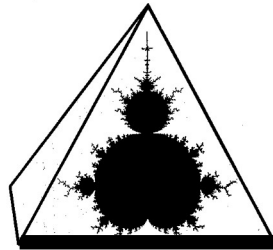


The Mathematics Education for the Future Project



Proceedings of the 14th International Conference

Challenges in Mathematics Education for the Next Decade

Sep. 10–15, 2017, Hotel Annabella, Balatonfüred, Hungary

**Edited by Janina Morska and
Alan Rogerson**

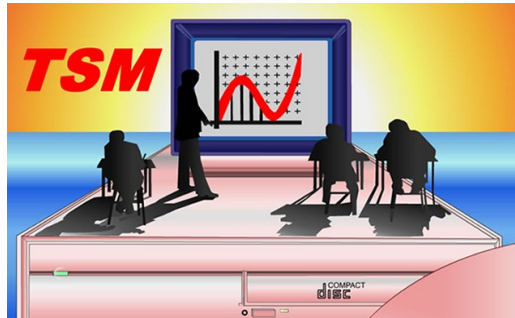
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Foreword

This volume contains the papers presented at the International Conference: *Challenges in Mathematics Education for the Next Decade* held from September. 10–15, 2017 at the Hotel Annabella, Balatonfüred, Hungary.

The Conference was organized by The Mathematics Education for the Future Project - an international educational project founded in 1986 and dedicated to the improvement of mathematics education through the publication and dissemination of innovative ideas. Many prominent mathematics educators have supported the project in the past: Hans Freudenthal, Andrejs Dunkels, Hilary Shuard, Bruce Meserve, Marilyn Suydam, Alan Osborne, Margaret

Kasten, Mogens Niss, Tibor Nemetz, Ubi D'Ambrosio, Brian Wilson, Tatsuro Miwa, Henry Pollack, Werner Blum, Roberto Baldino, Waclaw Zawadowski, and many others throughout the world.

We are pleased to mention two extra initiatives connected with the conference:

(1) Longer Research Papers were contributed by the following authors: Booker, Crabtree, Graumann, Haghverdi, Lousis and Whittles. Each of these longer versions expands on their papers in these proceedings and they can be found at: <http://directorymathsed.net/public/HungaryConferenceDocuments/> . In addition this webpage contains an extra workshop summary contributed by Hilary Povey but not presented at the conference.

(2) Deserving Scholars and Researchers were, for the third time in our conferences, invited to submit papers for peer review and publication in the proceedings even though they were unable to attend the conference itself. Instead their papers were included in the conference programme and were briefly presented by others. The papers involved in this initiative were those presented by lead authors Chadjipadelis, Cifarelli, Narayanan, Pourdavood, Radović, Takahashi and Whittles. Since we were unable to meet these authors face to face in Hungary we included their photographs and short CVs in the above Hungary Conference Documents webpage.

We sincerely thank all of the contributors for their time and creative effort. It is clear from the variety and quality of the papers that the conference has attracted many innovative mathematics educators from around the world.

We are especially grateful to Professor Martin Stein of Münster University, the Owner and Manager of the company that publishes these printed proceedings: WTM-Verlag (Wissenschaftliche Texte und Medien – scientific texts & media).

These Proceedings begin with the Plenary Paper and Workshops by Douglas Butler, followed by a contents list and then the papers & workshop summaries in alphabetical order of the principal authors.

Dr. Alan Rogerson

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Plenary Keynote Address: Is Technology Taking Mathematics Education in the Right Direction?

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There are numerous examples, in many fields, of spectacular advances in technology getting in the way of progress. In mathematics education, this can present itself through a plethora of new titles that can all too easily offer students clever short-cuts before they have a sound enough understanding. Most digital resources have their origins in the web, but the internet is a public

and totally unregulated network, allowing open source software, schemes of work and entire courses to appear without any moderation. This presentation will attempt to tread a cautious path through all this, suggesting standards by which students and teachers should evaluate digital resources before they use them. It is particularly important to choose dynamic software that has a sympathetic user-interface, allowing the mathematics to shine through.

Workshop 1. Douglas Butler: Using Digital Resources for Handling Large Data Sets and Understanding Basic Statistical Principles.

This hands-on workshop will start by exploring large data sets listed on Douglas’ website, TSM Resources, ranging from earthquake data to the late Hans Rosling’s “Gapminder”. We will practice extracting this data to Excel, and then to Autograph for statistical analysis. The basic principles of linear regression will be explored, starting with a small cluster of points. We will then consider the statistical principles involved in studying the Binomial, Poisson and Normal distributions. Finally, the Central Limit Theorem will be put to the test starting from a simple uniform data set, and leading on to a complex application involving a large set of passenger data in aircraft. Participants in this workshop will receive a complimentary copy of Autograph 4.

Workshop 2. Douglas Butler: Using Dynamic Software to Help Students Visualise Key Principles Using a Sympathetic User Interface

With the advent of widely available open source software it is important to discuss what represents a sympathetic user interface for dynamic software. This workshop will explore the user interface of the new Autograph 4, and consider its use in a number of key topics in secondary and college mathematics. Firstly, complex numbers: when explored dynamically on the Argand Diagram they will be ‘imaginary’ no more! The key principles of 3D vectors can be so much better understood by studying them in 2D first, but with 3D in mind. Another topic that benefits from Autograph’s interface is first and second order differential equations – so visual, so relevant and motivational. Further topics will draw on the need to introduce students to problem solving. Participants in this workshop will receive a complimentary copy of Autograph 4.

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