A Descriptive Analysis of Secondary Mathematics Students' Formal Report Writing

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Abstract
This study analyzed forty-nine formal mathematical reports written by secondary students participating in a 4-week intensive summer program. The goals of the study were to develop a detailed coding scheme for categorizing mathematical content and apply that coding scheme to describe these written reports. The coding system allowed for categorizing higher- and lower-level knowledge structures among other text features. Key findings included (1) the majority of statements were indicative of lower-level knowledge structures (description, sequence); (2) aspects of mathematics were predominately algorithmic and methodological; (3) higher level structures (classification, principles and evaluation) were frequently textbook based and (4) justification was absent from the majority of products. The findings indicate the need for additional research on students' writing in mathematics and have implications related to the role of writing in promoting higher-order thinking.

Introduction

Written mathematical communication is a filter allowing the exchange of mathematical concepts and ideas – a text-based view of the writer's mathematical thinking. Although there is a need for students to learn to communicate in mathematics, there have been few attempts to describe in detail students' writing, with a view to enhancing the learning of mathematics through the use of writing. The aim of this study is to develop a method to analyze expository writing of mathematics students in order to provide a coding scheme for the description of the content of their writing. More specifically this study will:

(1) discuss a detailed coding scheme developed to categorize the content of mathematical papers written by secondary school students and
(2) apply that coding scheme to describe these written products.

Theoretical Framework

Educators in varied disciplines have long acknowledged the connection between writing and learning. Vygotsky (1987) viewed writing as a tool which required deliberate and analytical action from the composer as he/she structured a web of meaning forming connections between current and new knowledge. In mathematics, writing is promoted as a means of supporting reasoning by providing a mechanism for students to think about and verbalize concepts and ideas. This generative process assists students in analysis, comparison of facts, and synthesis of information (Farrell, 1978). Boscolo and Mason summarize these ideas in saying that "writing can improve students' learning by promoting active knowledge construction that requires them to be involved in transforming, rather than a process of reproducing" (2001, p. 85).

Students often seem to write about mathematics in vague and unclear ways (Stonewater, 2002). In fact, students may reject their own mathematical thinking when they believe mathematics is best learned in a rote manner (Hamdan, 2005). One difficulty in such research has been the lack of a detailed method or coding scheme for analyzing the written papers of students (Shield & Gailbraith, 1998). Morgan (1998) reports that there is little research on writing in mathematics that focuses on the analysis of the written products themselves. Hamdan (2005) reported that written journal entries in linear algebra were a chaotic collection of ideas with no connection between them; however, writing became more structured and meaningful after students were engaged in labeling and arranging their collected thoughts into categories.

Ntenza (2006) noted that students' writing contained symbolic and mathematical writing. Symbolic writing is the product of routines and algorithms. Mathematical writing included direct transcriptions from other sources, translations of symbols into words, summarizing and translating where learners use their own words to explain text material or experiences, and creative uses of language where students explore and transmit mathematical information (see Davison & Pearch, 1988; 1990). Shield and Galbraith (1998) studied expository writing of 8th grade mathematics writing providing a coding scheme to describe the content of the writing, and formulating a general model for the expository writing. The authors suggested that by identifying and naming the parts of a presentation, teachers could assist students to write more elaborate presentations which stimulate deeper thought and understanding of the mathematical ideas.

Though results are mixed, writing-to-learn activities in mathematics classrooms, in general, have been reported to have a positive impact on learning of mathematics (Ntenza, 2006; Pugalee, 2005). Studies have shown improved metacognitive abilities (Pugalee, 2001) and improved mathematical performance on word problems (Pugalee, 2004). In addition to benefits for students, Hamdan (2005) argues instructors can also gain from observing students' processes of choosing appropriate methods and applications. Further, common misconceptions and difficulties with content become evident providing opportunities to readdress them in the future.

Methods

The writing samples analyzed were written by high school students enrolled in a summer enrichment program in science and mathematics. Students in this program were rising juniors and seniors in North Carolina high schools. Students applied for the program and applications were screened and rated based on academic performance, interest in mathematics, and recommendations. The highest rated students were offered slots in one of six campus sites participating in the program.

Students were instructed by master high school teachers and/or university professors in a science or mathematics area. The students in these four-week programs were in the same class each day for seven classroom hours. The programs were designed to enrich students' existing understanding of mathematics and expose students to the work of mathematicians.

Students met with their instructor(s) in classroom, laboratory, and field settings before choosing a research topic of interest. With the aid of instructors and peers, students then researched their topics, performed experiments, when appropriate, and wrote research-style reports to exhibit their understanding of their topic. The students were not provided explicit instruction in writing, but were each provided intermittent feedback during the writing process. Over the past five years of the Summer Ventures in Science and Mathematics program, 49 student research reports in mathematics have been produced; many of these reports having multiple authors. The 49 reports analyzed ranged between five and ten pages of text in length.

Researchers (two mathematics education professors and a doctoral student) chose an initial report at random to use for training before undertaking the analysis of the forty-nine research reports. After the researchers met to compare and discuss the initial coding attempts, the coding scheme was revised to incorporate what the team viewed as representative characteristics of student writing. The research team then randomly chose six more reports, at least one from each of the of mathematics courses, to be coded by each researcher. The research team then met to discuss and scrutinize the coding of each of the six reports. The purpose of these meetings was to further train and align the research team for consistency of future coding and to refine the coding framework. During this meeting, inter-rater reliability for the six training reports was 85%. The research team then divided the remaining 42 student reports among the team so each report would be coded by two researchers. After coding was completed, researcher pairs compared their results and discussed each instance of disagreement. Instances where reliability was less than 85% resulted in recoding by each researcher until the 85% threshold was met.

This analysis of the students' written reports followed qualitative procedures of searching for themes using a compare and contrast process focusing on the heterogeneity and homogeneity of the responses (Gay & Airasian, 2003). This allowed for the identification of written units usually one or more sentences. The coding scheme was then applied to these units providing a characterization of the students' written products.

Each report was categorized as belonging to one of seven forms or genres of student report writing (Wellington & Osborne, 2001). These categories included classification, decomposition, descriptions of functions or processes, listing of properties, explanation, experimental accounts, and argumentation. Classification papers begin with general classification then provide descriptions of the qualities, behaviors, and/or functions of particular phenomena. Decomposition papers explain a whole phenomenon or concept in terms of its constituent parts. Descriptions of functions or processes papers provide opening general statements or observations followed by a set of statements of various functions. Listing of properties papers begin with a general statement followed by a list of properties. Explanation papers consist of a general statement followed by a series of logical steps explaining how or why something occurs. Experimental papers provide a recounting or details of an investigation and explanation papers gave a general statement followed by a series of logical steps explaining how to why something occurs. Argumentation papers create a position based on use of evidence, using connectives which relate claims to the warrants or reasons for the beliefs.

A coding scheme was developed based on previous work of Shield and Galbraith (1998) and Huang, Normandia and Greer (2005). Shield and Galbraith's coding scheme, applied to expository writing of eighth graders, focused on explanation elaborations, aspects of mathematics and levels of language. Huang, Normandia and Greer's research focused on knowledge structures evident in student discourse in secondary mathematics classrooms. Though the codes developed were used in verbal discourse, the applicability to written work is promising providing a mechanism for analyzing students' mathematical knowledge structures. Identification of students' knowledge structures in written work may promote the researchers' goal of assisting teachers in pushing for higher-level knowledge structures. Higher-level structures were classification, principles, and evaluation whereas lower-level structures included description, sequence, and choice. Other codes included goal statements, description (general and particular), justification, link to prior knowledge or experience, examples, conclusions/results (empirical, theorems/definitions), exemplars (symbolic representation, diagram, graph, conventions, tables, procedures), and aspects of mathematics (theoretical, logical, algorithmic, methodological).

Results/Conclusions

The initial data analysis revealed several characteristics of students' written formal mathematics reports (1) the majority of statements were indicative of lower-level knowledge structures (description, sequence); (2) aspects of mathematics were predominately algorithmic and methodological; (3) higher-level structures (classification, principles and evaluation) were frequently textbook based (4) justification was absent from the majority of products; (5) mapping structures showed the most common pattern to be descriptive general statements followed by procedures; and (6) experimental accounts were the primary genre reflected in the reports.

Approximately 60% of the data pieces were descriptions, sequences, or choice statements. The prevalence of these lower-level knowledge structures is consistent with research on verbal discourse (Huang, Normandia & Greer, 2005). Descriptions were the most frequent among these three codes comprising approximately 75% of these lower-level codes. We differentiated between general descriptive (DG) statements and particular descriptive (DP) statements. DP statements were specifically related to the particular case or situation being discussed. DG statements were 8 times more likely to be used than descriptive particular statements. The majority (approximately 85%) of DP statements were found in recounting specific details of an experiment or investigation. Statements indicative of lower-level knowledge structures were primarily used to provide introduction and background of topics in all of the papers analyzed, but were also the most prominent form of statements used by students. The following excerpts from the data are provided to exemplify this finding.

Game theory, like lightning, existed since this world was born, long before a person forged the way to an explanation.

There are two basic types of bonds, which are vanilla bonds and zero-coupon bonds.

Pierre Fermat was born on August17, 1601 in Beaumont-de-Lomagne, France.

The prevailing aspects of mathematics revealed in the papers were algorithmic and methodological with approximately 95% of the papers containing 5 or more such blocks of text. Aspects of mathematics were more global in nature than the other codes and encompassed larger blocks of text. Algorithmic is best considered as writing that contains explicit methods or procedures while methodological is somewhat more general and heuristic in nature (see Shield & Galbraith, 1998). This finding isn't surprising considering that the focus of the papers was to present findings from an investigation or experiment. It should be noted, however, that few writers followed up with rich descriptions of their findings. Additional analysis is necessary to determine how such aspects of mathematics impacted students' mathematical communication and their presentation of important mathematical ideas. However, the findings raise questions about the prevalence of procedural orientations in students' mathematical understanding.

Statements of a mathematical nature were predominately algorithmic and methodological. The statements that follow are provided as exemplars of this finding.

The monthly income of a median wage family is calculated using the simple formula $M = \frac{(1-t)A}{12}$ *where M= monthly income, t= federal tax rate, and A= annual salary.*

*The calculation is as follows: (Price of Security after a 1% decrease in yield – Price of security after a 1% increase in yield)/(2*initial price of security)*

When higher-level knowledge structures were evident, they were frequently textbook based (in slightly more than 40% of these coded data pieces). Students frequently cited sources to provide definitions (coded as classification) and principles. It was also evident that many principle statements were contextualized in scientific ideas that were relevant to the experiment or investigation (such as friction and acceleration). Principle statements that were not textbook based were often presented in an if-then format. Evaluation statements (less than 10% of the higher-level data pieces) most frequently occurred in explaining results from an experiment or from working a particular mathematical example. Evaluation statements most often involved interpreting empirical results in light of a procedure or example. A small number of papers (8 %) contained evaluation statements that were related to the difficulty of applying a particular method or procedure. Evaluation is related to justification, but can be differentiated by the level of reasoning involved. Evaluation statements were often associated with reporting results or conclusions. In experiments or investigations, evaluation statements often involved interpreting findings by stating a type of relationship. Justification was coded when a statement provided a mathematical reason or argument for a decision or action. Evaluation was more commonly involved in verification of a result or step without offering a mathematical reason or argument as to 'why' the result was valid. Less than 2% of all coded statements involved justification.

The following excerpts show the nature of statements supporting this finding.

> C: *A force can act upon the electrons to speed it up called voltage (V).*
>
> P: *If i really can be represented by a point or a vector, then multiplying it by itself must equal -1.*
>
> E: *The history of complex numbers is perhaps most intriguing because, unlike the discovery of many other concepts, that of complex numbers is surrounded by controversy, unease, and secrecy.*

The following are some of the sparse number of justification statements made by students.

> *The reason that this method of finding a utility is considered weak is because it is strictly limited in that it does not give one any information as to the extent by which the subject enjoys one item over the other, just the order by which they enjoy them.*
>
> *Since the wheels all had the same amount of mass, the amount of friction did not increase or decrease enough to affect the results significantly.*

A mapping was done that showed which types of codes preceded and followed a particular code. The most common (18%) pattern was descriptive general (DG) followed by descriptive particular (DP). The preceding codes in this particular instance were not included since they primarily began after a paragraph or section break. This line of reasoning appears logical given that students provided a general description of a concept, idea, or process and followed up with a particular description related to the context of how that information was related to their topic. Procedures or algorithms (AL) frequently succeeded the DG – DP sequences. The following example from the data shows the nature of this common pattern.

> *The connection between music and math was more recognized in ancient times than it is now. (DG) The Greeks, often considered the pioneers of mathematics, studied four branches of math, which were: arithmetic, algebra, geometry, and music…(DG). The following calculations are summarized in Figure 1 (DP). This system involved the ratios of notes, with the simplest ratio, 2:1, being an octave (DP). By doubling the frequency of a note, a pitch an octave, or twelve semi-tones, above the original is produced…(AL).*

The most common genre for the reports was experimental accounts (61%) followed by explanations (19%). This might be expected given that there was an expectation that students would report on an investigation or describe their research on a mathematical topic or concept. There were no papers classified as arguments, listing of properties, or decomposition.

Implications

Students are often required to write formal papers or reports in mathematics; however, there is little research on what students actually produce in these assignments. While there are some materials available to guide general procedures and approaches (such as sections of a report or formatting), there is little guidance to assist teachers in developing high expectations for such products. This research provides some description of students' formal mathematical writing. The findings must be interpreted and reported in ways that will provide practitioners with ideas about how to improve students' mathematical communication and higher-level thinking through writing of formal reports. The researchers acknowledge that much work is required in this area including revisiting the written reports and assessing students' mathematical understanding within various domains such as conceptual understanding, procedural understanding, problem solving ability, and mathematical reasoning.

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