

# Mathematics Language Skills of Second Year Analysis Students

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## Abstract

The purpose of this study is to determine mathematics language skills of second year analysis students taking Analysis IV course in Elementary Mathematics Education Program at Anadolu University. The study was administered during the second semester of the 2006-2007 academic year. A test including four essay type items was prepared. This test whose items was on writing and reading mathematical sentences, and understanding mathematical notations was administered to 64 students. The results of this test are reported in this study.

## Introduction

“Mathematics” can be described variously, and one of them is that it is a language. Moreover, it is a universal language. As Aiken (1972) states, beside linguistic abilities affect performance in mathematics, mathematics itself is a specialized language. As many other languages, it has syntactical and rhetorical structures. Its rhetorical structure consists of indefinite terms, definite terms, axioms, and theorems. Furthermore, mathematics is also a discipline. It requires systematic thought. According to Jamison (2000), even if it is possible, systematic thought does not mean reducing everything to symbols and equations. He adds that systematic thought also requires precise verbal expression. We teach mathematical concepts in so many various mathematics courses with this language. Elementary courses include procedural calculations, or rather fewer symbols. By increasing symbols, definitions, theoretical notions; deeper mathematical thoughts are needed in higher level mathematics courses. Is it possible to grasp those higher level mathematics notions without understanding the language of mathematics? As Jamison states, if students understand how things are said, they can better understand what is being said, and then they have a chance to know why it is said. Moreover, is it possible to teach mathematics without using this language effectively? Esty and Teppo (1994) say many students can apply procedures, but few of them can express procedures. That is, students’ understandings of not only the mathematical concepts but also the syntactical and rhetorical structures of mathematics are important from the outset to gain deeper mathematical thoughts. Esty and Teppo’s study (1994) reports on The Language of Mathematics (Esty, 1994), a course at Montana State University. Esty had taught the course to many initially “math-anxious” college students who overcame the anxiety and their past failures when they studied the language (Esty, 1992). Both linguistic and logical aspects of the language of mathematics are discussed in this course (Esty and Teppo, 1994).

## Method

The purpose of this study is to determine mathematics language skills of second year analysis students in Elementary Mathematics Education Program at Anadolu University. We chose these students, because they had taken most pure mathematics courses of Elementary Mathematics Education Program. The study was administered during the second semester of the 2006-2007 academic year. A test including four essay type items was prepared and administered to 20 students taking a selective course for reliability. Authors assessed the results and checked if there is any misunderstanding due to wording. Because writing or reading in mathematics was evaluated with the test, assessments were consistent with coding writing, reading or meaning the

mathematical statements or symbols. The test whose items were as follows was administered to 64 students.

1. Write the following sets as  $\{x \mid \dots\dots\}$ .
  - a. The set of rational numbers which are bigger than -1 and less than 11.
  - b. The set of real valued decreasing functions whose domains are  $[a,b]$ .
  
2. Write the following propositions with mathematical symbol and mathematical statements.
  - a. The sum of two rational numbers which are less than zero is less than zero, and the multiplication of them is bigger than zero.
  - b. Let  $f$  be real valued continuous function on  $[a,b]$ . If the multiplication of values of  $f$  at  $a$  and  $b$  are negative, then there exist at least one root of the equation  $f(x)=0$  on the open interval  $(a,b)$ .
  
3. Write the verbal statements of the following mathematical statements.
  - a. Let  $f : \mathfrak{R} \rightarrow \mathfrak{R}$ . For  $\forall x \in \mathfrak{R}$ ,  $f(x) \neq 0$
  - b. Let  $f : \mathfrak{R} \rightarrow \mathfrak{R}$  and  $x_0 \in \mathfrak{R}$ .  $\lim_{x \rightarrow x_0} f(x) = L$ ,  $L \in \mathfrak{R} \Leftrightarrow \lim_{x \rightarrow x_0^-} f(x) = \lim_{x \rightarrow x_0^+} f(x) = L$
  
4. Write the meaning of the following mathematical statements.
 

a. $f(g(x))$	f. $\frac{\partial z}{\partial x}$	k. $\frac{d}{dx}(x^2 + 3)$
b. $f^{-1}(x)$	g. $\frac{\partial^2 z}{\partial x \partial y}$	
c. $f'(3)$	h. $\lfloor x \rfloor$	
d. $f'(x)$	i. $ x $	
e. $\frac{dy}{dx}$	j. $D_{xy}f$	

### Conclusion and Discussions

As known, the truth set of a mathematical expression which can be a mathematical open sentence, a proposition, a theorem, or a definition and the limitations which are properties of a mathematical object or a process are crucial. But unfortunately, it has been concluded in this study that most students ignore not only the truth set of a mathematical expression but also the limitations. In spite of the fact that the set which was required to be describe should be restricted to rational numbers, 20 students used “ $\mathfrak{R}$ ” symbol in the item of 1-b by ignoring the limitation of the number set. Moreover, some students (eight students) did not consider the structure of the truth set and they thought the condition to be decreasing function on real numbers as  $f(x) > f(x+1)$ . Another limitation which was ignored is to be continuous. 20 students neglected stating the continuity of function in the item of 2-b neither with verbal statement nor with symbols. That is, they were not aware of the importance of continuity for the truth of proposition. It is undeniable

that ignoring the truth set and/or its structure, and limitations are obstacles for mathematical deeply thinking.

To reflect mathematical knowledge in the language of mathematics, mathematical symbols such as quantifiers and connectives should be used in place and correctly. They are essential for communication in the language of mathematics. For instance, without using universal quantifier ( $\forall$ ) or existence quantifier ( $\exists$ ) in place and correctly, truth or meaning of a proposition, theorem, definition, or an open mathematical statement can be changed. If we consider the aspect of connectives, it is impossible to differentiate the hypothesis of the proposition from the conclusion. That is, to grasp the proof of a proposition is impossible without the connectives. But unfortunately, a lot of candidates for being mathematics teachers could not use these quantifiers and connectives in place and/or correctly in this study. 38 students didn't use the existence quantifier in the item of 2-b. Nineteen students did not use the symbol of "if...., then...",  $=$ , when they expressed the compound proposition in mathematical symbols in the item of 2-a. Moreover, when reading a compound proposition in the item of 3-b, 20 out of 64 students could not read the connectives of "if and only if",  $\Leftrightarrow$ , correctly. That is, they could not differentiate the hypothesis of proposition from the conclusion.

It is observed that 31 students stated "f(x) function" instead of "f function". Moreover some of them wrote  $f \in \mathfrak{R}$  for a function. Nineteen students out of 31 students used "f(x) function" statement only one time. We thought maybe they used this statement by mistake, but remaining students in the 31 students used this statement several times. Therefore, it is concluded that some students could not distinguish the function from the value of the function at x. In the aspect of function concept, while writing the set of real valued functions whose domains are  $[a,b]$  as  $\{x \mid \dots\dots\}$ , most students, 42 out of 64, used x for the elements of this set. In spite of the fact that denoting a function with x is not common in the language of mathematics, it can be done. But they use "f" symbol for functions in the right side of " $\mid$ " as usual. That is, these errors come from conceptual difficulty that most students could not consider a function as an object. Regarding a process as an object is the most important level of the learning. In terms of the function concept, for instance, regarding some functions which have common properties as elements of a set is a difficulty for most students. As Asiala, Brown, DeVries, Dubinsky, Mathews, and Thomas (1996) stated, functions with split domains, inverses of functions, composition of functions, sets of functions, the notion that the derivative of a function is a function are all sources of great difficulty for students, because of the same reason. We did not use the statements "defining property notation" or "set-builder notation" for the notation of set as  $\{x \mid \dots\dots\}$  in the expression of the question, because most students did not know these pronunciation.

To grasp the equation notion, conceptual and procedural knowledge about the roots of an equation should be related. In our study, it is observed that 19 students did not have the knowledge of the relationship between roots and the equation. It was seen that in spite of the fact that they had experienced various problems about roots of an equation, and we are sure that if we had required the roots of a specific equation, they could have got the roots easily, they could not state the sentence "there exist at least one root of the equation  $f(x)=0$  on the open interval  $(a,b)$ " in the mathematical language.

Beside difficulties mentioned above, it is seen that 17 students stated the proposition which was given with mathematical symbols word by word without considering the mathematical meaning. While expressing a mathematical statement verbally, it should be done according to the

syntactical structure of the mathematics language. Otherwise, some sentences not having mathematical meaning can arise.

According to the conclusions we obtained from this study, it is undeniable that something should be done to improve mathematics language skills of mathematics teacher candidates. Every person uses mathematical knowledge in their life in various levels. Therefore they must be able to explain their mathematical ideas. That is, every person needs to know some mathematics language according to their social position. But, it is obvious that a mathematics teacher should use mathematics language effectively. Therefore, we should ensure that students in education faculties who are candidates of mathematics teacher use this language effectively before graduation. It should be investigated whether it is sufficient to give importance to teaching also the language while giving the mathematical concepts to improve mathematics teacher candidates' mathematics language skills. We suggest offering a mathematics language course in education faculties for mathematics teacher candidates. Furthermore, we claim that students' both algebraic skills and attitudes towards mathematics are improved with a convenient language course. Moreover, the language course for mathematics teacher candidates should include some strategies to teach this language to a convenient level of students whom mathematics teacher candidates will teach mathematics to. In this way, mathematics teacher candidates can gain also self-confidence to teach mathematical knowledge before graduation.

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