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Investigating the Relationship between Mental Imaging and Mathematical Problem Solving
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Mental imagery and the internal representations subsumed within the imaging process have been demonstrated to play an important role in dynamic problem solving (Sadoski & Paivio, 2001). Although investigations of the effects of mental imagery on literacy tasks have represented a specific research focus (Gambrell, 1981; Gambrell & Bales, 1986; Pressley, 1976; Sadoski, 1983, 1985; Suzuki, 1985), little inquiry has been conducted in the effects of mental imagery on mathematics problem solving. This is unfortunate in light of the fact that mental imagery has been shown to enhance the type of deep level conceptual connections that facilitate task engagement and resultant recall and comprehension of verbal knowledge (Craik & Lockhart, 1972; Guthrie et al., 1996). Because mental imagery has been shown to support conceptual engagement in literacy learning, the strategy also holds promise for supporting students' conceptual engagement in mathematics problem-solving tasks. This paper will present the results of an exploratory investigation designed to determine the relationship between mental imagery strategies and the successful mathematics problem solving of elementary, middle school and secondary students.

Theoretical Framework and Purpose of the Study

Contemporary trends in education reflect a shift from a traditional teacher-centered approach to teaching and learning to one that that is characterized as constructivist and student-centered. A reflection of this trend is found in a focus on developing metacognitive awareness in a 21st century student population. This focus on developing a metacognitively-oriented, thinking-about-thinking approach to learning means teachers must provide their students with a variety of strategic learning tools designed to support achievement in the myriad academic tasks encountered across the curriculum. Within the literacy disciplines of reading and writing, metacognition is appropriately conceptualized as the self-monitoring of one's learning as well as ownership of a repertoire of learning strategies from which a student might draw depending upon the learning task at hand. Although the reading community is currently fraught with debate concerning exactly which strategies and approaches are most effective for developing reading achievement (Cowen, 2003; Roller, 2002; Flippo, 2001), there is universal agreement that students must have ownership of multiple strategies to deal with multiple literacy tasks. Unfortunately, an emphasis on providing students with multiple mathematics problem-solving tools has not been embraced universally within the educational community. This stance contradicts the National Council of Teachers of Mathematics (1991) recommendation that teachers intellectually empower each student in a self-construction of mathematics conceptual knowledge characterized by ownership of effective problem-solving strategies.

The idea that mathematics tasks do not require the same repertoire of strategies as literacy tasks is an uninformed one especially when viewed in light of the fact that the same percentage of the student population that experiences reading problems also experiences problems in mathematics (Sousa, 2001). Consideration of the issue of mathematics achievement and the need for student ownership of a repertoire of problem-solving strategies is even more compelling when one examines the type of text students are required to process in mathematics classes. Characterized by multiple abstractions, specialized symbolism and technical vocabulary, mathematics text has been found to be the most difficult content area material to read, even for students who do not experience reading problems in other areas of the curriculum (Schell, 1982). Coupled with the difficulty of reading mathematics text is the complexity of the tasks necessary for successfully constructing problem solutions. In order to accurately solve a mathematics problem, students first must be able to process the related verbal information (i.e., words and symbolic language). Secondly, students must be able to analyze the relationships among the facts stated in the problem and determine which facts express relationships to what is known and not known. Finally, a solution to the problem must be constructed. The multiple problematic aspects of the discipline of mathematics necessitate students' having ownership over specific strategies designed to support them through the problem-solving process.

A literacy strategy that has potential for facilitating mathematics achievement is mental imagery (Douville, 1998; Douville, Pugalee, Wallace & Locke, 2002). Sometimes referred to as the "mind's eye," mental imagery can be defined as the act of forming internal images/pictures of events or objects not

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present to the eye. Serving as a kind of mental blackboard or personal movie screen, mental imagery and its concomitant representations have been shown to play an integral role in dynamic problem solving by aiding learners in the memory and/or understanding of unfamiliar events or situations. Dual Coding Theory (Clark & Paivio, 1991; Paivio, 1971, 1983, 1986; Sadoski & Paivio, 2001; Sadoski, Paivio & Goetz, 1991) supports the notion of mental imagery as a strategy particularly well suited to mathematical problem solving. Conceptualized from the perspective of dual coding, knowledge is represented in two mental subsystems. In the verbal subsystem, verbal representations such as words are processed. Conversely, knowledge that is represented in non-verbal pictures or images is processed in the non-verbal subsystem. It appears that effective readers are able to effectively switch from one mental subsystem to another when they process verbal/word representations into non-verbal/image representations as they read. For example, when an accomplished reader encounters the word "C A T," an automatic non-verbal image of a cat is generated. The ability to use and connect both mental subsystems during the reading process has been shown to aid in the understanding and recall of text information (Gambrell, 1981; Gambrell & Bales, 1986; Hibbing & Rankin-Erickson, 2003; Pressley, 1976; Sadoski, 1983, 1985; Suzuki, 1985). It follows that if the dual coding of both verbal and non-verbal representations reflected in the imaging of text has been shown to facilitate reading recall and comprehension, then mental imagery also holds promise for facilitating mathematical achievement. Because mathematical problem-solving requires both reading and computational abilities, mental imagery can aid in the dual coding of both the verbal and non-verbal representations of mathematical concepts.

Methods

Eight teachers enrolled in a university graduate course volunteered to collect the data used in the study. Each of the teachers was responsible for teaching one or more mathematics classes at the elementary, middle school, or secondary level and none of the teachers reported instructing their students in the use of imagery strategies. Prior to the actual data collection, the researchers met with the teacher participants in order to explain the purpose of the study and provide an overview of the existing research in the effects of mental imagery strategies on memory and learning. Additionally, an explanation of a possible link between mental imaging and mathematical problem solving was discussed. For the purposes of the study, the teacher participants were instructed to administer a single mathematical word/story problem to each of the students in their respective mathematics classes. The mathematical problems were determined to be grade-level/course appropriate by each of the teacher participants, and did not represent word/story problems previously administered to the students. Teacher participants were specifically instructed not to prompt or cue their students to use any particular mathematical problem-solving strategies. For the purposes of data categorization and analysis, the teachers were also instructed to label all the completed work and imagery surveys with students' names. However, the researchers specifically assured the teachers that adherence to student confidentiality would be strictly enforced with respect to dissemination of study results. Table 1 shows the follow-up written directions that were provided for each teacher.

Table 1: Teacher Directions for Data Collection

• Assign a mathematical problem-solving activity (i.e., "word problem") to your students they have not previously encountered. Be certain to provide a copy of the assigned problem with the student work you return to me.
• Immediately following completion of the problem-solving activity, give the students the survey to complete. Be certain your students respond to all three questions, if possible.
• Collect all problem solutions and surveys from your students. Be certain all math work and imagery surveys are labeled with students' names. Grade the math problems and attach the surveys before you return them to me.

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In addition to the graded word/story problems, data sources also included three open-ended survey questions designed to elicit responses from the students indicative of whether mental imaging was used as a mathematical problem-solving strategy. The use of open-ended survey questions in a study designed to investigate the use of mental imagery strategies was particularly appropriate because, according to Patton (1990),

> The truly open-ended question allows the person being interviewed to select from among that person's full repertoire of possible responses. Indeed, in qualitative inquiry one of the things the evaluator is trying to determine is what dimensions, themes, and images/words people use among themselves to describe their feelings, thoughts and experiences (p.296).

Immediately upon completion of the mathematics problem, the open-ended survey questions were administered to each of the students. The teachers were instructed to tell their students there were no "right or wrong" answers to the survey questions. Instead, the students were told to respond truthfully to as many of the questions as possible. Table 2 shows the open-ended mental imagery survey questions that were administered to the students.

Table 2: Mental Imagery Student Survey

1. In your own words, describe what happens "inside your head" when you solve a math problem like the one you just completed.
2. Do you form pictures or images inside your head as a way of solving math problems? If yes, describe how pictures help you solve math problems.
3. If you used a "picture in you head" as one of the ways to solve the math problem you just completed, show/draw the picture below. Use as much detail as possible and label the parts of your picture as a way to explain what your picture means and how it helped you to solve the problem.

Two experienced educators employed as university graduate assistants were trained to serve as raters in the analysis and coding of the graded mathematics problems and open-ended survey question responses. The graded problems and responses to the survey questions were analyzed on the basis of homogeneity and heterogeneity of the data (Patton, 1990; Wersma, 2000). In order to ensure interrater reliability, each rater coded half of the data and then data sets were alternated in order to check for coding inconsistencies. The researchers discussed the coding with the raters and remained available to answer questions and resolve minor coding discrepancies when they occurred.

First, the solutions to the word/story problems were categorized either as "successful" or "unsuccessful." Partial credit was not extended for partial success. That is, mathematical problem solving was only categorized as "successful" based upon 100% credit. Because the study was designed to investigate the connection between achievement in mathematics and the use of mental imagery strategies, only the "successful" math problems were included in the initial analysis. Next, the open-ended survey responses that aligned with each of the successful math problems were analyzed in order to determine whether or not mental imagery strategies were reported. Based upon the students' self-reporting of imaging during the mathematical problem-solving process, responses to the survey questions were coded by the raters as either "imagery" or "non-imagery." For example, in response to question #2 that was designed to determine whether students reported using imagery as a mathematics problem-solving strategy, specific elementary, middle school and secondary responses that were "imagery" coded included (all responses are presented as written by the students):

- (2nd grade response) *Yes thay help me by helping my brain*
- (5th grade response) *Yes* (I form images as a way to solve math problems) *by picturing what the math problem is about. Then I try to picture exactly the way the problem describes it.*
- (7th grade response) *I draw the picture out if I want to make a problem easier so I can see it down on paper. If I don't draw it out, I'm not sure if I would get it right!*

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- (8th grade response) *Yes, pictures give you something to work with. It's always*(s) *helpful to draw a picture. It gives you clues to the problem you might not have picked up before.*
- (10th grade response) *Yes, and I draw the pictures out. Pictures help you solve math problems b/c you can see exactly what you're looking for or missing. You can see congruency, distance on a number line, how something is formed and what your possible answer can or cannot be.*

Alternately, specific responses to question #2 that were coded "non-imagery" included:

- (2nd grade response) No I just add.
- (5th grade response) No.
- (7th grade response) No.
- (10th grade response) No, I only think about the numbers.

Generally, only the students who reported using mental imagery as a mathematical problem-solving strategy were able to pictorially reproduce their images as a response to question #3. Figure 1 shows the pictorial image that a 2nd grade student reproduced as a strategy for solving the following problem: *There were 5 boys on the playground. Four girls came out. Then Mr. Love brought out his class with 23 children. How many children were on the playground?* It is important to note that the figure shows not only did the student include each of the essential aspects of the problem, but the she also incorporated the strategy of representing each of the addends with a distinctly different icon. The 5 boys in the image are represented as circles, the 4 girls as straight lines, and the 23 students in Mr. Love's class as curved "m-like" icons. Using the strategy of different icons enabled the student to represent each of the separate addends as a distinct feature of the total problem/solution image.
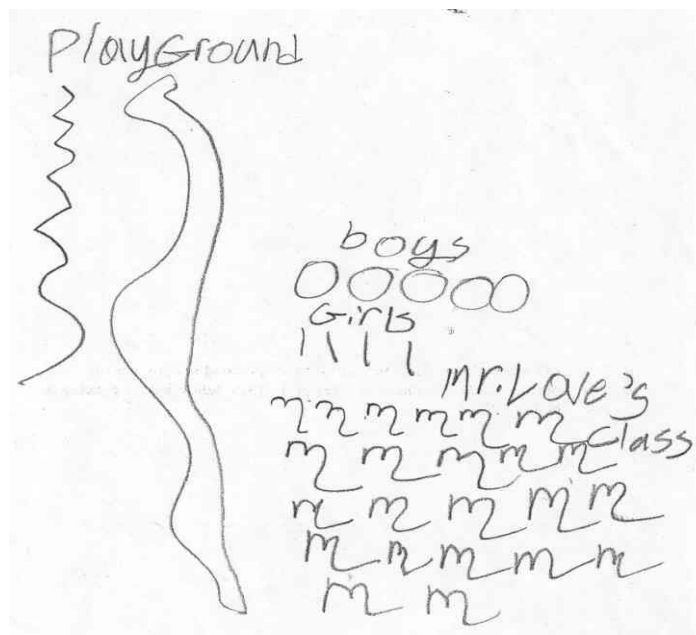

Figure 1: 2nd grade problem-solving image

Figure 2 illustrates how a 10th grade geometry student used a pictorial image to indicate how she used imagery to solve the problem: *In triangle XYZ, m<Z is 2 more than twice m<X, and m<Y is 7 less than twice m<X. What is the measure of each angle?* Like her second-grade counterpart, this student also represented each aspect of the total problem/solution image. However, this student also included a verbal explanation of how she used her image to solve the problem in an ordered set of sequential steps with directional arrows specifically placed to indicate which components of the image were used at particular steps. Using each of the components of her image enabled this student to compute the final solution as m<X=37°, m<Y=67°, m<Z=76°.

The final aspect of initial data analysis was a computation of the percentage of total "successful" mathematical students who reported using imagery as a problem-solving strategy compared to the

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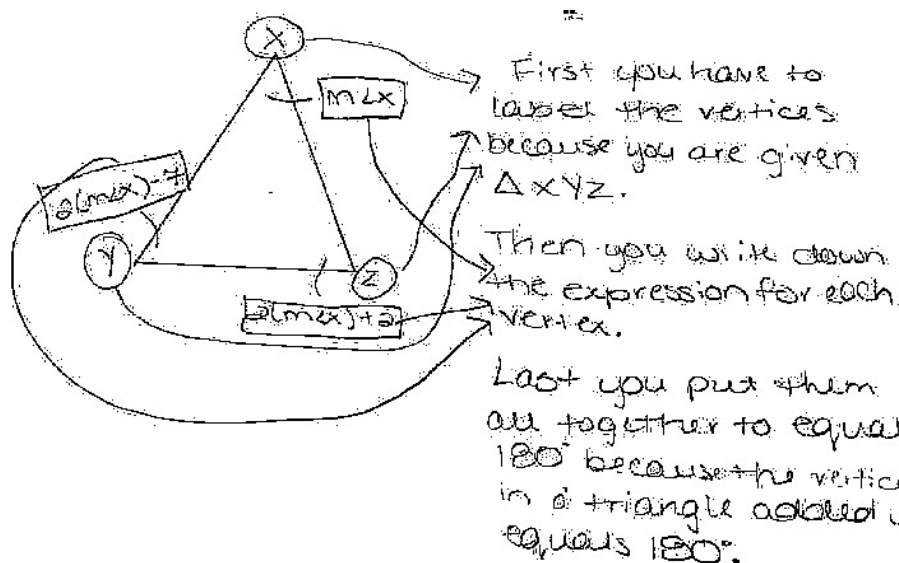**Brno, Czech Republic, September 2003**

Figure 2: 10th grade problem solving image

percentage of total successful students who did not use imagery. The results indicate that at each grade level almost twice as many successful mathematics students reported using imagery to problem-solve as those who did not report using imagery. Table 1 shows that 17, or .654, of the 26 successful elementary mathematics students used imaging as a problem-solving strategy compared to 9, or .346, of the students who did not use imaging. Results were similar at the middle school and secondary levels. 30 students, or .625, of the 48 total number of successful middle school students used imaging as a problem-solving strategy compared to 18, or .375, who did not use imaging. At the secondary level 28 (.651) of the 43 successful mathematics students used imagery to solve the problem compared to 15 (.349) who did not use imagery.

Table 1: Percentage of Subjects Reporting Imagery Use in Successful Mathematical Problem Solving

Subjects	Successful	Imagery	%	No Imagery	%
Elementary	26	17	(.654)	9	(.346)
Middle School	48	30	(.625)	18	(.375)
Secondary	43	28	(.651)	15	(.349)
Total	117	75		42	

Discussion and Conclusions

Results of the study reveal a number of findings. First, initial analysis of the data indicated that a greater percentage of successful mathematics students at the elementary, middle school and secondary levels reported using mental imagery as a problem-solving strategy than those who did not report using imagery. Not only did a majority of successful students report using imagery, but they were also able to externalize the problem-solving imaging process in individual pictorial representations that aligned with the assigned mathematics task. Secondly, responses to the open-ended survey questions revealed that non-imagers appeared unable to explain why imagery was not used as a strategy. The non-imagers also failed to offer alternative mathematics problem-solving strategies (*"No I just add"*). However, the responses of the imagers were characterized by explanations of how the imaging process contributed to mathematics success (*"It's always helpful to draw a picture. It gives you clues to the problem you might not have picked up before"*). Although the imagery students were able to provide explanations of how imagery contributed to successful problem solving, the developmental levels of the students appeared to affect the depth of metacognitive awareness of precisely how the imaging process supported learning. Younger students' explanations were less specific and reflected only a vague notion of how imagery facilitated problem solving (*"…thay help me by helping my brain"*). However, older students were able to explicitly describe how the imaging process supported learning in responses that were more elaborated (*"Pictures help you solve math problems b/c you can see exactly what you're looking for or missing. You can see congruency, distance on a number line, how something is formed and what your possible answer can or cannot be"*). The elaborated explanations are indicative of a deeper level of metacognitive awareness of how the imagery strategy specifically assisted mathematics problem solving. This level of metacognitive awareness is representative of the type of task engagement that facilitates deep-level conceptual understanding (Craik & Lockhart, 1972; Guthrie et al., 1996).

Even in the absence of scaffolded instruction in the use of mental imagery strategies, successful mathematics students at the elementary, middle school and secondary levels report using imaging as a problem-

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solving tool. These students appear to have a tacit understanding of how images serve to concretize abstract mathematical concepts in ways that facilitate learning. Teachers and students of mathematics must have knowledge and control over a repertoire of effective instructional and metacognitive strategies, respectively, to process the symbolic language and abstract concepts reflected in the discipline of mathematics. Rather than depending on students to independently generate and apply effective strategies, educators must continue to investigate how mental imagery can support students' mathematics learning in future in-depth investigations.

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