The Mathematics Education into the 21st Century Project
Proceedings of the International Conference
The Decidable and the Undecidable in Mathematics Education
Brno, Czech Republic, September 2003

Deciding Optimal Experiences in Coordinate Geometry for Pre-Service Elementary School Teachers

Jack A. Carter, California State University, Hayward, CA 94542-3092 USA
Beverly J. Ferrucci, Keene State College, Keene, NH 03435-2001 USA

Abstract: This study was undertaken to determine if more advanced coordinate geometry topics could be integrated into a mathematics course for prospective elementary school teachers. The topics were incorporated into a series of interactive computer activities that pre-service teachers completed as part of their mathematics coursework. Results showed that the introductory activities with dynamic graphs and concluding activities with iteratively defined functions were less challenging for the future teachers than were activities involving functional attributes and composite functions.

Several recent research studies have demonstrated the usefulness of technology as a powerful tool that can enable students to: (1) make meaningful connections between symbolic and graphic representations of functions; (2) develop a greater understanding in generating and coordinating multiple representations of functions; and (3) exhibit greater evidence of a transition from an operational to a structural understanding of functions (Kordaki and Potari, 2002; Hollar and Norwood, 1999; O'Callaghan, 1998). Other studies (Chinnappan, 1998; Adams, 1997) have related the difficulties experienced by their subjects, many of whom were prospective teachers, on a multitude of tasks related to characteristics of functions. Implications in these studies have emphasized the need to devise learning environments that make functional relations more prominent. Abramovich and Brouwer (2003), Pandiscio (2002), and Cedillo (2001), in research pertaining specifically to prospective teachers, found a tendency for these students to explore problems more deeply with mathematical software and to espouse the belief that such computer applications help in understanding situational concepts more fully.

The present study built on this research base by investigating the efficacy of using computer-generated dynamic representations of functions as a means of enhancing prospective teachers' comprehension of the characteristics and compositions of functions. The participants in this study were future elementary school teachers enrolled in a senior level (fourth year) mathematics education seminar. Prior to this course, the prospective teachers had taken three other mathematics courses designed for elementary school teachers. Topics in these courses included linear, probability, statistical, and spreadsheet functions, as well as statistical plots. In additional to working with spreadsheets, the future teachers in the study had also had hands-on experience working with graphing calculators to investigate linear functions, statistical plots (histograms, scatter plots, and box plots) and fitting linear equations to data.

Two of the prerequisites for the course in which the prospective teachers were enrolled were semester-long courses: one focused on the real numbers and its subsets; and the other emphasized applications of the real numbers. The third prerequisite course was a semester-long geometry course in which the students gained experience with dynamic geometry software. Consequently, this study also aimed to connect the future teachers' prior work in geometry with dynamic representations of geometric functions.

The venue for this study was a laboratory classroom where each future teacher worked individually at a computer on Geometers Sketchpad activities that were designed to investigate aspects of functions and their properties. The prospective teachers had five 110-minute class sessions to complete the activities that were presented on worksheets with introductory, sketch-making, and exploratory sections. The introductory section described the activity, while the sketching and exploring sections provided opportunities to follow construction steps and to answer questions about functions and their properties. Table 1 (below) lists the activities during the five class sessions.

Note that all activities in Table 1 are numbered except the first activity of class session 2. This activity was not numbered since it was the only one that did not require the future teachers to answer questions. The future teachers' work on this unnumbered activity was neither scored nor analyzed for this report.

The activities for the first class served as an introduction to the types of questions associated with dynamic coordinate geometry. Dynagraphs, a type of graphical representation that was developed to facilitate the process of understanding functions, were used in activities 3 to 7. The developers of

Class Session 1
1. Circumference As a Function of Diameter 2. Segment Length As a Function of Distance in a Triangle
Class Session 2
•. Choosing Variables, Predicting Graphs, and Using Loci to Graph Functions 3. Introduction To Describing and Identifying Variables in Dynagraphs
Class Session 3
4. Identifying, Transforming, and Comparing Graphs 5. Domain and Range in Graphs
Class Session 4
6. Odd and Even Functions in Graphs 7. Evaluating Function Composition and Modeling Composed Functions with Dynagraphs
Class Session 5
8. Iteration of Arithmetic Operations on Cartesian Coordinates of Points

Table 1. Descriptions of Activities during Each Class Session

dynagraphs intended for these representations to provide students with a graphical expression of the input-output view of functions (Goldenberg, Lewis, and O'Keefe, 1992). As a result, dynagraphs may be viewed as an intermediate step between input-output machine models of functions and graphs of functions on a Cartesian coordinate system (Figure 1).

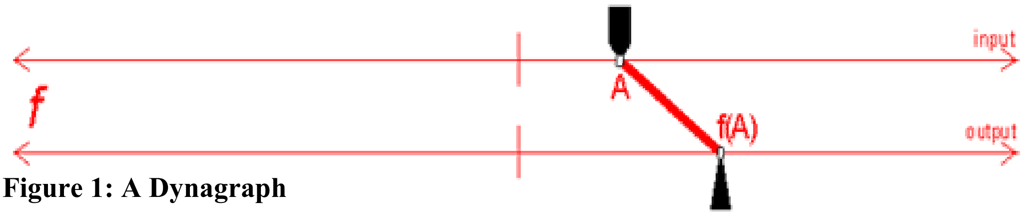

Figure 1: A Dynagraph

Dynagraphs were used in each class except the first and the fifth as a means of investigating the domain and range of functions, odd or even functions, and function composition. During the first activity of class session 2 the future teachers experimented with choosing independent and dependent variables from an assortment of geometric figures, predicted the graphs of the resultant functions, and checked these predictions by using loci of plotted points to sketch the graphs of the functions. In the second activity of class session 2, the students described dynagraphs and identified their numerical inputs and outputs.

In other class activities, the prospective teachers identified and transformed dynagraphs to Cartesian graphs and compared dynagraphs to Cartesian graphs. Other activities provided opportunities to study the concepts of domain, range, odd, even, and composition as they apply to functions. A general objective of these activities was to use coordinate geometry to expound the nature of functional properties. In particular, the future teachers had opportunities to explore symmetries in Cartesian graphs and dynagraphs to determine a function's parity (whether a function was odd, even, or neither), to evaluate specific function compositions, and to use dynagraphs to model and find the values of other composed functions. The activities also employed iteration of arithmetic operations on Cartesian coordinates of points and the loci of these iterated points as a means of representing functions and their graphs. These iterative activities provided an approach that aimed to use technology to bridge the symbolism of functions and the visual nature of their accompanying graphs.

A graduate student grader evaluated each future teacher's performance on the worksheets and calculated a score for each activity. Table 2 shows the future teachers' mean percentage scores and descriptive statistics for each of the numbered activities.

The first and second activities produced the highest and lowest average scores (means: 85.71 and 42.65; medians 92.86 and 25, respectively). Activities 3 and 4 that introduced dynagraphs yielded average scores close to 80%. These values contrast with the average scores on activities 5, 6, and 7, in which the future teachers used dynagraphs to investigate the domain and range concepts, as well as the notions of odd or even functions and function composition. The mean scores on these particular function properties indicated some evidence of improvement from the earlier to the later activities. It is also notable that the

	ACTIVITY ONE	ACTIVITY TWO	ACTIVITY THREE	ACTIVITY FOUR	ACTIVITY FIVE	ACTIVITY SIX	ACTIVITY SEVEN	ACTIVITY EIGHT
Mean	85.71	42.65	78.68	82.24	58.04	63.72	70.59	77.78
Std Dev	18.33	42.67	21.97	13.66	26.59	22.15	22.53	22.57
Median	92.86	25	81.25	76.32	64.29	55.56	80	79.17
Count	17	17	17	16	16	17	17	18

Table 2: Students' Average Percentage Scores on the Eight Activities

average scores on the last activity (8), the iterative activity with no dynagraphs are similar to those on activities 3 and 4, the initial dynagraph activities. The overall activity averages and the overall class averages along with the differences of the two means are displayed in Table 3.

	OVERALL ACTIVITY AVERAGE	OVERALL COURSE AVERAGE	OVERALL COURSE MEAN MINUS OVERALL ACTIVITY MEAN
Mean	69.98	79.61	9.7
Std Dev	17.78	8.46	13.85
Median	75	77.37	5.67

Table 3: Overall Activity and Class Averages and Mean Differences

A histogram and a normal-quartile plot that represents the overall course mean minus the overall activity mean for each future teacher are displayed in Figure 2. The histogram has an appearance that suggests the bell-curve shape of a normal distribution and the approximately linear shape of the accompanying normal-quartile plot of the mean differences reinforces this indication. A single sample t-test for the null hypothesis of $\mu \leq 0$ to the difference data produced $t = 2.89$ and a p-value of .005 indicating that the null hypothesis could be rejected in favor of the alternative that $\mu > 0$. This was evidence that the prospective teachers' overall course means were higher than their overall activity means. Comparable analyses for differences between scores on consecutive activities gave no plots that reflected distribution normality and no basis for further tests of the means.

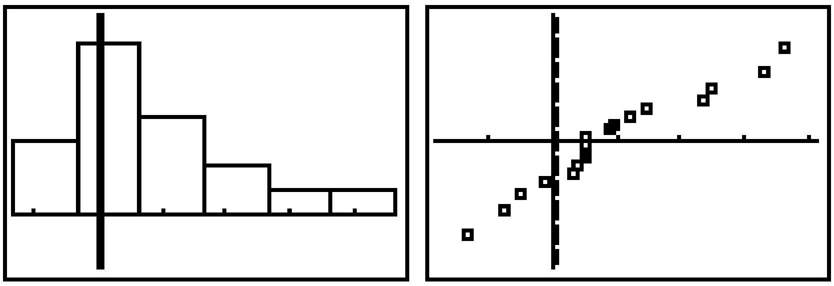

Figure 2: Histogram and Normal-Quartile Plot of Prospective Teachers' Overall Course Means Minus Their Overall Activity Means

The future teachers' scores on the first two activities showed considerable variations. In particular, their mean scores on the first activity were the highest of all the activities, while the mean scores on activity 2 were the lowest of all the activities. This disparity may reflect the fact that only some of the prospective teachers may not have fully understood how to use the software from their previous class experience.

Activities 3 to 7 were the focus of the study in that they provided the future teachers with a systematic development of key aspects of functions and their operations. Mean scores on activities 3 and 4 (78.68 and 82.24, respectively) were comparable to the overall class mean for the entire course (79.61).
Activities 5, 6, and 7 formed the core of the explorations of functions and their compositions, and as such might be regarded as the most intellectually challenging of the activities. Further analysis of future teachers' overall mean on these activities showed that they were lower than the overall course mean (p = .03). Activity 8 provided an alternative dynamic representation of functions and students' scores on this activity demonstrated a mean average (77.78) close to the overall mean for the entire course (79.61).
As noted earlier, the future teachers who participated in this study completed two prerequisite courses on the real number systems and their operations, properties, and applications. One section of these courses dealt with relations and functions with a specific emphasis on domain, range, and function composition. Also, each prospective teacher was required to enroll in an informal geometry class that included work with dynamic geometry software. As indicated previously, the fact that some of the future teachers did not fully understand how to use the software may have affected their performance on the dynamic coordinate geometry activities in this study. In addition, the prospective teachers did have the same professor for the informal geometry course, but they may not have the same professor for the number system courses. As a result, some of them may not have studied the topics of domain, range, and function composition in as in-depth a manner.
The issue of the effectiveness of these activities for enhancing understanding of the characteristics and compositions of functions remains an important one for further study. Multi-line calculators and graphing utilities are becoming commonplace in middle schools and teacher candidates for positions in these schools need an increasingly sophisticated knowledge of the coordinate systems that are a basis for functions and their graphs. Dynamic geometry software has the capability of providing a powerful context to enhance the development of teachers' skills as well as to support new types of pedagogy (Conference Board of the Mathematical Sciences, 2001). One means of exploiting this capability is to insure that prospective teachers at all levels have opportunities to use dynamic mathematical software in undergraduate courses. With the currently increasing number of educational technologies available, mathematics educators are apt to have many options for making these opportunities come to fruition.

References

Abramovich, S. and P. Brouwer. "Revealing Hidden Mathematics Curriculum to Pre-teachers Using Technology: The Case of Partitions." *International Journal of mathematical Education in Science & Technology* 34 (January 2003): 81-94.
Adams, T.L. "Addressing Students' Difficulties with the Concept of Function: Applying Graphing Calculators and a Model of Conceptual Change." *Focus on Learning Problems in Mathematics* **19** (Spring 1997): 43-57.
Cedillo, T.E. "Toward an Algebra Acquisition System: A Study Based on Using Graphing Calculators in the Classroom." *Mathematical Thinking and Learning* **3**(4) (2001): 221-260.
Chinnappan, M. "The Accessing of Geometry Schemas by High School Students." *Mathematics Education Research Journal* **10** (2) (September 1998): 27-45.
Conference Board of the Mathematical Sciences (2001). *The Mathematical Education of Teachers, Issues in Mathematics Education, Volume 11*. Providence, RI: American Mathematical Society.
Goldenberg, P., P. Lewis, and J. O'Keefe (1992). "Dynamic Representation and the Development of a Process Understanding of Functions". In Harel, G. and E. Dubinsky (Eds.) *The Concept of Functions: Aspects of Epistemology and Pedagogy*, MAA Notes, Volume 25. Washington, DC: Mathematical Association of America.
Hollar, J.C. and K. Norwood. "The Effects of a Graphing-Approach Intermediate Algebra Curriculum on Students' Understanding of Function." *Journal for Research in Mathematics Education* **30** (March 1999): 220-6.
Kordaki, M. and Potari, D. "The Effect of Area Measurement Tools on Student Strategies: The Role of a Computer Microworld." *International Journal of Computers for Mathematical Learning* **7**(1) (2002): 65-100.
O'Callaghan, B.R. "Computer-Intensive Algebra and Students' Conceptual Knowledge of Functions." *Journal for Research in Mathematics Education* **29** (January 1998): 21-40.
Pandiscio, E.A. "Exploring the Link between Preservice Teachers' Conceptions of Proof and the Use of Dynamic Geometry Software." *School Science and Mathematics* **102**(5) (May 2002): 216-225.