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Development of a framework to assess primary students' number sense in Malaysia

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Abstract: Number sense refers to the ability to use numbers and quantitative methods as a means of processing, interpreting and communicating information. It requires an understanding of the number system, a repertoire of computational skills and an inclination and ability to solve number problems in a variety of contexts. The early years of schooling are deemed crucial in providing the kind of positive start to students' number sense learning that is needed to develop confident and capable lifelong learners. This paper will discuss a research that was undertaken with the main purpose of developing the number sense framework in the following components: counting, addition and subtraction, multiplication and division and place value. The framework was developed based on existing framework from other researches and modified to suit local context. In addition, feedbacks were obtained from elementary school teachers on the suitability of the framework in relation to the Malaysian elementary school mathematics curriculum. One hundred and eight elementary school children from years 1 to 3 were then interviewed to ascertain their strategies with respect to the various components of the framework. The framework that was developed will be presented together with the preliminary findings from the interviews.

Introduction The early years of schooling are crucial in providing the kind of positive start to students' number sense learning that is needed to develop confident and capable lifelong learners. Many studies have shown that students' experiences related to the learning of number concepts at primary school level are of central importance in instilling their beliefs and values they associate with mathematics. If these experiences are meaningful they will further lead to attainment of positive attitudes, values and beliefs about number concepts. On the contrary, experiences that are not mathematically meaningful will lead students to believe that mathematics learning consists of memorizing activities devoid of meaning (NCTM, 1989). The main concern for most mathematics educators are that many students demonstrate little understanding of numerical situations in instances where they have to solve number problems (Leutzinger & Bertheau,1989; Burns,1989). For example, Kamii and Livingston (1993) found that children were able to know that 6x 6 = 36 but they were not able to answer the question 7x 6. This showed that children were not able to relate the product of 6 x 6 with the product of 7 x 6 or how the product of 7 x 6 is actually six more than 36 based on the multiplication order. Yang (1995) suggests that this could be due primarily to the mindless application of the standard written algorithms which students learned in school. Students are good rule followers but unfortunately do not always understand the procedures they learned (Hiebert, 1986). They are adept at manipulating and following symbol rule but are less able at making sense of numerical situations.

While emphasis on computational skills may produce high computational scores, the extent to which these processes transfer to students' understanding is unknown. A number of mathematics educators seem to agree that the difficulties experienced by students in solving mathematics exercises is closely related to the development of number sense thinking (Leutzinger & Bertheau,1989; Burns,1989). Reports by The Examination Syndicate MoE Malaysia (1995; 1996) highlight the issue as to whether children exhibit number sense while doing mathematical problems. The Malaysian Examination Syndicate, MoE (1995; 1996) reported that secondary school students are not able to acquire the basic concepts of mathematics and they could only guess the answers but were not able to actually work out the problems (Lembaga Peperiksaan, 1996). Some very weak students even did the working only by using numbers in the given problem without understanding the problem properly(Lembaga Peperiksaan, 1996). Research by Sharifah Norul Akmar (1997) showed that many students represent integers as a point on the number line and use the formula "subtracting a negative number is equal to adding the numbers" by rote without having a deep understanding of the formula.

Researches by Behr, Hare, Post dan Lesh (1992) and Greer (1992) show that students do not display number sense for problems involving fractions and decimals. Many students think that decimals are two different numbers separated by a dot(Threadgill dan Sowder, 1984; Hiebert dan Wearne, 1986; Resnick, Nesher, Leonard, Magone, Omanson dan Peled, 1989). For example, there are students who believed that 1.25 is greater than 1.3. Students add two decimal numbers in the same manner as they would add two whole numbers. There are primary school children who think that the denominators and numerators of a fraction are two separate entities (Behr, Wachsmuth, Post dan Lesh, 1984; Kerslake, 1986), therefore 6/8 is thought of as bigger than 6/7. Although there are children who can recognize fraction as an entity, often the fraction representation is not understood (Peck dan Jencks, 1981). In multiplying 4.5 x 1.2, a student carefully lined up the decimals and then multiplied, obtaining the answer 54.0 (Reys et.al, 1991, p.3). When children are asked why they say 17 is larger than 13, they respond that "it just is." They are unable, when asked, to give any further justification (Sowder & Wheeler, 1987). Student know the answer to 6 x 6 but cannot multiply 7 x 6. There are many examples of such errors, which are said to reflect a lack of 'number sense'. Responses to questions such as these reveal the level of understanding of number meanings, operations and computations.

Recent curricular reform documents (such as National Council of Teacher of Mathematics, 1989;Australian Education Council, 1991;Cockcroft,1982, MoE 1991) emphasize the importance of number sense based on the rationale that numbers sense will be very helpful to understand numbers in general. Relatively, the focus on the term "number sense" in the mathematics curriculum is quite recent and most has targeted their arguments to the primary school level (Burns,1994; Hiebert,1984; Plunkett,1979; Skemp,1982, MoE 1989). Even though many good teachers are undoubtedly teaching mathematics in ways that lead their students to develop good understanding in numbers, operations, the relationships between numbers and operations, and computations, the researchers believe that the development of number sense will play an important role in elementary mathematics education in Malaysia. Despite efforts by the mathematics education community to move away from the traditional conception about mathematics, many researches have strongly indicated that most children have not attained the understanding demanded by the new curriculum (Noor Azlan Ahmad Zanzali, 1995). This paper shall discuss the process of developing a framework to assess children's number sense particularly counting, addition and subtraction strategies, multiplication and division strategies and knowledge of place value. Efforts to define number sense revolve around making mathematical manipulations and situations involving number sense as meaningful. (NCTM, 1989a; Sowder, 1992; McIntosh et.al, 1992), having an understanding about relative and accurate number magnitude (Sowder dan Schappelle, 1989; Case, 1998), mental computation and estimation (Sowder dan Schappelle, 1989; Greeno, 1991; Sowder, 1992; Case, 1998). All the above definitions of number sense consistently refer to the needs of the students to understand mathematical operations and computations that they carry out. NCTM (1989) argues that children must understand number meanings if they are to make sense of the way numbers are used in real life situations.

Objectives of the project

1. To develop number sense framework for primary school students in years 1 –3.
2. To assess years 1-3 students' number sense using the developed number sense framework in (1).

The framework focused on basic mathematical concepts: counting, place value, strategies for the four basic operations (addition, subtraction, multiplication and division). It is assumed that, the framework if mastered by a student would allow him or her to understand higher level mathematics.

The framework was refined through pilot interviews with selected students from year 1,year 2 and year 3. This stage also took into account the students strategies. The number sense framework was used in follow up interview with students were chosen according to their ability.

Methodology Building the number sense framework

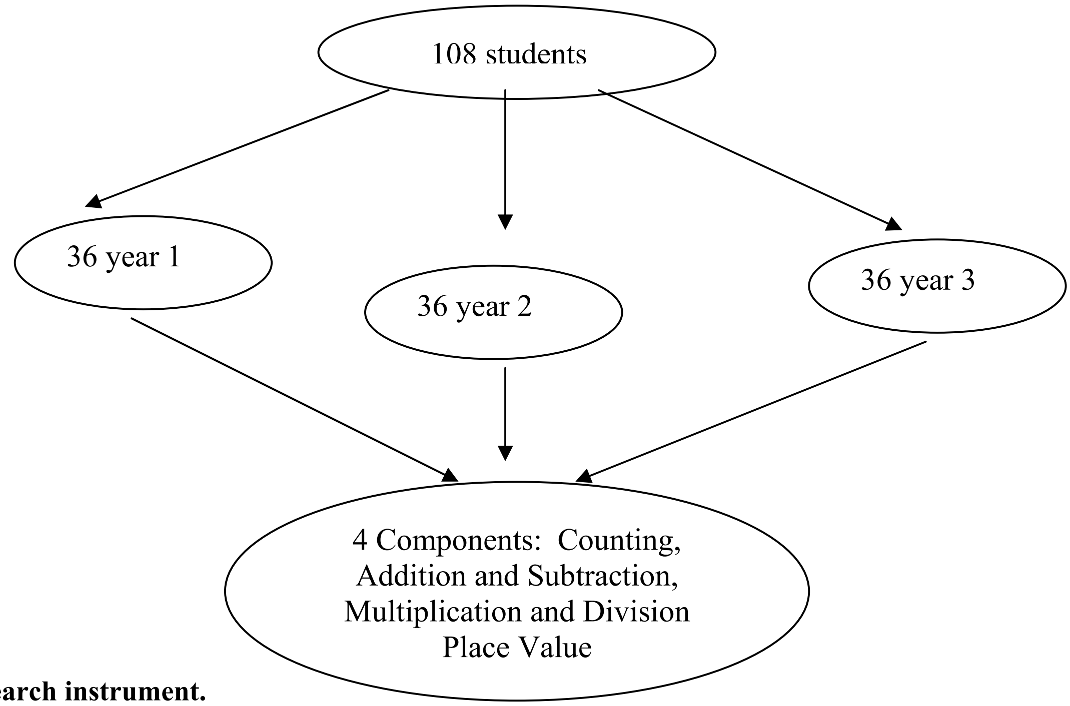
1. Literature review of projects relating to number sense and numeracy such Leverhulme Numeracy Project (UK), Supporting Literacy and Numeracy (Australia) and compare the finding of these researches with the current situation in primary school mathematics. After reviewing the literature, the researchers will build a draft of the proposed number sense framework.
2. Refinement of the number sense framework through workshops with experts in primary school mathematics (teachers, lecturers in Universities and officers from the Centre of Curriculum Development as well as Educational and Research Planning Unit of Ministry of Education, Malaysia).
3. Second refinement of the proposed framework through workshops with teachers and an international consultant from University of Tasmania. A draft of the proposed framework will be used to assess year 1-3 students' number sense.

4. Refining the framework through pilot interview with 30 students ie. 10 each from year 1,year 2 and year 3. This stage will also take into account the students strategies.
5. Assessment of years 1-3 students number sense through interview of 90 students
6. Building the interview protocols
7. assessment of students number sense

Research Sample

Students were identified to fall into three groups of competencies namely good, average and weak. A 30 item test consisting of the 4 components were drawn up and administered to 480 primary one, two and three students. A total of 108 students were then selected for the interviews.

Students involved



Research instrument.

Table 1 shows a summary of the instrument that was developed to assess number sense. Four separate instruments were produced for the different components: counting, addition and subtraction, multiplication and division and place value. The items in each component are categorized into three main representations: contextual, pictures/objects/concrete and symbolic. The items for the two operation components (Addition and Subtraction, Multiplication and Division) included word problems, mental computation, written computation as well as creating problems from a given written computation.

Place value is slightly different from the other components in the sense that it is knowledge that is applicable across the other components. This knowledge serves as an indicator of number sense. Therefore the focus of the place value component is on the assessment of students knowledge about place value which is based on three levels:

Level 1: Developing place value.
Level 2: Expanding place value.
Level 3: Essentials of place value.

Development of the number sense framework and discussion

Referring to the existing similar frameworks from researches done elsewhere in Australia and New Zealand and based on some preliminary studies and excessive review of relevant literatures, we came up with a draft framework in which we categorized the number sense framework into four basic components, namely knowledge of counting, addition and subtraction, multiplication and division and knowledge of place value. The respective representations with which to build the instruments for each component were also discussed such as pictorial, symbolic, contextual, verbal and concrete. While other frameworks focused on the knowledge in terms of number sense that students should have, this framework aims at describing children's strategies in those components. The uniqueness of this framework as compared to other existing ones is that it is concerned with the elicitation of children's strategies in specific domains of mathematics that indicates their number sense.

This draft framework was further refined through a workshop with Years One, Two and Three teachers. In this workshop, the teachers discussed and prepared tasks for the four components of our framework. The tasks include questions for each representations and indications of expected strategies. With this draft framework, a pre-pilot interview was done on 36 students from Years One, Two and Three. This sample was selected from the 480 students who took the entry skills test and were subsequently classified as good, average and weak students. This is to ensure that the students selected were equivalent and comparable in ability across the different schools. Following this, further refinement was done to the framework and a second draft was produced after a workshop with our international consultant who has extensive experience doing research in number sense in Australia and New Zealand. The workshop clarified some of the excessive items of the framework such as repetition of activities, unnecessary elaboration of tasks and so forth. Most importantly, the workshop differentiated between a task and an activity. Amendments were made to the framework and activities were turned into tasks which were considered more meaningful in the context of this research.

A final framework was produced and will be used to interview another 12 students with respect to strategies in counting, addition and subtraction, multiplication and division and place value knowledge. The cycle of refinement of the framework allowed us to shed some light onto specific details of the interview sessions, for example tackling students' shyness in answering questions, adding more strategies into the framework, refining skills in asking questions and probing answers and the like. Technical aspects of the interview (for example, the counting marbles activity) was also looked into, where some objects were found to actually restrict students' movement in employing their preferred strategies. The researchers will observe the interview sessions to get hindsight for further improvement of the framework. Through a cycle of refinement, a final framework was developed, which will be used to interview 108 students from Years One, Two and Three.

Strategies

A categorization of "emergent", "beginning" and "competent" was formulated to analyze the strategies employed by the students. For each of the four components, "emergent" strategies are those strategies which display naïve methods of doing the mathematics problem. For example, a child who gives no answers, counts or adds wrongly or displays premature strategies is considered an emergent learner. "Beginning" strategies refer to methods that exhibit some indication of thinking and employment of strategies of solving the problem that gives the correct answer but with no expert-like strategies. In the counting component, for example, these strategies may be point counting with fingers, counting in twos (with fingers or counters) and so on. Strategies that are considered "competent" are those which demonstrate expert-like method of doing the mathematics. In counting a group of objects, for example, a child who subitises or employs multiplication of rows and columns to get the answer is categorized as competent. Similarly, a child who utilizes mental computation of making tens in adding two two-digit numbers is categorized as competent learner. Therefore, a strategy categorized as competent in the counting strategy may be categorized as 'beginning' in the addition and subtraction component.

Conclusion The framework will focus on basic mathematical concepts namely counting, addition and subtraction, multiplication and division and place value. All the years one to three students in this study will be tested using the same framework. All the four components of the framework booklet consist of questions which will be used in interviews with the years one to three students. What we hope to bring forth is a visible matrix of students' capabilities defining different levels of the strategies employed. The matrix would be able to seek out advanced level of operations as well as the normal band and also those legging behind. This is imperative for mathematics educators to take the cue as to where improvements and innovations can be taken so as to enhance the overall performance to a desired national level.

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