The Effect of Using Prediction Questions in the Middle School Algebra Classroom

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Abstract

This paper presents prediction as an instructional means to develop students' mathematical understanding and reasoning. In order to determine the effect of using prediction questions, data from two middle school algebra classrooms taught by the same teacher were gathered during one school year. According to the state test results, the two classrooms were considered comparable at the beginning of the study. During the school year data were collected while two algebra units were taught: in one classroom prediction questions were posed and discussed at the launch of each lesson, and no prediction questions were exposed to the other classroom. After each of the two units, students were given a unit assessment evaluating students' mathematical understanding and reasoning on linear and exponential relationships. These unit assessment data show that the treatment classroom outperformed the non-treatment classroom, which indicates that students with prediction questions developed better mathematical understanding and reasoning.

Introduction

The importance of understanding and reasoning in the teaching and learning of mathematics has been recognized by the mathematics education community all over the world, and yet developing students' ability to think and reason mathematically has not been a trivial task. This paper highlights an initiative to utilize prediction questions as a means of helping students develop mathematical understanding and reasoning.

Prediction is an aspect of reasoning to which researchers in mathematics education have paid less attention compared to other aspects of reasoning, such as justification. Our analysis of the U.S. state standards revealed that prediction was the most prevalent reasoning expectation across grades as well as content strands (Kim & Kasmer, 2006). This suggested that prediction could be an important component of reasoning that could be easily pursued at all grade levels and in all mathematics content strands. It also implicated the sound rationale to investigate the potential of prediction in the mathematics classroom for the development of mathematical understanding and reasoning. As such, we began to explore to what extent and in what ways prediction questions could help students develop understanding and reasoning in the context of middle school algebra.

Related Literature

Prediction is a type of reasoning that can lead to a generalization of patterns and also be derived from a generalization (Kim & Kasmer, 2007). Peirce's (1998) notion of abduction supports the importance of prediction in developing knowledge. According to Peirce, abduction is forming a prediction without any positive assurance, is the only way in which people are introduced to a new idea, and makes a logical connection between deduction and induction.

In various disciplines, prediction has been investigated as a means of helping students' learning. In reading education, a body of research has investigated prediction in the area of reading (e.g., Block, Rodgers, & Johnson, 2004; Palincsar & Klenk, 1991). In such research, students were asked to make a prediction in a reading activity using questions, such as "what do you know about this character that helps you predict what he or she will do next?" and "given the situation in the story, what will possibly happen next?" The results revealed that asking students to make such a prediction helped increase students' comprehension of reading. Gunstone and White (1981) incorporated prediction in science teaching and suggested a prediction-observation-explanation model, which was utilized in several other studies (e.g., Palmer, 1995). Similarly, Lavoie (1999) explored the effect of using prediction in high school biology lessons and suggested a prediction-and-discussion phase be integrated into learning cycles. Interestingly, de Bruin, Rikers, and Schmidt (2007) asked college students to make a prediction when they learned how to play chess in a computer game setting. They found that students who were asked to predict and explain their prediction learned chess principles better than students who only predicted a chess move and students who observed games without making predictions.

In mathematics education, a few studies bring attention to prediction. Battista (1999) found benefits of having students make predictions in 3D geometry lessons. According to him, a discrepancy between predictions and actual answers made students reflect on their strategies and helped build useful mental models. Buendía and Cordero (2005) viewed prediction as a social practice that supports the construction of meaning. They noticed that students developed the meaning of a periodic function while making predictions. Cordero (2006) also found similar results in a context of calculus. Zur and Gelman (2004) found that asking young children to predict answers to arithmetic problems and to check their predictions helped them develop their abilities to add and subtract numbers.

Kim and Kasmer (2007) provided a conceptual framework of using prediction in mathematics education. According to them, prediction not only motivates students' interest, but also provokes prior knowledge. Prediction can help students engage in sense-making (making sense of a problem situation and related concepts); discussions of prediction can encourage alternative perspectives to look at a problem; prediction helps make connections between concepts; making a prediction can provoke visualization of a problem situation and related concepts; prediction can be a useful tool to assess students' thinking.

Methodology

This study incorporated a quasi-experimental design. The data were gathered from students of two middle school classrooms when they learned algebra (linear and exponential relationships) during one school year. These classrooms were taught by the same teacher and the state test results showed that the two classrooms were comparable. In one classroom, prediction questions were asked and discussed at the launch of each algebra lesson. Examples of prediction questions posed in this classroom are, "Which student will get to the frozen yogurt shop first? Where will Terry be when Jade reaches the yogurt shop?" "Would graphs, tables, and equations of this problem look similar to what we have done before?" Such prediction questions were purposefully not incorporated in the other classroom. These algebra classrooms were observed 5-9 times throughout the year. In order to confirm the effectiveness of utilizing prediction questions, students' mathematical understanding and reasoning in the two classrooms were compared using unit assessments. Two unit assessments were administered: one at the completion of a linear unit and the other after an exponential unit. The unit assessments items were drawn from the curriculum the classrooms used and some items were modified to capture each mathematical understanding and reasoning indicator (see Table 1 below). We developed these indicators based on the curriculum (Lappan, Fey, Fitzgerald, Friel, & Philips, 1998a, 1998b) and other literature (e.g., Mullis, Martin, Smith, Garden, Gregory, Gonzalez, Chrostowski, & O'Connor, 2001; NCTM, 2000). Note that the mathematical understanding components specifically align the content, i.e., linear and exponential relationships in the middle school level, and yet the reasoning components are applicable in other content strands.

Additional data were gathered from the teacher. In order to capture her perspective on using prediction questions, we interviewed the teacher four times throughout the study. The teacher also kept weekly journals reflecting on her algebra lessons with prediction questions. These data were also used to confirm or discard inferences made from observation data.

Table 1.
Indicators of mathematical understanding and reasoning

Mathematical understanding	Mathematical Reasoning
U1. Represent patterns in tables, graphs, words, and equations.	R1. Formulate, evaluate, support, and use generalizations. Formulating a generalization is defined as making a statement about something true for any case.
U2. Understand and recognize patterns as linear, exponential, etc.	R2. Construct, evaluate, and support/dispute mathematical arguments. Constructing an argument is defined as making an informal or formal statement about a specific or general case; one form of this is making a conjecture that may lead to a generalization in the end.
U3. Understand the meaning of a representation (an equation, a table, or a graph) as a whole and parts of it.	R3. Analyze/evaluate a problem situation. Analyzing and evaluating a problem situation is defined as making information from the problem useful for solution.
U4. Understand and use the relationship among a table, an equation, and a graph (e.g., a constant in the equation is the y-intercept of a graph and the initial value in the table.)	R4. Use inductive/deductive reasoning to establish/support mathematical relationships. Using inductive reasoning is defined as searching for mathematical relationships through study of patterns while using deductive reasoning is defined as utilizing an established mathematical relationship to support a pattern found in a specific case.
U5. Use equations, graphs, and tables to solve problems and relate the answers to problem situations	R5. Make sense of others' thinking/ideas/approaches and provide rationale behind them. This indicator means understanding others' logic from a critical/evaluative perspective (what the claim is and how it is supported) whether or not the reasoning is acceptable.
U6. Find a pattern (linear, exponential, etc.) in a table/graph and use the pattern to predict for a particular incident	R6. Ask questions and raise challenges in situations of misunderstanding or disagreement. This indicator is about seeking clarification or providing an opposing opinion.
U7. Identify and compare characteristics of tables, graphs and equations of algebraic relationships	R7. Draw and support conclusions in varied topics. This indicator is about making a statement that summarizes the findings that is not necessarily a generalization or an argument.

Students' responses to the unit assessment items were scored by each of us. As the curriculum suggested, for each item, one point was given for a correct answer and two points were for supportive reasoning. Before determining scores for reasoning we discussed expected responses and set the criteria for scoring. We also reconciled our scoring case by case when a question arose. After completing each set of the unit assessments, we examined and discussed each other's scoring to maintain consistency. Once the scoring was completed, independent T-tests were conducted between the two classrooms. In addition, the items were clustered by indicator of mathematical understanding and reasoning, and additional T-tests were administered to see whether or not the treatment students performed better in each indicator.

Results

The results indicate that the prediction questions influenced students' ways of thinking and their approaches to problems. The teacher explicitly acknowledged that prediction questions enabled students to provide their thinking and reasoning without being concerned about the correctness as well as providing opportunities to connect mathematical ideas previously taught with new topics. Prediction questions also helped students understand the problem situation before engaging in the mathematics of the problem, and make sense of the solution in the context of the problem.

The unit assessment results revealed that the treatment classroom performed significantly better, which indicates that they had better conceptual understanding and reasoning (see Table 2). The unit assessment after the first unit on linear relationship showed that the treatment classroom outperformed the non-treatment classroom, and yet it was not statistically significant (t_{38}=1.439, p= 0.079). After the second unit assessment on exponential relationship, the difference between the two classrooms was more evident (t_{36}=2.552, p= 0.015).

Table 2. Results of unit assessments

Unit assessment	Class	*N*	Mean	*SD*	*t*-value	*p*-value
Linear	Treatment	20	16.98	3.39	1.439	0.079
	Non-treatment	20	15.13	4.64		
Exponential	Treatment	19	23.74	3.22	2.552	0.008*
	Non-treatment	19	20.55	4.38		
Combined[#]	Treatment	19	40.61	4.86	2.258	0.015*
	Non-treatment	19	36.16	7.08		

*Significance ≤ 0.05. [#] Combined scores were only from those who took both assessments.

In order to identify specific indicators in which the treatment classroom performed significantly better, assessment items and scores were clustered in terms of mathematical understanding and reasoning indicators. Then, a T-test was conducted for each indicator (see Table 3). Test results helped identify six such indicators: three understanding indicators and three reasoning indicators while the differences in the other indicators were moderate.

Table 3. Combined unit assessment by indicator

Indicator	Class	Mean	*SD*	*t*-value	*p*-value
U1	Treatment	14.45	2.64	2.792	0.004*
	Non-treatment	12.08	2.59		
U2	Treatment	11.55	2.44	2.077	0.023*
	Non-treatment	9.84	2.63		
U3	Treatment	23.92	3.52	1.965	0.029*
	Non-treatment	21.45	4.21		
U4	Treatment	13.39	2.38	0.446	0.329
	Non-treatment	13.02	2.70		
U5	Treatment	10.79	1.99	1.647	0.054
	Non-treatment	9.47	2.86		
U6	Treatment	6.21	1.55	1.353	0.092
	Non-treatment	5.47	1.80		
U7	Treatment	8.18	1.57	1.287	0.103
	Non-treatment	7.53	1.58		
R1	Treatment	24.11	3.69	1.387	0.087
	Non-treatment	22.18	4.78		
R2	Treatment	18.87	2.59	1.773	0.042*
	Non-treatment	17.11	3.47		
R3	Treatment	20.34	2.06	1.601	0.059
	Non-treatment	18.68	4.02		
R4	Treatment	30.84	4.07	1.805	0.040*
	Non-treatment	27.76	6.22		
R7	Treatment	17.05	2.84	2.846	0.004*
	Non-treatment	14.26	3.19		

*Significance ≤ 0.05. Unit assessment items did not incorporate R5 and R6.

Conclusion

The results of the study support that using prediction questions routinely helps develop students' mathematical understanding and reasoning. The two classrooms in this study were taught by the same teacher using the same curriculum. They also were comparable in terms of state test results. However, engaging in making and discussing predictions during algebra lessons throughout one school year, students in the treatment classroom performed better in mathematical and reasoning assessments on linear and exponential relationships. Now, we need to investigate how prediction questions helped these treatment students.

References

Battista, M. (1999). Fifth graders' enumeration of cubes in 3D arrays: Conceptual progress in an inquiry-based classroom. *Journal for Research in Mathematics Education, 30*, 417–448.

Block, C., Rodgers, L., Johnson, R. (2004). *Comprehension process instruction: Creating reading success in grades K-3.* New York: The Guilford Press.

Buendía, G., & Cordero, F. (2005). Prediction and the periodical aspect as generators of knowledge in a social practice framework. *Educational Studies in Mathematics, 58*, 299-333.

de Bruin, A., Rikers, R., & Schmidt, H. (2007). The effect of self-explanation and prediction on the development of principled understanding of chess in novices. *Contemporary Educational Psychology 32*, 188–205

Gunstone, R. F., & White, R. T. (1981). Understanding gravity. *Science Education, 65*, 291-299.

Kim, O. K., & Kasmer, L. (2006). Analysis of emphasis on reasoning in state mathematics curriculum standards. In B. Reys (Ed.), *The intended mathematics curriculum as represented in state-level curriculum standards: Consensus or confusion?* (pp. 89-109). Greenwich, CT: Information Age Publishing.

Kim, O. K., & Kasmer, L. (2007). Prediction and mathematical reasoning. *Proceedings of 2007 Hawaii International Conference on Education* (pp. 3016-3034). Honolulu, HI.

Lappan, G., Fey, J. F., Fitzgerald, W., M., Friel, S., N., & Philips, E. D. (1998a). *Moving straight ahead* (an eighth-grade unit of Connected Mathematics). Menlo Park, CA: Dale Seymour Publications.

Lappan, G., Fey, J. F., Fitzgerald, W., M., Friel, S., N., & Philips, E. D. (1998b). *Growing, growing, growing* (an eighth-grade unit of Connected Mathematics). Menlo Park, CA: Dale Seymour Publications.

Lavoie, D. R., (1999). Effects of emphasizing hypothetico-predictive reasoning within the science learning cycle on high school student's process skills and conceptual understandings in biology. *Journal of Research in Science Teaching, 36*, 1127–1147.

Mullis, I., Martin, M., Smith, T., Garden, R., Gregory, K., Gonzalez, E., Chrostowski, S., & O'Connor, K. (2001). *TIMSS assessment frameworks and specifications 2003*. Chestnut Hill, MA: TIMSS International Study Center, Boston College.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Palincsar, A. S., & Klenk, L. J. (1991). Dialogues promoting reading comprehension. In B. Means, C. Chelemer, and M. S. Knapp (Eds.), *Teaching advanced skills to at-risk students*. San Francisco: Jossey-Bass.

Palmer, D. (1995). The POE in the primary school: An evaluation. *Research in Science Education, 25*(3), 323-332.

Zur, O., & Gelman, R. (2004). Young children can add and subtract by predicting and checking. *Early Childhood Research Quarterly, 19*, 121–137.