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WHAT ROLES DO MATHEMATICS TEACHERS PLAY WHEN THEIR STUDENTS SOLVE PROBEMS?

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Abstact: This study examined mathematics teachers' roles and interventions with their students' work while engaged in problem solving (before, during, and after). The sample for the study consisted of 10 elementary mathematics teachers from Abu Dhabi Educational zone. Based on literature, an observation form was developed and used in the study. Teachers were videotaped while monitoring students' work on a problem. Videotapes were analyzed based on the observation form. The results of the study showed that generally teachers did not play roles that support problem solving or help students become problem solvers. These results suggest that mathematics teachers need to be extensively trained on teaching mathematics via problem solving.

Introduction

The national council of teachers of mathematics NCTM requires instructional programs to enable all students from building new mathematical knowledge through problem solving; solving problems that arise in mathematics and in other contexts; applying and adapting a variety of appropriate strategies to solve problems; and monitoring and reflecting on the process of mathematical problem solving. NCTM recommends that students in grades 3–5 should have frequent experiences with problems that interest, challenge, and engage them in thinking about important mathematics (NCTM, 2000). Problem solving is not a separate topic, rather it is a process that should permeate the study of mathematics and provide a context in which concepts and skills are learned (NCTM, 2000).

Teachers help their students become problem solvers if they select rich and appropriate problems, orchestrate their use, and assess students' understanding and use of strategies. Students are more likely to develop confidence and self-assurance as problem solvers in classrooms where they play a role in establishing the classroom norms and where everyone's ideas are respected and valued. These attitudes are essential if students are expected to make sense of mathematics and to take intellectual risks by raising questions, formulating conjectures, and offering mathematical arguments. Since good problems challenge students to think, students will often struggle to arrive at solutions. It is the teacher's responsibility to know when students need assistance » and when they are able to continue working productively without help. It is essential that students have time to explore problems. Giving help too soon can deprive them of the opportunity to make mathematical discoveries. Students need to know that a challenging problem will take some time and that perseverance is an important aspect of the problem-solving process and of doing mathematics.

Solving problems usually passes through 4 stages: understanding the problem, making a plan, carrying out the plan, and reflecting on the solution. Teachers need to assume certain roles in each stage. This study aimed at studying the roles teachers tend to really take at each stage in the problem solving process.

Teacher's roles before, during and after solving the problem

The teacher has a very important role to play in problem solving instruction. This role starts before students begin to work on the problem and continues until the end of the process.

Understanding the problem (the Before stage):

If the student is to solve a problem successfully, he/she should fully understand the problem and determine what exactly he/she should answer. This requires that students carefully read the problem or listen to the one who reads it. This is particularly important to students with reading difficulties. This means that teachers should underscore the importance of reading problems carefully (Baroody with Coslick, 1998). Teachers can read the problem themselves or have a students read the problem. Teachers also should make sure that students understand the problem before they begin to work on it. This can be done by asking students to identify what is asked for in the problem; discussing any unfamiliar terms in the problem; and/or asking students to restate the problem using their own words. They can go further by encouraging brainstorm possible solution strategies (Cathcart, 2001). Teachers should clarify to students the task at hand.

Making and carrying out the plan (the During stage):

While students work on the problem, teachers should keep circulating among them. They should be listening carefully to students' thoughts and discussions; observing the dynamics of group work; and questioning individuals or groups about the strategy they are using, and their findings. Teachers should ask students appropriate questions to help them clarify the direction they are taking in solving the problem (Vane; Cathcart; and Baroody with Coslick, 1998). The should also provide hints to students who need them. Teachers should watch for students who finish their work early and encourage them to solve the problem in a different way. Moreover, it is so important for teachers to encourage students to make extensions or generalizations of the solutions (Van De Walle,).

Reflecting on the solution (the After stage):

Solving the problem is not the end of the problem solving process. So, after students solve the problem, teachers should encourage them to reflect on their solutions and on the processes they used. They should ask them to justify their solutions. Effective teachers engage the class in productive discourse by letting students communicate their ideas in words, diagrams, or manipulative materials; and letting students share their ideas and strategies. They also ask the students to think of how the problem is similar to and different from problems they solved previously. To help students consider the impact of special features, teachers should have them discuss the important aspects of the problem (Baroody with Coslick, 1998). Finally, teachers should summarize the main points of the discussion and make sure that all students understand them (Van).

Method

Sample:

The sample for the study consisted of 10 elementary mathematics teachers from Abu Dhabi Educational zone. Those teachers voluntarily agreed to participate in the study.

Instrument:

The instrument of the study was a researcher-developed observation form (see Appendix).

Procedure:

Based on the above discussion of teacher's roles at the different stages of problem solving, the researcher summarized these roles in specific points and used them to develop the observation form shown in tables A-C in the Appendix. This instrument was validated by experts in the field.

Then, the researcher asked two teacher supervisors from Abu Dhabi Educational zone for volunteered teachers to conduct the study. Ten teachers volunteered to participate in the study. The researcher met with those teachers and arranged with each of them a time for videotaping a lesson. Also the researcher arranged with each teacher to have students work on a problem relevant to the topic scheduled for the time of videotaping. The problem was made by both the researcher and the teacher. In the specified day, the teacher was videotaped for the whole class session. Two persons watched the video tapes and used the observation form. After watching each videotape, the observers discussed the teacher's actions and gave a score they both agreed on. The emphasis was on the roles the teacher played during the different stages of problem solving. The observers gave each teacher 3 scores (one for each stage). Whenever there was a disagreement point, the observers played certain portions of the videotape and discussed the point until they reached an agreement. It was decided that if the two observers did not reach a consensus on a score, the mean of their scores will be given. However, there was no need to use this rule at all.

Results and discussion

As mentioned above, each teacher was given a score on each stage of the problem solving (before, during, and after). For each teacher, a mean score was calculated. Also, the mean for all teachers was calculated at the stage level and the mean score level. Scores on stage Before ranged from 50 to 90 with a mean of 66 and SD of 11.97. Scores on stage During ranged from 60 to 90 with a mean of 71 and SD of 11.0. Scores on stage After ranged from 20 to 60 with a mean of 34 and SD of 13.29.These results are shown in table 1 (maximum score is always100).

Table 1: results

	Mean	SD
Before	66	11.97
During	71	11.0
After	34	13.29
Total teacher's score	57	11.35

At the Before stage, most teachers distributed a work sheet with the problem written on it. They did not intentionally clarified the problem nor checked if students understood it. Students themselves led the teacher to clarify the problem by their frequent questions. This might reflect the teachers either were not serious about using the problem solving instruction or they lacked experience in such method of instruction. In both cases, students of these teachers are deprived from having meaningful problem solving instruction.

At the during stage, teachers walked around the class observing students' work and answering students' questions. In most cases, teachers' interventions were not effective. Sometimes, teachers gave help more than needed. In other times, teachers gave hints that students could not understand.

At the After stage, teachers did the worst. Most teachers did not seem to consider this as a stage. Actually, once students reached an answer, teachers considered the process done. Except few exceptions, teachers asked students if their answers were identical to those the teachers have.

The results of the study showed that generally teachers did not play roles that support problem solving or help students become problem solvers. Teachers did not seem to recognize the importance of the first stage in the problem solving process (understanding the problem). During solving the problem, teachers tended to intervene inappropriately with students' work. At the final stage, teachers generally did not encourage students to reflect on their work. The did not push students to make generalizations or solve more complicated problems. The results of the study suggest that teachers need to be extensively trained on teaching mathematics via problem solving.

References

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Appendix

Table A: the observation form (stage Before)

Stage	What teacher should do	What teacher really did (evidence/ indicators)	Comments
Before	- Pose the problem to students (orally or in a written form). - Read or have a student read the problem. - Make sure that students understand the problem before they begin to work on it. This can be done by asking students to identify what is asked for in the problem; discussing any unfamiliar terms in the problem; and/or asking students to restate the problem using their own words. - Brainstorm possible solution strategies. - Clarify the task at hand.		

Table B: the observation form (stage During)

Stage	What teacher should do	What teacher really did (evidence/ indicators)	Comments
During	- Circulate among students. - Listen carefully to students' thoughts and discussions. - Observe the dynamics of group work. - question individuals or groups about the strategy they are using, and their findings. - Ask students appropriate questions to help them clarify the direction they are taking in solving the problem. - Provide hints to students who need them. - Encourage students who are finished to solve the problem in a different way. - Encourage students to make extensions or generalizations.		

Table C: the observation form (stage After)

Stage	What teacher should do	What teacher really did (evidence/ indicators)	Comments
After	- encourage students to reflect on their solutions and on the processes they used. - Ask students to justify their solutions. - engage the class in productive discourse by letting students communicate their ideas in words, diagrams, or manipulative materials; and letting students share their ideas and strategies. - Ask the students how the problem is similar to and different from problems they solved previously. - Have students discuss the important aspects of the problem. - Summarize the main points of the discussion and make sure that all students		

	understand them.		