

Using a Touchscreen Calculator in a Calculus Course to obtain Multiple Representations of Functions

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Abstract

It is well known that calculus teaching is enhanced if students have access to technology that facilitates multiple representations of functions (i.e. symbolic, graphical and tabular). The Advanced Placement Calculus program emphasizes representations beyond the symbolic. A touchscreen calculator, with drag and drop functionality, makes multiple representations easy. We will demonstrate first the capability of a particular touchscreen calculator, the ClassPad 300, and then illustrate how it can be used to good advantage in solving and discussing problems from recent Advanced Placement Calculus examinations.

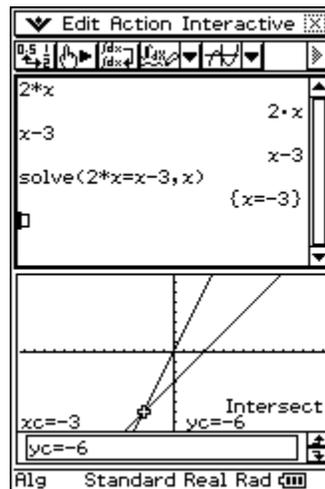
General comments about the ClassPad 300 and its emulator

The ClassPad 300 is a handheld graphing calculator that offers its user a large number of applications from a menu that is displayed when the calculator is turned on. Among these applications are *Main*, *Statistics*, *Graph&Table*, *Geometry*, *3D Graph*, *DiffEqGraph* and *Spreadsheet*. The functions of most of these applications should be self-explanatory from their names, but we note here that the *Main* application offers a powerful computer algebra system (CAS) coupled with robust numerical routines. The numerical and symbolic functionality is enhanced by the calculator's helpful editing capabilities. In this paper, we will focus on the *Main* and *Graph&Table* applications.

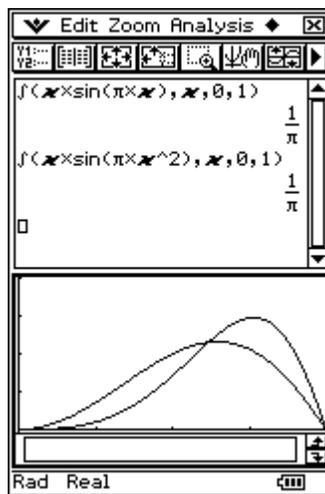
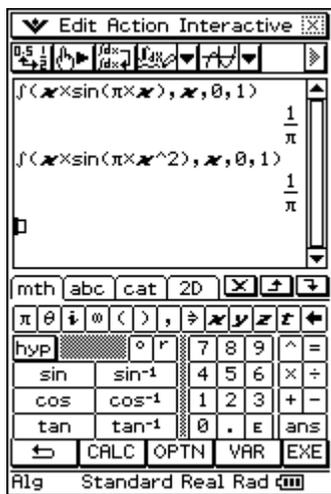
The operating system of the ClassPad 300 is replicated in the ClassPad Manager software. The software, which runs on Windows computers, is useful for classroom displays since it allows the user to project simultaneously both an image of the physical calculator and an enlarged image of its display. The software has a screen capture feature which is very helpful for preparing documents. Indeed, the images in this paper are screen captures from the ClassPad Manager.

A quick overview of some relevant features of the ClassPad 300.

Example 1: The screen capture displayed at the right shows, in a simple situation, how the ClassPad 300 can be used to illustrate the interplay between algebra and (analytic) geometry. Before going into the specifics of the example, we need to introduce some terminology. We began in the *Main* application. The corresponding window is the upper half of what you see at the right. We then tapped the *Graph & Table* button to display the window containing the coordinate plane that is the lower half of what you see at the right. We will refer in the following to the “Main window” and the “Graph window”. Having set up the two windows, we entered the two linear expressions shown in the Main window and then “dragged and dropped” the two expressions into the Graph window. Next, in the Graph window, we used the sequence Analysis>G-Solve>Intersect to find the coordinates of the point common to the two lines. Finally, we went back to the Main window and used the *solve* function to find the *x*-coordinate of the point of intersection.



Example 2: This example would be more relevant in a calculus class than the one above. Consider the two following screen captures.



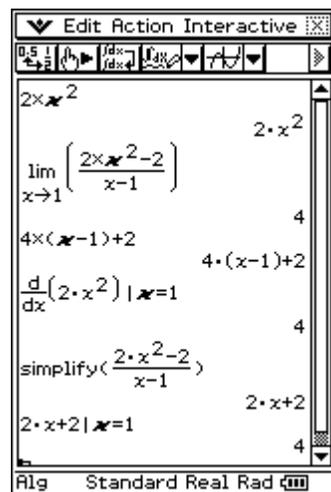
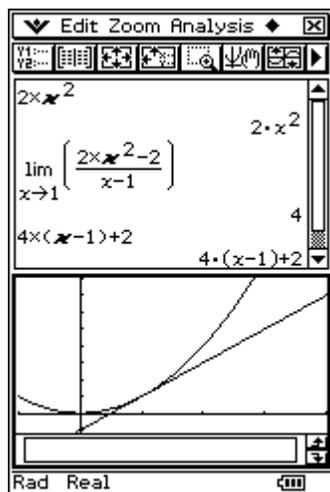
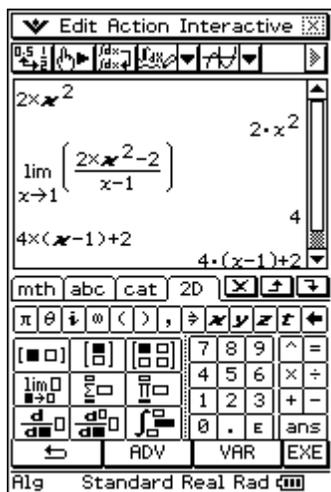
The screen capture at the left reveals a somewhat surprising result, i.e.

$$\int_0^1 x \sin(\pi x) dx = \int_0^1 x \sin(\pi x^2) dx.$$

Of course, the powerful editing features of the ClassPad 300 make it easy to enter the second definite integral once the first has been entered. Note also that the values of the integrals are given exactly, not numerically, and that we used one of the many keyboards offered by the ClassPad 300 to enter our integrals. [In our next example, we will use the 2D keyboard.]

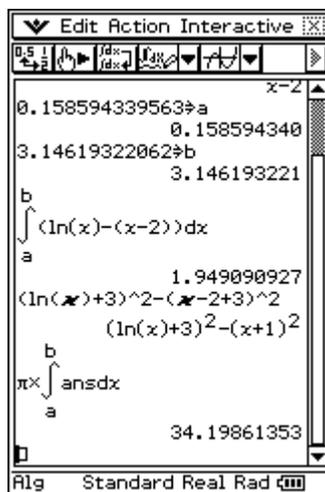
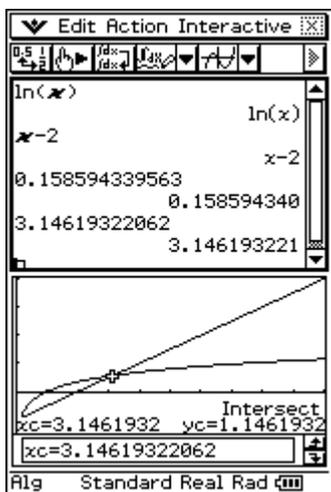
Having observed that the two integrals have the same value, an instructor should then ask his class why this should be the case. The screen capture at the right shows the graphs of the two integrands over the interval $[0,1]$ and reveals that it is certainly plausible that the areas under the two graphs are equal and hence the integrals should be equal as well. We note that we obtained the graphs by dragging and dropping the integrands from the Main window into the Graph window.

Example 3: In this final example in this section, we illustrate how the ClassPad 300 might be used in a classroom lesson on the geometric significance of the derivative of a function at a point. In the screen capture at the left below, using the 2D keyboard, we introduce the function $f(x) = 2x^2$, compute the limit of $\frac{f(x) - f(1)}{x - 1}$ as x goes to 1, and then produce the linear function whose slope is equal to the value of the limit and which also goes through the point $(1, f(1))$. In the screen capture in the middle below, we have used the drag and drop feature to graph our function and what we believe is its tangent line at the point $(1,2)$. The graph shows that the line almost certainly does have the desired properties. Finally, the screen capture at the right below extends the algebraic computation from the screen capture at the left. First, we used the derivative function from the 2D keyboard and the $|$ function to find the derivative of our function at $x = 1$, reconfirming that this value is 4. We then algebraically simplified the difference quotient $\frac{f(x) - f(1)}{x - 1}$ observing then that it was obvious why the limit of the quotient is, indeed, equal to 4, as x goes to 1.



The Classpad 300 in the context of two problems from the 2006 AP Calculus Free Response Examination

Problem AB-1: In this problem, the student was asked several questions about the region R which is bounded above by the graph of $y = \ln(x)$ and below by the graph of $y = x - 2$. The student was given the graphs of the two functions and the region R was labeled. However, as the screen capture at the left below shows, the region is easily found using the ClassPad 300. We used the Zoom>Auto feature to obtain the optimal y-Range for the (overly generous) x -Range we used, i.e. $(0,10]$. We also used, in the Graph window, Analysis>G-Solve>Intersect (twice) to find the x -coordinates of the two points of intersection, and copied and pasted these values into the Main window. [The larger of these two values is not π , as some students who took the examination seemed to believe.] Now, to find the area of the region R , which is part (a) of the given problem, we can copy and paste the integrand and the limits into the appropriate integral, as shown in the screen capture at the right below. [We first stored the limits in the variables a and b .] This second screen capture also contains the solution to part (b) of the problem, i.e. to “find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.” [There was a part (c) in the problem, but it did not require any computation.]



Problem AB-2: This problem was much more interesting than problem AB-1. Students were told that the rate at which cars turn left at a certain intersection is given by

$$L(t) = 60\sqrt{t} \sin^2(t/3) \text{ cars per hour over the time interval}$$

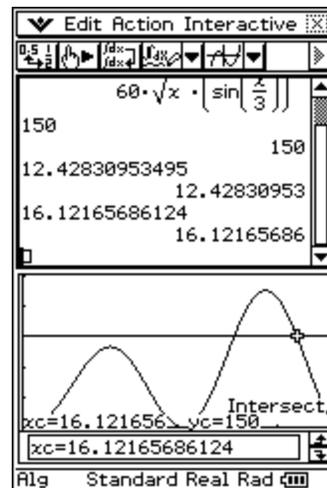
$0 \leq t \leq 18$ hours. Part (b) of the problem required students to find the values of t for which $L(t) \geq 150$ and then compute the

average value of L over the resulting time interval. The ClassPad 300 handles this problem very nicely. The screen capture at the right shows the graphs of the given function and the horizontal line $L = 150$. As in the previous example, we found the t -coordinates of the points of intersection and pasted them into the Main window. Note how convenient it is to see the graph and the relevant t -values at the same time. To

complete the problem, the student needed only to store the t -values as a and b and then

evaluate $\frac{1}{b-a} \int_a^b L(t) dt$. This computation is not shown, but the ClassPad 300 quickly

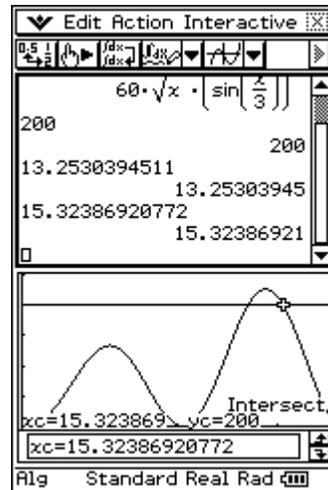
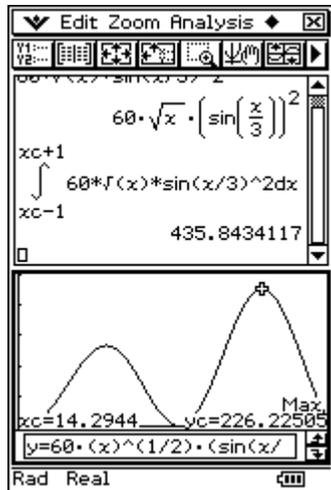
returns the necessary answer, i.e. 199.4261161 cars per hour.



Part (c) of this problem was the most interesting of all three parts. Students were asked to determine whether there is a “two-hour time interval during which the product of the number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000.” Students were told that “in every two-hour time interval, 500 oncoming cars travel straight through the intersection.” Thus, in effect, students were asked whether there is a two-hour time interval during which the number of cars turning left is 400 or more. In other words, students needed to decide

whether there is a value of h such that $\int_h^{h+2} L(t) dt > 400$. In the screen capture at the left

at the top of the following page, we used Analysis>G-Solve>fMax to discover that the maximum value of $L(t)$ occurs at $t = 14.2944$. [The ClassPad 300 stores this as xc .] Then using this value as h we discovered that a time interval with the required properties does indeed exist. However, the screen capture at the right at the top of the following page provides a much more elegant solution, one which is relatively easy to find using the features of the ClassPad 300. The alternative solution is based on the observation that if the average value of L over some two-hour time interval is greater than 200 (i.e. half of 400), then the number of cars passing through the intersection during this time interval will be greater than 400. This train of thought suggests that we should graph $L(t)$ and $L=200$ simultaneously, which we did in the screen capture at the right, and find the t -coordinates of the points of intersection. If these t -values differ by 2 or more, then $L(t)$ exceeds 200 over an interval of length greater than 2, and hence the average value of L over this time interval is greater than 200. The screen capture at the right thereby confirms our original answer to the given question.



A final comment

We have not discussed or demonstrated the table functionality of the ClassPad 300, but it is at least as powerful as any other graphing calculator on the market. However, the ClassPad 300 also has a built-in spreadsheet, with very powerful functionality, including the ability to plot the values contained in, or computed in, a table.