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MENDEL: BIOLOGY, MATHEMATICS AND HISTORY OF SCIENCE

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Abstract: Brno is the place where the Augustinian monk Gregor Mendel carried out his studies in genetics, initiating with that a new and important branch of biology. In the small orchard of the Augustinian Monastery-Abbey of this city, he experimented with about 27.000 pea plants for nine years, from 1854 to 1863. Nevertheless, Mendel would not have been able to come to the magnificent conclusions of this work without the help of mathematics. In this way, he applied a simple but effective mathematical method in the analysis of the empirical data obtained by means of those experiments with plants.
Thus, all these elements together, Mendel, Brno, mathematics, biology..., in sum, interdisciplinarity and history of science, represent a good opportunity to be discussed in a conference on mathematics, like this one, hold in the city of Brno.

Introduction

When we hear the name "Mendel" we always associate it to the laws of inheritance in the living beings. The theory of heredity always appears in secondary school biology. So, it is a typical theme that, if it is presented in an adequate way, becomes very attractive for the students of biology. However, by going deeper into this theory, we can appreciate that it has an important relation with mathematics. On the other hand, it is also a good example of the history of science to be used as a teaching aid in the classroom.
Thus, by treating this subject from that perspective some advantages can be found from a didactic viewpoint:

- To show the students of biology how this biological research could not have been done without the help of mathematics.
- To show the students of mathematics how important the contribution of mathematics in the development of other sciences can be.

All that also offers an image of the interdisciplinary nature of science. And the use of history of science, in turn, has some more advantages, such as:

- To place the students in some problematic situations that the scientists have lived, so that they can know how these scientists solved the problems.
- To promote the reflection of the students about how the scientific ideas were built.
- To motivate the students to study sciences, that in this way will become more attractive and pleasant to be learned.

Nevertheless, there are another reason to discuss here this topic: this conference on mathematics is being held in Brno, the same city where Mendel carried out his experiments about heredity.

Brief biography of Mendel

Johann Gregor Mendel was born in 1822, in Hyncice, that belonged then to the Austrian-Hungarian Empire and now to the Czech Republic. He was the second child of a couple of farmers in Brno (Moravia), of limited economic resources. He was a very good student at school, but his parents could not afford a higher education for him. So, the only way -very common in those days- that made possible to keep studying was to enter in a religious institution. In fact, he entered in an Augustinian monastery-abbey in Brno, a centre where theology, natural sciences and philosophy were learned, and there he started a teaching career. At the age of 21, in 1843, Mendel became a friar and at 1868 he reached the position of abbot in his abbey.
For a period of two years he stayed at the university of Vienna, and this fact represented a very important moment for his future research. It was a time of hard discussions over the theories of evolution and the ideas about the influence of environment. And probably the great interest of Mendel in the evolutionism comes from those days. Other influence that Mendel received in that period, also very significant for his studies, was the idea of the importance of statistics, the combinatory analysis in the scientific research and the possibility of reducing all the natural phenomena to mathematical expressions. That is to say, the necessity of applying mathematics to the study of nature.

The experiments of Mendel

Mendel carried out all these experiments in the orchard of his own Monastery. Actually he was not a scientist, but only a secondary school teacher of natural sciences. He began this study about inheritance moved by his deep fondness to nature. By growing different varieties of plants in this orchard, he found that the essential traits of the parents were kept in the new individuals of the offspring, without any influence of the environment. Thus, these traits were inherited, but he wondered why and how they could be transmitted. This question was posed by many scientist before Mendel, but he was the first who built a successful theory, by means of designing his experiments according to some magnificent ideas.

Which were these ideas? First, he had realized that the traits were inherited in certain ratios. So, he experimented with different plants and even with mice. But after he decided to experiment with pea plants, the common garden pea (*Pisum*), because they presented a great advantage: they have some traits that occur either in one variation or another, but not in between. And that fact simplified the problem very much. Those traits were, for instance, form of the ripe seed (smooth or wrinkled); colour of the flowers (violet or white); colour of seed albumen (yellow or green); form of ripe pods (inflated or constricted), etc.. Mendel chose seven of this kind of traits in the peas, but he studied only one trait at a time: this was other of his great ideas.

Thus, for nine years, from 1854 to 1863, he experimented with about 27.000 pea plants, studying the results in the inheritance of that simple traits along several and successive generations. This was also a quantitative study, so that the results of his observations in a so high number of samples could be statistically expressed and mathematically treated. In sum, the conditions were:

- To study with a very big sample of individuals: so a statistical, and not individual, behaviour can be studied.
- To generalize the transmission of the traits in successive generations: so a mathematical development can be applied.
- To employ a very simple system of symbols, that made easier this study.

Mendel decided to began crossing two purebred plants (in a total amount of 70 different purebred peas), very carefully, with his own hands, so that the plants were protected from the influence of foreign pollen. When breeding two different purebred plants (the parents, P_1, that were different) then the result was hybrid new individuals (first generation of offspring, F), and a particular variation of one trait did not appear in this first generation. But this variation would appear again in the next generation (F_1), produced by breeding the first offspring generation with itself. For instance, by crossing two purebreds pea plants with different colour of the flowers (but all the rest of simple traits being the same) he observed that all the hybrids of the generation F had violet flowers. He supposed that each trait is defined by two factors or characters, each one coming from one of the parents. And one of those characters was *dominant* -it is the one that "appears" externally- and the other one was *recessive*. Mendel represented the dominant character, violet colour of the flowers, with the symbol V, and the recessive one, white colour, with the symbol v. Thus, the individuals of the generation F received two different factors, V and v: they were *hybrid* or *heterozygous* for that character, Vv, whereas the parents were *purebred* of *homozygous*, VV and vv.

All these results and observations in Mendel´s experiments led him to define the first law of inheritance: the individuals of that first generation look like only one of the parents and never the other one.

The next experiments of Mendel consisted on the study of the following generation (F_1), the offspring of these hybrids (F). But it was necessary that the ovules of one flower were fertilized by the pollen of the same flower, being absolutely protected from all foreign pollen. Then he observed that the white colour of the flowers was exhibited again and so this trait re-appeared in this generation. Mendel concluded that one trait was masked in F -but not disappeared-, because it was exhibited in this following generation (F_1). He counted the ratio of the peas with violet flowers and with white flowers, resulting a ratio of 3:1, respectively.

In order to explain all these results Mendel supposed that when two plants breed, the variation of their traits are combined. And he explained this combination by supposing that the two pieces of information that describe each trait are separated in each parent before the fecundation. With all that Mendel posed his second law of inheritance (law of segregation).

Elaboration of the mathematical development

By employing that simple symbols with two letters for a trait, capital letters for the dominant information, and small letters for the recessive one, the problem of calculating the possible cases became easier for Mendel. By using a Punnet square all the possible combinations created can be drawn. Thus, for the generation F_1 the possible cases are:

	V	v
V	VV(pure)	Vv (hybrid)
v	vV(hybrid)	vv (pure)

The ratio of hybrid individuals and the bredpure ones is 2:1:1 (50% Vv, 25% VV and 25% vv) and the ratio of violet: white flowers is 3:1 (75% of violet and 25% of white flowers) is now easily explained: the dominant character (violet colour) would come from the pure individuals VV (25%) and from the hybrid ones Vv (50%). The first ratio corresponds to *genotype* frequencies and the second one to the *phenotype* frequencies.

In the same way Mendel studied the results of the next generation, F_2 , produced by the auto breeding of the individuals of F_1 He observed that 75% of violet flowers peas and 25% of white flowers peas were produced, deducing from that the following distribution of hybrid and pure individuals:

25% VV – 50% Vv – 25% vv…. ratio 1:2:1

Finally, a mathematical expression was found for the *behaviour of hybrids* in the auto breeding:

- First generation (F_1) : Vv + Vv $\Rightarrow$ ratio 1VV , 2Vv , 1vv
- Second generation (F_2) : The auto breeding of the individuals of the first generation will produce:
  1 VV x 4 $\Rightarrow$ 4 VV
  2 Vv x 4 $\Rightarrow$ 8 Vv (distributed as 4 Vv, 2 VV and 2 vv)
  1 vv x 4 $\Rightarrow$ 4 vv
  The total amount of pure or hybrid individuals will be:
  6 VV (4+2), 4 Vv , 6 vv (4+2) , being then the ratio 3 : 2 : 3 , that can be represented as : $(2^2 – 1) : 2 : (2^2 – 1)$
- Third generation (F_3) : The auto breeding of the second generation will produce
  6 VV x 4 $\Rightarrow$ 24 VV
  4 Vv x 4 $\Rightarrow$ 16 Vv (distributed as 8 Vv, 4 VV and 4 vv)
  1 vv x 4 $\Rightarrow$ 24 vv
  Being the final sum: 28 VV (24 + 4) , 8 VV , 28 Vv (24 + 4) , ratio 7 : 2 . 7 , represented as $(2^3 – 1) : 2 : (2^3 – 1)$
- N th generation:
  Generalizing that mathematical expression, the ratio of the individuals in the **n**th generation will be: $(2^n – 1) : 2 : (2^n – 1)$

These experiments and their results, expressed by means of this simple but efficient mathematical development, demonstrated that the hybrids never disappear although their number decreases in relation to the bredpure individuals.

				Ratios
Generation	VV	Vv	vv	VV : Vv : vv
First	1	2	1	1 : 2 : 1
Second	6	4	6	3 : 2 : 3
Third	28	8	28	7 : 2 : 7
Fourth	120	16	120	15 : 2 : 15
Fifth	496	32	496	31 : 2 : 31
Nth	……	……	……	$2^n – 1 : 2 : 2^n – 1$

All that corresponds to the transmission of only one trait. In the cases of the inheritance of two or more traits the calculations are more complicated, but it is also possible to determine the genotype and phenotype frequencies.

Conclusions

The study of Mendel and the discover of the laws of genetics have had a huge influence in the life of mankind The consequences for us are many and very varied. In medicine, because many diseases, that are known to be inherited by the descendants, can be controlled by means of determining the probability of being transmitted. In agriculture, because new plants with desired characteristics can be designed in the laboratory. And the same can be said about animals...

Thus genetics has a great influence on the way we live, but we must remember that in the development of the theory of inheritance the mathematics played an essential role.

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