Project CSUMS at Rensselaer

Mark H. Holmes
Professor of Mathematical Sciences, Rensselaer Polytechnic Institute, Troy, NY 12110 holmes@rpi.edu

Abstract
This paper describes a recently established undergraduate research program at Rensselaer, Project CSUMS. The program includes a new upper-division course aimed at preparing students for research in mathematics, establishment of undergraduate research awards to fund their research over twelve months, an undergraduate colloquium and a working research seminar, and a series of workshops to help students apply to graduate school. We are in our first year and this paper describes what has worked and what has been modified from what was originally proposed to the NSF.

Introduction

As with all scientific disciplines, mathematics is undergoing dramatic changes due to the technological advances made over the last couple of decades. Mathematical education has been affected, for example, by the use of computers in calculus and the capabilities of classroom interactive visualization systems. Mathematical research has been more profoundly affected, and examples are the data mining problems arising from the genomic and gps technologies, and the problem of trying to simulate large complex systems (e.g., the climate or materials design). The latter were, in part, the stimuli for the National Science Foundation to initiate a new educationally related program, titled Computational Science Training for Undergraduates in the Mathematical Sciences (CSUMS). The stated goal is "to enhance computational aspects of the education and training of undergraduate students in the mathematical sciences -- mathematics and statistics -- and to better prepare these students to pursue careers and graduate study in fields that require integrated strengths in computation and the mathematical sciences." This is a joint effort of the Education and Human Resources, and the Mathematical and Physical Sciences directorates. The solicitation can be found on the NSF web-site (1).

NSF started the CSUMS program in 2006, with funding beginning in 2007. The institutions in the first funded group included Rensselaer, New Jersey Institute of Technology, George Mason University, University of Wyoming, and Brigham Young University. Each is in the middle of the first year of funding, so the jury is still out on the effectiveness of the program. However, at least at Rensselaer, we have learned a great deal, both in terms of what works, and what was proposed that needs to be modified in subsequent years. This paper describes what we have learned to date and our vision of a successful undergraduate research program.

Organization

Our CSUMS program is a collaborative effort involving Rensselaer and Howard University. As with all CSUMS programs, the objective is for the undergraduates to undertake a substantive research project that contains both mathematical and computational components. The key word here is "substantive." To achieve this our program is organized into three overlapping components. They are Preparation, Research, and Dissemination. Another important aspect is that, unlike REU programs, our students are supported for twelve months and usually participate for 16 months. This gives us time to prepare them for their research and, as a consequence, enable them to produce work that can be presented at national meetings. The ongoing activities and background of our program can be found on the web-site (2).

Preparation

Our target recruiting audience is junior math majors. Given the heterogeneous backgrounds of these students, we have created a new spring semester upper-division course, "Undergraduate Research in Applied and Computational Mathematics." Aspiring CSUMS students are required to enroll in the course, and other students are encouraged to participate. The course begins with one-hour presentations by each team of senior CSUMS students on their research projects. This serves to help connect the junior and senior cohorts, and help the juniors focus on the course by seeing concretely the possible outcomes of the research experience. After this some common research skills are covered, such as efficient literature searches, useful computational and analytical methods, and LaTeX. Concurrently, they select a research paper from three different categories listed in the CSUMS library (3) and write a short report on each. They are required to select one of these topics to serve as the basis of their class research project. During the remainder of the semester they work on this project, meeting weekly with a small research group, and meeting weekly with their faculty mentor. It is this aspect of the project that has been revised from what we originally proposed in the grant. The issue is time. We had assumed they would be able to undertake at least two such research projects, the goal being to provide them a broad perspective of contemporary research in computations and mathematics. Given their schedules (courses, work-study, etc.) this is unrealistic. It would be possible if they were simply producing a library report, but we want them to explore, tying to push past what they had read in the papers. This takes time and for most students one project is all they can handle. At the end of the semester they made posters and presented their work to an audience of students and faculty.

One outcome of the course is that it helps identify who should be supported by CSUMS. It also helps the students decide if this is something they would like to continue. We were fortunate that every student who wanted to continue was also someone who the faculty considered as capable of continuing in CSUMS. These students were given research awards, with a funding period of twelve months that began in the summer. Specifically, the award consists of a 10 week stipend in the summer, a stipend during the upcoming academic year, and a travel budget. Moreover, the grant pays for their room and board during the summer. This turned out to be important as many of the students in the program normally work in the summer, while living at home, to help pay for their education. Without the room and board they would not have been able to participate.

Research

The summer is when the majority of the research is completed. During this period the students meet at least biweekly with their faculty advisors. They also attend the CSUMS research seminar, which meets weekly. This is informal and overlaps with lunch, which is provided. In the seminar, they give a 40-minute presentation of their work, and they do this every other week. The purpose is to show what they have accomplished, to discuss problems that they are not yet able to solve, and explain what will be coming next. We also use this to help them improve their communication skills. For this reason, we limited them to blackboard presentations. At the first of the summer they usually attempt to rewrite numerous, rather long, formulas on the board from notes, trying to give an exhaustively thorough presentation. They find that the seminar environment is lively, and they are continually peppered with questions from the audience and this causes the discussion to take multiple directions. Most of them soon learn how to express the ideas from their work more abstractly, both analytically and graphically. They also learn, very quickly, that research involves uncertainty and the faculty can have no idea on how to solve some of the problems presented. They also learn how these more experienced researchers parse, or reformulate, a difficult problem to produce a solution. Finally, the seminar served to produce a cohort of students and faculty, who work on separate problems but interact closely as a group.

Dissemination

This component includes presentations, publications, and communication. As evident in the description of the CSUMS seminar, a portion of this effort occurs during the research stage of the project. It continues into the Fall, when the trainees give a presentation on their project to a general audience at CSUMS Day (which is used to recruit the group for the next year). This gives them the confidence and ability to give presentations at more formal events, including: Hudson River Undergraduate Mathematics Conference, outreach talks at local high schools and colleges, and minisymposia at national meetings.

Current Projects

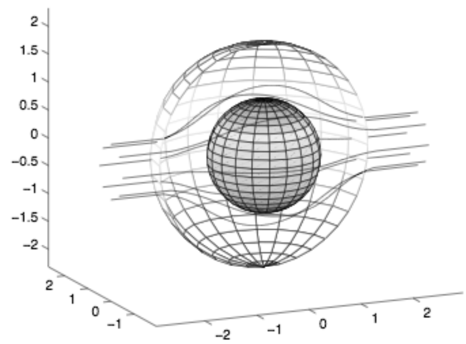
Below are the projects for this year.

1. Invisibility in Metamaterials

   Description: A recent paper by Schurig, et al. (4) discusses the bending of light rays in materials in which the index of refraction is allowed to vary continuously. We are exploring the ideas set forth in this paper, computing specific examples (see figure below), analyzing the theory and pursuing possible extensions.

   Students: Andrew Nixon, Miles Crosskey, Leland Schick

   Faculty Mentor: Kovacic (Mathematical Sciences)

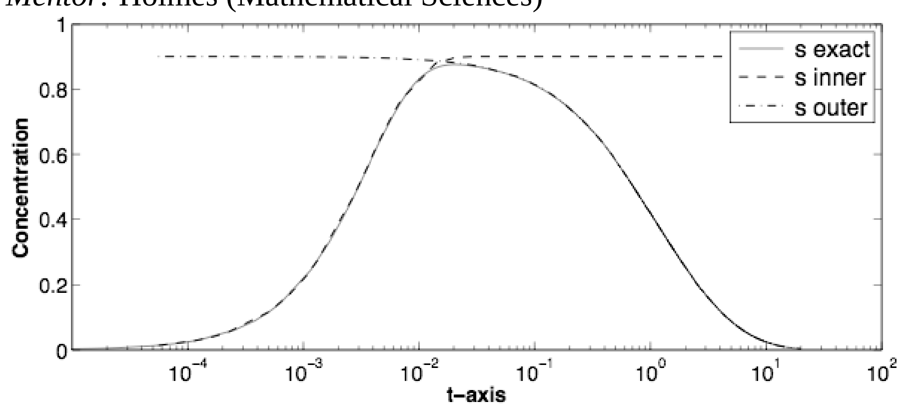


2. Kinetic Modeling of the Maillard Reaction

   Description: The Maillard reaction has an important role in both food science and medicine. The goal of this project is to construct and analyze a model based on a general chemical mechanism that has been constructed from various experimental papers. Non-dimensionalization has led to a reduced system of inner/outer solutions (see figure below) that accurately describe the behavior of the reaction and enables us to determine the kinetic parameters from experiment.

   Student: Michelle Burke

   Faculty Mentor: Holmes (Mathematical Sciences)



3. Neuronal Bursting

Description: Neuronal bursting is a series of spikes or action potentials fired in succession. It is thought that neurons communicate with each other in this way. The purpose of this research is to add noise to the Hodgkin-Huxley equations, which model the neuron, and then to perform analysis in terms of slow-fast dynamics and mixed-mode oscillations. A mathematical explanation for bursting is that the model of the neuron is near the Hopf bifurcation this affects the spiking pattern by making it chaotic. Noise being 10% of the signal is apparently the "correct" amount. It is not known whether the regime with noise has the Markov property.

Student: Joshua Baldwin

Faculty Mentor: Roytburd (Mathematical Sciences)

4. Finding a Width of a Perfectly Matched Layer for Elastic Wave Propagation in the Frequency Domain

Description: The goal of the project is finding a suitable width for an artificial layer to absorb and attenuate incoming waves in elastic media in the frequency domain. This width should take into account reflection coefficients and aim to minimize numerical reflection at the boundary between the medium and perfectly matched layer. The width will ultimately be validated computationally through spectral methods to determine its accuracy.

Student: Ben Baker

Faculty Mentors: Roecker (Earth and Environmental Sciences), Herron (Mathematical Sciences)

5. Using PCA on Biological Motion

Description: This experiment uses principal components analysis (PCA) to break down biological motion and decide which components are the most useful for classification purposes. We are using these methods on penalty kicks in soccer, tennis serves, and pitching in baseball.

Student: Dennis Ehlinger

Faculty Mentors: Fajen (Cognitive Sciences), Herron (Mathematical Sciences)

6. Structure of a More Realistic Food Web Model with Speciation

Description: Many of the current food web models (specifically those which include speciation) do not incorporate very biologically realistic functional responses or restrictions. This is due to the fact that the papers primarily focus on network structure and development rather than population sizes. In our work we desire to create a more realistic food web model and measure to what degree the simplifications that other papers use has on network properties with specific attention paid to scale-free properties. In addition we wish to examine how these properties change with territory size.

Student: Thomas A. Wentworth

Faculty Mentors: Lister (Biology), Kramer (Mathematical Sciences)

7. Low-Reynolds Number Swimming with Thermal Fluctuations

Description: Various swimming strategies have been studied for swimmers (single-celled organisms, and hypothetically, man-made swimmers) in low Reynolds number fluids. We are studying how thermal fluctuations affects the performance of microscopic swimmers and, possibly, how swimmers can be modified to improve performance, in particular.

Student: Sam Hughes

Faculty Mentor: Kramer (Mathematical Sciences)

Evaluation

As part of our formative evaluation of the program we asked the students in the spring semester research course to provide their comments on the course. One question on the evaluation was

the following: Please score each of these questions 1 to 5, 1=strongly agree, 2=agree, 3=neutral, 4=disagree, 5=strongly disagree. If you have no opinion just skip the question.
a. The training component (LaTeX and Matlab) was helpful.
Results: 1 - 3 votes, 2 - 5 votes, 3 - 2 votes, 4 - 2 votes, 5 - 2 votes, No Answer - 4 votes.
Comments: They generally liked Matlab but some really did not care for LaTeX. Given the preeminence of LaTeX in mathematics, we will repackage how we present the program so it is more appealing.
b. The math related lectures were helpful.
Results: 1 - 5 votes, 2 - 7 votes, 3 - 4 votes, 4 - 2 votes, 5 - 0 votes, No Answer - 0 votes.
Comments: We tried to cover too much material, thinking that the more they know the better off they are. The topics were numerical differential equations and Matlab, probabilistic (stochastic) simulations, and Hamiltonian mechanics. We will eliminate one or more of them next year.
c. The external lectures were interesting.
Results: 1 - 3 votes, 2 - 6 votes, 3 - 6 votes, 4 - 3 votes, 5 - 0 votes, No Answer - 0 votes.
Comments: We invited four outside speakers to discuss their research (i.e., we have a math colloquium for undergraduates). The somewhat lukewarm response reflects their individual interest in the topics. We had a diverse set of speakers and their opinion of what was presented varied significantly in the class, with no agreement on who they liked.
d. The homework assignments helped achieve the goals of the class.
Results: 1 - 1 vote, 2 - 9 votes, 3 - 2 votes, 4 - 5 votes, 5 - 1 vote, No Answer - 0 votes.
Comments: This is related to Question (b).
e. You enjoyed your research group experience.
Results: 1 - 8 votes, 2 - 6 votes, 3 - 0 votes, 4 - 4 votes, 5 - 0 votes, No Answer - 0 votes.
Comments: We will expand this part of the course next year. The faculty mentors varied somewhat in how they ran their groups, and we are working on developing a best practices model for this.
f. Your interest in graduate school in mathematics increased as a result of this class.
Results: 1 - 3 votes, 2 - 4 votes, 3 - 7 votes, 4 - 2 votes, 5 - 1 vote, No Answer - 1 vote.
Comments: This question is for NSF's benefit. Most of the students were seriously considering graduate school before enrolling in the course, so our affect on them is unclear.

Conclusion

Although we are only six months into our first year, the affect of Project CSUMS has been significant. The undergraduate colloquium alone has generated campus-wide interest in mathematics, and the availability of NSF-style undergraduate research awards has made the program very attractive. How well we actually succeed, however, will need to wait another year before we have more quantitative data.

Acknowledgements

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References

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(2) http://eaton.math.rpi.edu/CSUMS
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(4) D. Schurig, J. B. Pendry, and D. R. Smith, "Calculation of material properties and ray tracing in transformation media," Opt. Express **14**, 9794-9804 (2006).