What is the future of Wholemovement in the development of mathematics Education?

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Abstract
This paper developed from discussions about " Reform, Revolution, and Paradigm Shifts in Mathematics Education " at Johor Bharu, in Malaysia at the 2005 conference. The question came up; " Is Wholemovement a revolution or evolution in Mathematics Education?" Understanding this question is in the individual experience of folding circles. Initial observations about the richness of information generated by folding circles will be discussed within the Wholeness of the circle. The importance of an early hands-on experience in folding circles and the observations make learning mathematical functions, constructing images and models, and generalizing theories about relationships, easier to understand because of the contextual unity, multiple functions, and the inclusiveness of the transformational nature of the circle.

Definition and overview
Our thinking is constrained to the past if we do not periodically update the words we use to express current experience, ideas, and ideals. For thousands of years we have been measuring the earth and the things around us with an idea towards understanding universal laws of organization and relationships. We call this study geometry, meaning *earth measure*. Recently we have gone into space, looked back to earth from the moon, and now measure universes far beyond our own. Our experience and thinking has progressed from local, to global, to universal, and now must move beyond; past in present extending forward. Similar to the Fibonacci progression of numbers, we do not know the value of the past until it fully gets here, bringing everything forward as the means for continuation.

Geo means earth. The earth is spherical. The sphere is the only form demonstrating complete unity; it is Whole. *Metery* means measure, and measuring is about movement. Comprehensively, and in keeping with progressive experience, geometry means Wholemovement; self-referential movement of absolute unity. The word Whole is capitalized to mean comprehensive, distinguishing the inclusive from all parts and pieces we tend to call whole. Within the word "geometry" is a greater meaning that reflects more accurately the ongoing development of human understanding. We struggle with local to become global. Universal derives meaning from the greater cosmic Whole.

Revolutions overthrow governments, ideas, and processes already in place. They are always messy and wasteful, with little to build on, mostly starting over again on the destruction of, or ignorance about, and always, only part of the past. Revolution is more about local change than meaningful large-scale growth. By observing the interrelated functions of life and the profound effects of our interactions that we have on each other and the planet environment, we have to seriously question the idea of revolution in light of long-range human survival and the evolutionary development of potential beyond local boundaries and guarded interest.
Wholemovement is the only movement there is; within are endless subsets of interaction. Every individual action is an interaction of relationship determined by the relative meaning and value we find within the greater Whole. Separation is a concept, not reality. Nothing exists by itself without interaction. Mathematics is the generalization of functions observed about the movement patterns inherent to the interactions forming our universe. Mathematical thinking provides a formal abstract frame by which to attempt to accommodate the unknown. There are many different sets that have no apparent function were it not for the unseen extended connections of interaction. The parameters of mathematics assures some measure of consistency giving meaning to formulation, but hardly anticipates the discovery of all possible combinations of the known and undiscovered potential of the beauty and complexities of life expression.

Math understanding is a struggle towards consistency always building on what has come before, attempting to weave the truth of what we know through where we are now. The Whole is the only place everything will eventually happen, the place where there is room and time to reveal potential. It is a place to experience, to demonstrate in the context of everything we know, all we don't know. The circle is Whole; origin, context, potential.

Compressing the sphere
Folding the circle is direct expression of spherical unity. To truncate or cut the sphere destroys unity, limiting potential to a few parts. Compressing any two of an infinite number of opposite points along any axis of the sphere, reforms the sphere to a circle disk at right angle to the direction of compression. There has been a transformation without destroying unity or losing spherical information. The sphere compressed to a circle disc in space is Whole and a great circle part, functioning simultaneously as both, in ways nothing else can demonstrate. The circle is a compressed distortion of spherical potential. The drawing of the circle has its origin with the compass, not the sphere. Geometry has become flat over the years, images that represent what spatially exist. The image has become a symbol, and a tool of construction. To acknowledge compressed spherical origin reconnects us to concrete functional experience. Folding circles is a process that decompresses spherical information, requiring interaction, attention, and observation.

We know by analogy and metaphor, and now by demonstration, the circle is both Whole and part. A finite circle has a specific measurable diameter; making it as all other quantifiable shapes and forms. The circle as Whole is infinite by virtue of properties and cannot be quantified or qualified except in the absolute. The finite can never be perfect and the infinite will never be less than perfect; the two are mutually necessary.

Folding the circle
1.) Use a paper circle, any will do. Draw a circle and cut it out, notice the difference. I use paper plates; they are cheap and tolerate a lot of folding. Two points located anywhere along the circumference when touched together by curving the circle will form an open circle plane perpendicular to the axis of the circle surface. A cone pattern is formed; the conic form is incomplete. Nobody picks the same two points, so each cone has a different angled slope and each open circle plane differs in size to all others. If the two selected points are the exact end points of the axis to the circle, then a cylindrical pattern is formed. The cylinder is a special case cone that happens when the diameter is perpendicular to the curving movement of the circle. In this same way the square is a special case quadrilateral.

2.) Holding the same two points on the circumference together, crease the circle. This flattens the open circle plane making a crease that divides the circle in half, at right angle to the movement of touching two points together. Only if the points are exactly opposite will it show the square relationship of two perpendicular diameters. There are three sequential movements to the sphere; compression, curving, and creasing, all right angle functions. Nobody chooses the exact same two points in folding. While all diameters look the same, each is individually formed from an infinite number of diameter possibilities.

Principled Information
This first fold in half is principle to all subsequent folding. A lot of observable information has been generated. First we note: the <u>Wholeness</u> of the circle/sphere, then through <u>movement</u> is a <u>division</u> forming <u>duality</u> in <u>triangulation</u>, where all parts are <u>consistent</u> to the movement, and are <u>inner-dependent</u> to the Whole. It is the dependency of parts to Whole that determines the interrelationships between parts. These seven observations are inherent to the generation of information that sequential directs and regulates the order and arranging of the reconfigurations and joining of multiple circles. These principles are reflections of what we observe in the nature world.

The crease divides the circle in two equal parts without separation. The semi-circles are congruent because one fits exactly on to the other as a function of any two points on the circumference touching. The diameter is multifunctional as a divider, a line of symmetry, bisector, and axis. The circumference moving in two directions is 360° forming a spherical pattern, origin of the circle. Each semicircle is 180°; the sum of interior angles in a triangle. Any one point on the circumference in relationship to the end points of the diameter is always a right-angle triangulated half circle. The spherical rotation of the axis shows an inside and outside reciprocal function of the circle surface. Only half of the circle can be seen at any one time from any orientation, as with the spherical surface.

This first fold allows us to explore the relationship of parts. Counting four points, two touching points and the end points of the diameter, shows four points, two in space moving at right angle off the flat plane. This is a tetrahedron pattern of movement. Counting the diameter as one of the six straight lines connecting the four points, four individual triangle planes are formed. The properties of the tetrahedron are discernable in the first movement of the circle in half. Full axial movement shows a reciprocal function turning the tetrahedron inside out, alternating positive and negative positions. When the circle is flat we see the properties of the tetrahedron compressed to a kit shape on a flat plane. By drawing lines connecting each of the four points a fifth point of intersection appears revealing four individual triangles, showing eight triangles in combination. There are both left and right-hand right triangles, isosceles triangles, and scalene triangles, as well as complimentary, acute, obtuse, and inverse angles. Everything about triangles can be discovered through observation, reflection, and discussion of the first fold in the circle.

The advantage of not cutting the circle/sphere is that everything is understood in the context of everything else. Nothing is reduced to parts in isolation. The circle remains Whole. By observing interrelationships between selective parts, the connective functions become apparent. Ever part is multi functional to all others. Nothing has only one function, except when removed from context, thereby limiting possible connections.

Two points with one path in two directions is generally notated, (ab=ba). The folded circle shows us three individual paths (one diameter and two on the circumference), six individual directions between points (a) and (b). Three specific areas are defined; the circle and two semicircles. Folding the circle in half allows us to see and experience functions that have been “algebratized” into a symbolic language. Many functions can be observed and understood just by discussing relationships we see folded within the circle. We can use either common language or an algebraic language of symbols. The advantage of one language over the other is greater economy and clarity in observing relationships between parts with consistency and number that otherwise often go unnoticed.

There is nothing in folding the circle that denies any of what we teach in mathematics, it simple puts curriculum information into a context that gives greater meaning and helps clarify the relationships that have been separated, abstracted, and formalized. The circle provides an experiential context allowing students to approach mathematics from their own observations. The above information is only the first step into what students can discover by reflecting and sharing what they observe. The circle generates all necessary information. There is no need for prior knowledge, only folding and observation is required of students. Each fold sequential reveals information for the next fold, and all subsequent folding and reforming to a systematic process developed from that first fold in half. The diameter folded circle, the ratio of one whole to two parts suggests three; triangulation is principle. There are three options to fold one diameter again in the ratio of 1:2; three, four, and five diameters. Three comes first and is the obvious place to start. Each fold increases options and expands available choices. Once you understand the principles and the systematic development of the forming process, the only choice is one of direction. One has only to pay attention, to discover the consistent nature of evolving information and changing movements of appropriate interactions to assure continuation.

Folding the circle three times in a ratio of 1:2 generates three diameters equally placed around the circle; a hexagon pattern of seven points. We must again ask; what has been generated that was not there before? We need only to observe what happens through the functional interactions between parts, within the movement of the circle, to learn the symmetries and functions of geometry that are foundational to developing a mathematical sensibility. Three diameters offer two choices in directions to fold, where the progression of each will eventually lead to the same folded grid matrix from which all regular polygons, polyhedra, and irregular angulations are derived. Nothing is cut from or added to the circle, it is all through patterned transforming movement. Truncation and stellation are right angle movements into and out from a centered location. Concentric function suggests that the center is only a smaller circle without any inner boundary limit. There is no outer boundary limit to the circumference. The center point and circumference are part and total of the Wholemovement of the circle.

Conclusion

In teaching mathematics we are limited by the parameters of mathematical pedagogy about functions that are defined by symbols, formulas, and rigid boundaries. Math is not dynamic for most students, unless they find a greater meaning within themselves.

Mathematics has developed from observation, thinking about and reflecting on what we observe, making generalizations with some form of modeling to see if we have it right. This is how we learn anything. Understanding develops from experience, observing and then some kind of interaction with those reflected observations. To just observe the results is not enough. Learning is a structural process that engages four specific areas; practical, progressive, meaning, and experience.

No formula is going to cover all learning experiences; we all learn differently. In the last fifty years storage, retrieval, and transmission of information has changed dramatically. We now have greater access to tools and fragments of information without much connection or personal experience. The context that gives meaning to any information is not technology; it is in the greater experience. By observing more closely and more comprehensively, to question with greater depth our own experiential understanding of mathematics will afford greater meaning to the information we have available.

The principled and comprehensive nature of the circle allows students to experientially find information to their level of interest and understanding. This is always more meaningful than listening to someone else's mathematical story. Without personal experience of discovery there is little progress in the dynamics towards mathematical development. Five-year-old students can fold circles in half and they can talk about what they see happening, discovering mathematical functions for themselves, with guidance from the teacher. Folding circles gives them a hands-on, interactive, comprehensive, and principled experience of doing for themselves what they can understand and talk about.

Understanding starts with individualized experience relative to the largest context, the grandest concept, within the perceived Whole. The circle holds unlimited information about order, pattern, interactions, relationships, balance, proportions, and symmetries of generalizations about life; some of that information we call mathematics. Our planetary condition requires that we begin to context what we teach young students in terms of the Whole, to give them every possible access and demonstration of the importance of the interconnected functions between all parts. Only when students begin to experience the practical, progressive nature of expanded mathematical functions will they begin to make their own meaningful connections. The Wholemovement of geometry provides a principled, hands-on, experience that supports the on going development of mathematical thinking. The question about Wholemovement and its place in the evolution of mathematics education is not mine to answer; it is yours, and your students. It can only be answered through individuals folding circles, making connections, and finding value in the Wholemovement of the circle.