The Use of Research-Based Methods and Materials for Preparing to Teach Mathematics with Technology

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Abstract
Preparing middle and high school mathematics teachers for technology-equipped classrooms is a complex task. This paper discusses our integrated approach to develop materials for mathematics teacher education, provides sample materials, and shares initial research results.

Technology in Mathematics Teacher Preparation

In its most recent document, the National Council of Teachers of Mathematics (NCTM, 2000) states, "technology is essential in teaching and learning mathematics; it influences what is taught and enhances students' learning" (p. 24). Whether technology will enhance or hinder students' learning depends on teachers' decisions when using technology tools that are often based on knowledge gained during a teacher preparation program. The Association of Mathematics Teacher Educators asserts teacher preparation should "provide opportunities [for teachers] to acquire the knowledge and experiences needed to incorporate technology in the context of teaching and learning mathematics" (AMTE, 2006, p. 1). We believe these opportunities in teacher preparation should focus on developing teachers': 1) knowledge of mathematics, 2) knowledge of technology, 3) pedagogical and pedagogical content knowledge, and 4) understandings of students' mathematical understandings when using technology tools.

Teacher education and research on teachers has been greatly influenced by Shulman's (1986) idea of teachers' pedagogical content knowledge (PCK). For example, Simon (1995) uses PCK to describe components of a mathematics teaching cycle that includes a teacher's knowledge of: mathematics, activities and representations, students' learning of content, and their hypotheses about students' current knowledge. More recently, Koehler and Mishra (2005) and Niess (2005) have described technological pedagogical content knowledge (TPCK) as the integration of teachers' knowledge of content, pedagogy and technology and is needed to effectively use technology to teach specific subject matter (Figure 1).

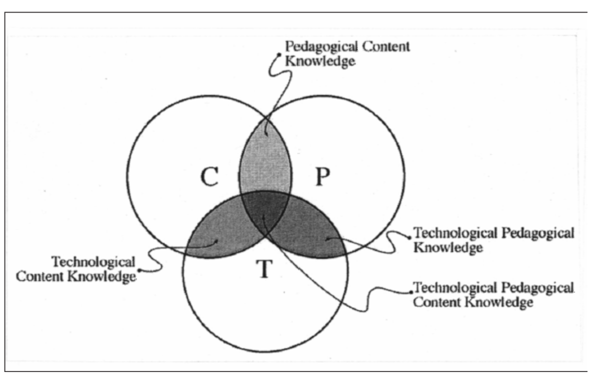

Figure 1. Components of TPCK (Koehler & Mishra, 2005, p. 133).

With a focus on the intersection of the three components of technology, content, and pedagogy, Niess (2005) describes four different aspects that comprise teachers' TPCK:

1. an overarching conception of what it means to teach a particular subject integrating technology in the learning process;

2. knowledge of instructional strategies and representations for teaching particular topics with technology;
3. knowledge of students' understandings, thinking, and learning with technology; and
4. knowledge of curriculum and materials that integrate technology with learning.

These four aspects of TPCK essentially extend Simon's (1995) components of teachers' knowledge in a mathematics teaching cycle by incorporating a focus on technology.

Given the changing nature of technology, it is important that teachers develop a model of teaching and learning that goes beyond the specifics of a technology tool so that they are able to make informed decisions about appropriate uses of technology in mathematics. Following from a model of the components of TPCK (Koehler & Mishra, 2005; Niess, 2005), we believe such a model should integrate mathematics, technology, pedagogy, with a focus on student thinking. Thus, a key feature in preparing teachers to teach mathematics with technology is to integrally develop teachers' TPCK. Teachers need to understand that instructional decisions they make are grounded in their understandings of each domain (technology, pedagogy, and content) and influenced by their beliefs and conceptions.

Curriculum Materials for Developing Mathematics Teachers' TPCK

By integrally developing teachers' understanding of mathematics, pedagogy, technology, with a focus on student thinking, we hypothesize that teachers will develop a more complete picture of what is needed when teaching mathematics with technology, and in turn be prepared to make informed decisions about appropriate uses of technology. In the materials, *Preparing to Teach Mathematics with Technology: An Integrated Approach,* our project[1] intends to create three modules that could be distributed separately and used in college mathematics education methods courses, mathematics courses, or in professional development workshops to prepare teachers to teach mathematics with technology. The modules will provide opportunities for middle and secondary mathematics teachers to develop: 1) deeper conceptual understanding of school mathematics topics, 2) proficiency in using technology tools, 3) effective pedagogical techniques, and 4) abilities to analyze students' thinking when using technology tools to solve mathematical tasks. The modules are not designed to be used directly by teachers with their grades 6-12 students. Rather, it is anticipated that when teachers complete the modules they will have the knowledge needed to create their own activities to meet the needs of their students.

The three modules focus on the teaching and learning of: 1) Data Analysis and Probability, 2) Geometry, and 3) Algebra. Thus far, we have completed the development and evaluation of *Module 1: Data Analysis and Probability*. *Module 1* is currently under review for possible commercial publication and several components have been field tested at other universities. The three modules will include the use of technology tools such as graphing calculators, computer algebra systems, spreadsheets, *Fathom*, *TinkerPlots*, *Geometer's Sketchpad*, and *Probability Explorer*. The three mathematical domains are selected because of their amenability to the use of technology tools, the current need for teachers to have a deeper understanding of concepts in these areas, and the importance of these topics in the learning and teaching of middle and high school mathematics. Each module is designed to be completed in approximately 5-6 weeks (about 15-20 hours). The three modules could together form the curriculum for a course on "Teaching Mathematics with Technology."

Research suggests providing teachers with a mathematics problem they need to solve as a learner of mathematics and asking them to reflect upon their thinking and consider their work from a student's perspective is an effective strategy that we employ in our instructional modules (Simon & Tzur, 2004). However, prospective teachers often lack experience working with students using technology that enable them to envision how a student may solve a mathematics problem with a tool in ways that may differ from their own solution path and anticipate difficulties they may encounter. The inclusion of pedagogical questions in the text and students' work via videocases enables us to direct teachers' attention to how students are thinking when they have access to technology. A central feature in all three modules will be the inclusion of videocases depicting students' work with technology tools to provide opportunities for teachers to analyze students' work and develop understandings about the way in which technology tools, tasks, and teacher interventions influence students' mathematical thinking.

In Figure 2, an excerpt from Chapter 3 in *Module 1* is provided where preservice teachers (PSTs) are analyzing data from a sample of vehicles released in 2006. As PSTs are learning to use the technology (*Fathom*) to analyze data and answer questions, they are introduced to how certain approaches can help students. Also, they are explicitly asked a pedagogy question focused on how a graphical representation could influence students' data analysis. Although PSTs may struggle in responding to this question, the presence of such questions throughout our text create opportunities for pedagogical perturbations that can prompt reflection and critical thinking. These perturbations and reflections may help prospective teachers develop knowledge that lies at the intersections in the Venn diagram in Figure 1.

We believe the modules could be used in courses in mathematics education or mathematics, and that the "modularized" approach could facilitate a wider impact. For example, a mathematics department may want to use the Data Analysis and Probability module within a course on "Statistics and Probability for Teachers." Institutions that are limited to having only 1-2 mathematics methods courses may choose to integrate several sections from a particular module in a course. In addition, a single module could form the basis of a professional development course for practicing teachers.

Chapter 3-Section 3: *Comparing Distributions Using Center and Spread*

Thus far, we have explored the City mpg for the entire aggregate of vehicles. From our analysis, we observed that some types of vehicles have better City mpg than others. In particular, we previously noticed that the four cases considered as outliers were all Hybrid engines. Our data set contains vehicles of three different Engine types: Standard, Diesel, and Hybrid. When students make an observation like this about a data set, it often prompts them to explore a new question. This is an important feature of EDA [Exploratory Data Analysis]—analysis of data leads to more questions, which leads to further analysis. Consider the following question:

Which type of engines give vehicles the best fuel economy in the city?

To examine this question, we need to use two attributes in the data set: City mpg and Engine type. We now have a question that needs us to use bivariate data with one quantitative attribute (City) and one qualitative attribute (Engine). Having students examine one quantitative and one qualitative attribute together in a data set can provide a transition into them working with bivariate data (two attributes) to answer a question.

One way to begin examining the data with attention to the two attributes is to overlay the qualitative attribute on top of the dot plot of the distribution of the City mpg. This action will recolor the icons according to the categories of the qualitative attribute and display a legend explaining the coloring.

To overlay a legend attribute to a graph:

1. click and drag the name of an attribute form the case table and point to the interior of the plot window. Directions will appear as shown in Figure 3.10. You only need use the Shift or Ctrl keys if it is not clear which type of attribute you are dragging, or if you want to purposely use an attribute a specific way (e.g., if the categories of a qualitative attribute have been entered using numeric codes such as 1, 2, 3, you may have to use the Shift key to force *Fathom* to recognize the data as categorical).
2. Release the mouse and notice the appearance of the legend and that different shapes and colors are represented (see Figure 3.11). If the legend attribute is qualitative, shapes and colors will be used, if the attribute is quantitative, a color gradient will appear.

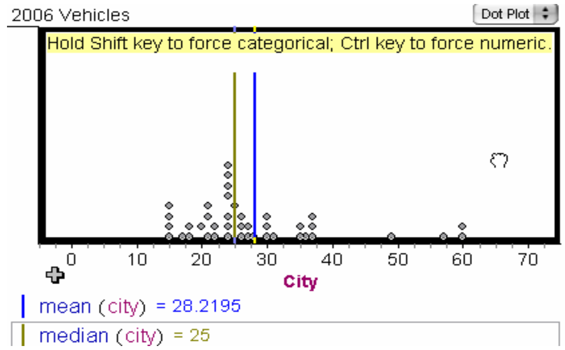

Figure 3.10

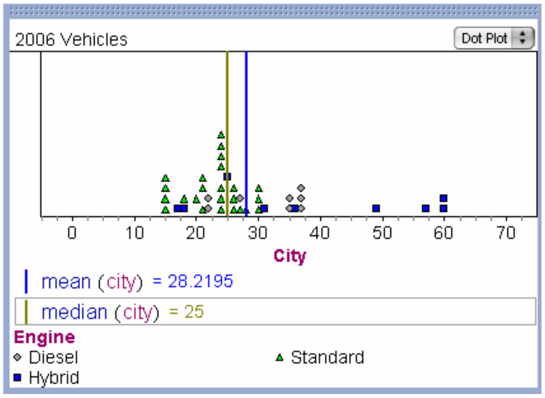

Figure 3.11

FOCUS ON MATHEMATICS

M-Q8. Viewing Figure 3.11, what can you say about the City mpg for each of the three Engine types?

FOCUS ON PEDAGOGY

P-Q7. How can overlaying a categorical (qualitative) attribute on a dot plot of a numerical (quantitative) attribute influence students' ability to examine data?

The graph in Figure 3.11 is good way for students to begin to coordinate two attributes in a data set, and thus is a first step in learning to conduct bivariate data analysis where one variable is quantitative and the other is qualitative. The graph can help students begin to compare three distributions.

Figure 2. Excerpt from *Module 1* that illustrates our integrated approach.

Results from Field Tests of Materials

As we began the development of *Module 1*, we used students enrolled in the course "Teaching Mathematics with Technology" in Fall 2005 (n=15) as the control group. This course is offered 3-4 times per year in a computer lab and regularly serves middle and secondary prospective teachers and a few beginning graduate students with little experience using technology. During the five-week unit on Data Analysis and Probability, the instructor in Fall 2005 (not one of the project PIs) used the pre-existing established curriculum for the course. The students took a pretest and posttest designed to assess content, pedagogical, and technology knowledge related to data analysis and probability. Over the next three semesters, the materials for *Module 1* underwent a cycle of implementation and revisions. In each of the subsequent semesters (Spring 2006, Fall 2007, Spring 2007), the five-week unit on Data Analysis and Probability was taught by the same instructor as Fall 2005. In addition, in Spring 2007, the Module was implemented in a section of the course taught by a different instructor. In each semester, written work was collected from students and pre and post tests were given. During some semesters, the class sessions were videotaped and several students were interviewed. What follows is a brief discussion of the comparisons across semesters as only measured and indicated by the gains in students' scores from pre to post test (see Table 1).

		Control Fall 2005 n = 15	Experimental I Spring 2006 n = 18	Experimental II Fall 2006 n= 15	Experimental III Spring 2007 n = 32
Overall	Mean	10.5	12.583	10.4	10.781
50 points	Median	10	11.75	8	12
	StDev	5.17	4.876	6.65	5.857
Content	Mean	0.333	1.833	1.733	2.281
subsection	Median	0.5	1.75	2	2
20 points	StDev	1.665	2.307	2.897	2.426
Pedagogy	Mean	2.367	2.305	3.13	2.656
subsection	Median	2	2	2.5	2
14 points	StDev	2.662	2.641	3.73	2.497
Technology	Mean	5.6	8.444	5.53	5.844
subsection	Median	7	8.5	8	6
16 points	StDev	4.27	3.569	3.72	3.593

Table 1. Center and Spread for Gain scores in Control and Experimental Groups

Due to the small sample sizes, a Wilcoxon Rank Sum test was used to compare the differences in gains from the pre to posttest with an alpha level of 0.10. The gains experienced by students in Experimental I (n=18) were significantly higher (p=.10) than the Fall 2005 Control group (n=15), specifically with items related to content knowledge (p=.007) and technology (p=0.058). While the effects of the materials may have been mitigated in part by the composition of the students (i.e. graduate students who are practicing teachers) in the control group, a post hoc analysis of the materials revealed places where additional pedagogical issues could be raised and discussed. These issues were addressed in the revisions during the summer 2006 which included integrating more text and questions directly related to issues of teaching and learning data analysis and probability. The newly revised materials were tested in Fall 2006 (Experimental II, n=15) with a drop in overall, content and technology gain scores, and an increase in pedagogy gain scores (although not significantly different). An interesting component to the gain scores in Fall 2006 is the wide variation among students' scores (reflected in the standard deviations in all sub sections and overall) that indicate that several students had much greater gains after using the module than some of their peers in the same class.

Several minor revisions were made to *Module 1* before implementation in Spring 2007. Spring 2007 was the first semester that all students received a bound copy of the module to serve as a textbook for reference in and out of class. The overall gain score in Experimental III (n=32) was barely above the 0.1 significance level (p=0.134) in comparison to the Experimental II group. In addition, the gain scores in the content subsection for Experimental III students were significantly higher than those in Control (p=0.001), Experimental I (p=0.048) and Experimental II (p=0.074). Although there was a slight drop in the pedagogy gain scores from Experimental II, the Experimental III pedagogy scores were still higher than those in the Control or Experimental I groups, with slightly less variability. The small sample sizes in all our groups contribute to the difficulty in making any claims based on this data. However, the qualitative data (video, written work--including lesson plans, and interviews) suggest that students' understandings about learning and teaching data analysis and probability with a variety of technology tools is improved by their use of the *Module 1*, and we plan to continue tracking students' results as the Module is implemented in subsequent semesters at many universities.

Conclusion

With technology becoming a ubiquitous part of our daily experiences, it is important for mathematics teachers, many of whom are "digital immigrants," to build on the experiences of "digitally native" students (Prensky, 2002, p. 1). To do so, teachers need to know how to capitalize on the power of technology to create lessons that assist students in developing understandings of mathematics. We believe an instructional model that engages prospective teachers in solving mathematics tasks using technology tools and encourages them to reflect on those experiences from the perspective of a teacher provides an integral learning experience that is similar to what they will encounter when placed in a classroom. By developing prospective teachers mathematical TPCK, we believe that we preparing them for the classrooms of today and equipping them with knowledge and skills to navigate within the classrooms of tomorrow.

Notes

[1] The *Preparing to Teach Mathematics with Technology* project is supported by the National Science Foundation under Grant No. DUE 04-42319 awarded to North Carolina State University. Any opinions, findings, and conclusions or recommendations expressed herein are those of the authors and do not necessarily reflect the views of the National Science Foundation. More information can be found at http://www.ncsu.edu/project/ptmt.

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