes from research in cognitive therapy, particularly the work of Beck (1976) on cognitive therapy for emotional disorders. Although the focus of Beck's work is on negative emotional states such as anxiety and depression, self-statements can also be used to understand an important cognitive component underlying positive emotional states. Beck's theory posits that negative self-verbalizations or internal dialogue on particular themes accompany negative emotional states. A person's internal dialogue can be conceived of as comprising momentary and fleeting voluntary cognitions, automatic thoughts, and images, that are easily accessible to awareness. Cognitive therapists and cognitive behavioral therapists use a variety of approaches to enable patients to explore their self-statements, in order to cope with anxiety and depression (e.g., Hollon & Beck, 1994).

"Check This Out"

In this section, we describe some features of the "Check This Out" structure in the context of mathematical behavior, as a way of illustrating the interactions among some of the different components of an affective structure.

The type of stimulus situation likely to initiate this archetypal structure includes either cognitive complexity or perceived utility. The complex mathematical problem triggers emotions such as curiosity, puzzlement, being intrigued, and feeling challenged. The meta-affect is positive – the student feels a sense of pleasure and anticipatory joy in the feeling of challenge. Encoded in these feelings is information regarding the person's standing in relation to the problem – being at the beginning of an exploration. Self-talk at this stage may include some internal mathematical dialogue, but also such statements as, "This problem seems interesting. There are many possibilities. I think I can do this. This is exciting."

The continuing observable behavior is typical of concentration and intense engagement. "I'm really into this." The student's belief is that he is capable of figuring out the solution, if he works hard at it, and having that kind of capability is valued. Heuristic strategies evoked by these feelings are forward-moving, exploratory, and evaluative. Guided by internal self-talk, the student experiences an aspect of his self-identity that could be characterized as "mathematical problem solver," "engaged thinker," or "serious student." He may act to draw others into his sphere of involvement.

Should frustration or a sense of impasse occur, it may serve to heighten the student's interest in the problem (an example of negative affect becoming enjoyable when the meta-affect is positive), evoking more complex heuristic processes for understanding or simplifying the problem. But here is a possible "branch point" (see also the discussion of affective pathways in Goldin, 2000): In an alternative affective structure, frustration could evoke avoidance and disengagement, leading to a different outcome entirely.

The student has further emotional feelings in response to making progress in the problem. His interest grows, he is excited about rising to the challenge, and he feels pride that encodes a sense of getting closer to a solution. He also has feelings about his feelings – he likes what he is experiencing, and likes thinking of himself in this way – a way that may be very different from how he typically thinks of himself in other situations. Ultimately he feels a sense of satisfaction and accomplishment.

The description here is, of course, highly idealized, as are many possible idealized variations on the "Check This Out" structure. For example, a student whose interest is stimulated by the perceived utility of being able to solve the problem (rather than its complexity or novelty) may value the experience for different reasons, and engage in different self-talk. Here she may say, "Figuring this out will be useful. I'll be able to know the best deal, and save money. In the future I'm likely to deal with situations like these, and if I'm good at them, I'll be more powerful." Such internal dialogue guides her to intense engagement with the problem, and mediates the affective pathway of emotional states of curiosity, puzzlement, being intrigued, as so forth, culminating in pride and satisfaction. Her self-identity structure may include features such as "smart," "savvy," "street-wise," or "having a good head for business," with accompanying positive feelings.

Concluding Comments

Much of the study of affect and motivation in mathematics education has tended to emphasize students' attitudes toward mathematics, or teachers' and students' beliefs about mathematics or about themselves in relation to mathematics. We propose that important and fruitful results may ensue from the study of students' individual and classroom mathematical behavior through the lens of archetypal affective structures.

As we come to identify the most important such structures and characterize them more precisely, we should learn what to take as persuasive evidence that a particular structure is present and functioning. We should learn how to recognize and influence the choices students' make at the most critical branch points. And we should learn teaching strategies that create emotionally safe environments for engagement in conceptually challenging mathematics. Our work to this point in inner-city classrooms suggests the importance and value of such a research program.

Endnotes

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