The Relationship between Preservice Teachers' Conceptions of Randomness and their Pedagogical Content Knowledge of Probability

Sarah Ives
SarahIves@gmail.com North Carolina State University, Raleigh, USA

Abstract
This report presents the findings of a study on the relationship between pre-service mathematics teachers' understanding of randomness and their pedagogical content knowledge of probability. Task based interviews and participant reflections were analyzed using a framework based on the work of Batanero, Green, and Serrano (1998). Findings indicate a relationship between preservice teachers understanding of randomness and their perspective of the meaning of probability; both of which influence pedagogical decisions.

Introduction

In the past decade the teaching and learning of probability and statistics has been receiving increased attention by the mathematical community. Empirical as well as psychological research details the complexities and difficulties of understanding probability by students at grade levels varying from elementary up to the college level (Fischbein & Schnarch, 1997; Jones, Langrall, Thornton, & Mogill, 1999; Lee, Rider, & Tarr, 2006). Additionally, research has been conducted regarding teachers' understanding of certain probability concepts and how to teach probability (Begg & Edwards, 1999; Mojica, 2006; Watson, 2001). However, little research has been done specifically on preservice mathematics teachers understanding of probability and the teaching of probability.

As curriculum reform requires more attention be given to the learning of probability and statistics, research on preservice teachers' pedagogical knowledge is needed (National Council of Teachers of Mathematics, 2000). This study will advance the current research by giving teacher educators a better understanding of preservice mathematics teachers' knowledge of randomness and probability. With this knowledge, teacher educators can improve the teaching and learning of probability by offering pedagogical tasks that explore randomness and how it relates to probability. The results in this paper are taken from a larger study into the relationship between preservice teachers' beliefs, content knowledge, and pedagogical content knowledge of probability. The focus of this paper is on preservice teachers' understanding of randomness, how it relates to their perspective of probability, and how these two aspects impact their pedagogical decisions.

Theoretical Frameworks

One of the fundamental aspects of probability is the notion of randomness and chance. The work done by Batanero and colleagues (Batanero, Green, & Serrano, 1998; Batanero & Serrano, 1999) offers a way to describe students' understandings of randomness. The word 'random' is used in everyday language as well as in mathematics classrooms. The definition of randomness is not clear and this ambiguity can increase the possibility of students having difficulty with this idea. Random is usually used as an adjective such as random number, random experiment, random variable; definitions of these tend to concentrate on the object being described as random rather than a definition of random itself (Batanero et al., 1998).

In addition to the definition being unclear, the meaning of randomness varies depending on a person's perspective of the meaning of probability. There are multiple perspectives that one can have for the meaning of probability; these perspectives fall into two categories: objective and subjective. Within the objective category there are the classical and the frequentist perspectives of probability. From a classical view of probability, one understands the meaning of probability as a ratio of favored outcomes over total outcomes. This view is limited to finite number of

outcomes and thus someone with this perspective would believe randomness is tied to equiprobability. “In the classical conception of probability we would say that an object (or an event) is a random member of a given class if there is the same probability for [it as there is for] any other member of its class” (Batanero et al., 1998, p. 115). Someone with a frequentist perspective considers probabilities to be assigned based on the long run behavior of random outcomes. Within this perspective the equiprobability principle need not apply and a preconceived theoretical probability may not be known. Batanero et al. state, “here, we might consider an object as a random member of a class if we could select it through a method providing a given *a priori* relative frequency in the long run to each member of this class” (p. 115). This view of randomness is also an objective property of an event, yet it is based on relative frequencies.

Another view of probability is that it is subjective, meaning that the probability one assigns to an event is subjective to that individuals' beliefs. This view requires an understanding of randomness that is also subjective. What may be random to one person may not be considered random to another. This view is applicable when we know something that may affect our judgment of the randomness of an event.

Methodology

This study is a qualitative case study focusing on 3 preservice teachers from a large university in the southeastern United States. The subjects are juniors and seniors in a mathematics education major with a focus on high school. The participants are purposefully sampled from a 400-level mathematics methods course on teaching with technology. Multiple data sources used for this study include interviews, classroom observations, and documents. Each student participates in an initial interview, a second interview, reflection, and a final interview. The interviews are task based and focus on the nature of the preservice teachers’ understanding of probability: their beliefs, content knowledge, and pedagogical content knowledge. Data collection and concurrent analysis occurred during the Spring 2007 semester.

In the initial interview participants were asked, “What does random mean to you?” The responses were then analyzed using the framework by Batanero et. al. (1998, 1999) to determine a possible perspective towards the meaning of probability. After participation in a pedagogically oriented interview where the participants were asked to analyze student work, critique a teaching episode involving discussion of the corresponding student work. The task that was presented in the second interview was designed for middle school students, but could be used at the high school or college level. In the task students worked in pairs to determine if a particular company’s dice were fair by running simulations using the software program *Probability Explorer* (Stohl, 1999-2002). Once the students gathered enough evidence they created a poster to display conclusions. In the second interview of this study, the preservice teachers analyze these posters to assess students’ understanding of probability.

After the second interview the participants wrote a reflection on what they had learned about the teaching of probability. In the final interview, which occurred approximately 4 to 5 weeks after the initial interview, participants were asked again, “what does random mean to you?” Their responses were then analyzed using the same framework to determine if there was any change in their understanding after having participated in the second interview.

Results

In order to look at the relationship between one’s understanding of randomness and their pedagogical knowledge first we need to ascertain their conception of random. The first section of

results discusses the preservice teachers' conception of random and their perspective of probability. The second section shows how these perspectives can impact pedagogical decisions.

Preservice teachers understanding of random

The following table shows the participants responses in the initial interview and in the final interview to the question "What does random mean to you?" Key words that indicate a possible perspective of probability based on the framework are bolded.

Participant & Perspective	**First Interview Response**	**Final Interview Response**
Brad [frequentist, classical]	Um it just means, wow, usually use random in the definition… Um, it just means its **messed- jumbled up, uh there's no real order to it at all** its just, I don't really know what else to say, just there's no order to it, it just happens, [pause] randomly [laughs].	I guess just anything that … man, its hard not to use random in the definition of random. So, something that happens by, as **having an equal chance**, well I wouldn't say equal, but there's a chance of **anything happening or anything could come up** in a certain set. Yeah, random would be anything that would come up in that set.
Jeff [frequentist and classical]	Well if you think about it … **blindly picking something out of a pile**. It is hard to put into words as far as random goes, **not repetitive** … I don't know, its hard to put into words.	Its hard to put into words, random is out of like a series of possible results **any one result is possible**, there could be a greater chance of some over another, I mean **there's a chance for any one of them** because it's random.
Pam [subjective and classical]	Uh, I guess, in terms of probability when I think of random I think of a **random sample** so it would just be so if you were to have a completely um you know like **not hand picked group of people but just kind of like uh really open sample** and you were to just randomly pick someone from it.	random would mean **having no bias** in place at all, no factors that would make **one outcome more likely to occur** than another one. And I guess that's it, picking a **random sample** would be not really, um, not focusing in on one thing but having a whole group of something.. I don't know this is a really hard thing for me to describe.

Table 1: Preservice teachers understanding of randomness

Definitions of random included – no order, not repetitive, not hand-picked, equal chance, having no bias. In the initial interview, Brad and Jeff displayed a frequentist perspective; their definitions of random referred to the order of outcomes being non-repetitive. Whereas, Pam, did not define random but instead defined what a random sample means. Later in the interview Pam said, "probability is all personalized, I don't know if I'd actually be that great at teaching it because I think about it as more of a choice: you're given information then you do what you want with it." This comment suggests she has more of a subjective perspective. All three subjects explicitly said they had difficulty defining the meaning of random: "It's hard to not use random in the definition of random," "it's hard to put into words," and "this is a really hard thing for me to describe."

In the final interview, we can see that Brad, although still expressing difficulty, adds to his previous definition, "having an equal chance." This indicates that he may also have a classical perspective of probability. Jeff as well expands on his definition by adding, "any one result is possible," indicating a classical perspective. However his understanding of equiprobability is not

clear as he says, “there could be a greater chance of some over another.” Pam, in the final interview, displays a stronger understanding of random by adding “having no bias… no…one outcome more likely to occur than another.” This also indicates that Pam has a classical perspective of probability.

Pedagogical decisions

In the second interview we can see how the preservice teachers’ perspective impacts their pedagogical decisions. One of the tasks they were given was to analyze posters of student work. They were asked to describe what the student knows about probability based on what the students put on their poster. On one poster the students had displayed a frequency bar chart similar to the following figure:

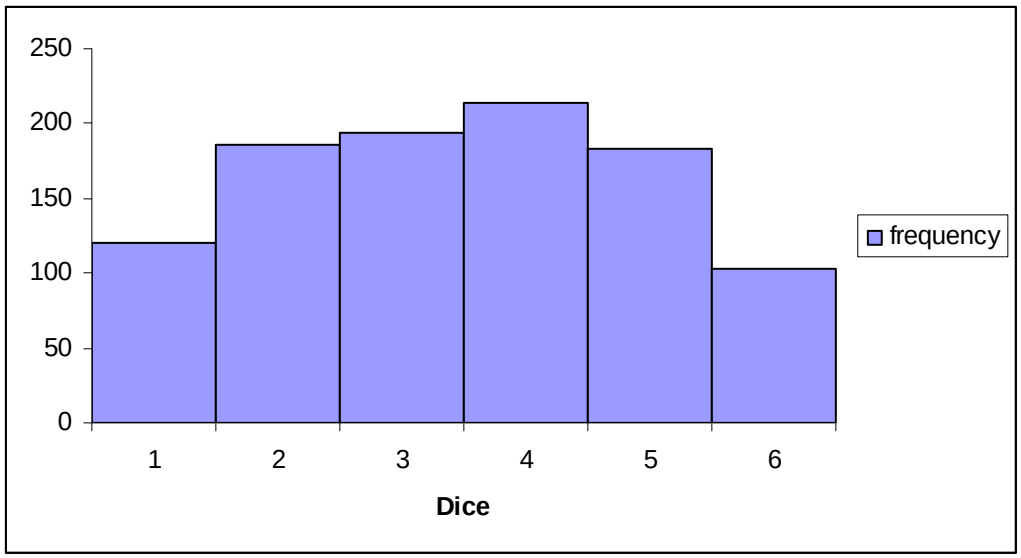

Figure 1. Frequency of each number rolled

The results of the student’s simulation were 120,186,194,214, 183, and 103 for 1, 2,3,4,5, and 6 respectively. The students claimed that this dice was unfair because “out of 1000 trials 2 numbers have very low percents and the others have high percents.” In response to this Pam states she disagrees, “I wouldn’t consider that difference to be extremely low and extremely high, maybe they think because it’s out of 1000 that is the case. Maybe they think there’s a huge difference between 250 [guessing the height of the bar] and 150. I would say those are pretty fair die.” This indicates that she thinks this variability is due to the randomness of the die. Having more of a subjective view of probability Pam believed the dice were fair because the frequencies to her seemed random. Even when the frequencies were displayed graphically she could not see that 1 and 6 were rolled less often than the other 4.

In her reflection on this interview Pam expressed that she learned a lot about how to teach probability. Such as “open-ended poster creation allowed teacher to see what and how students were thinking.” She also mentioned that when assessing student work, teachers need to be aware of students’ background. They may give input that is not accurate and this can make teaching difficult. Again, this points to a subjective view, she believes some students base their claims on inaccurate background knowledge.

Discussion

These findings support the claims made by Batanero et. al. (1998) that students’ have difficulty understanding randomness. In addition, one’s understanding of randomness is related to one’s perspective of probability. After participating in the first and second interviews, writing

a reflection, and completing their methods course (which had a unit on probability and statistics) all of the three preservice teachers were able to give a more complete definition of random. However, their confidence in their abilities to define random remained low.

Through the analysis of the second interview we can see that, for this particular preservice teacher, her perspective of probability influenced her pedagogical decisions. In addition her understanding of randomness led her to over look the pattern in the data. Results of this study indicate a need for more emphasis on the concept of randomness in teacher education. Through the use of pedagogically oriented tasks teacher educators can increase understanding of randomness as well as an awareness of the different perspectives of probability. By being knowledgeable of the perspectives future teachers will be better able to assess student understanding.

Teacher educators can use the tasks from these interviews to stimulate discussion within mathematics education classes. The use of these tasks could possibly raise further questions into the nature of beliefs of randomness and direct improved instruction in this area. Questions for further research include: 1) what experiences in teacher education could foster a deeper understanding of randomness? and 2) how does a teachers' understanding of randomness affect students' understanding?

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