Assessment of General Aims like Ordering, Structuring and Analogising – a decidable task?

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Round the whole world it is normal that in mathematics education one has to learn mathematics. I think this must be so as far as we think of basic mathematical concepts and skills. However the real goal of mathematics education in school can not be the gaining of mathematical knowledge. Beyond some applications of mathematical knowledge in everyday world *the main aim of mathematics education for most people* (that means people who don't want to work in a job where mathematics is fundamental) *lies in the general aspects we can learn while dealing with mathematics*. There are a lot of such generals aims which we can split up e. g. into five dimensions: a) pragmatic dimension (that is: what we directly need or what can help us in everyday world), b) enlightenment dimension (that is: for better understanding of our world), c) social dimension (that is: becoming an active member in groups and working with on social problems), d) dimension of personal factors (that is: all those abilities we need to become an integrated person and a person which can master life), e) dimension of critical reflection (that is: reflecting our acting and the limits of techniques we are using)[1].

Let's now look at such general aims which can be reached in mathematics education. First of all there are many *mathematical concepts which help us to understand the world around* though we have to learn that mathematical concept not always have the same meaning in everyday world. But for men it is typical that we throw our net of concepts over the phenomenons and occurrences for getting comprehension of the world around and to *order our thoughts*. Especially in our modern technological world a precise knowledge of mathematical concepts are necessary. But separate concepts are not sufficient. We also must see important connections, real and logical interlockings as well as systematic backgrounds. Traditionally mathematics concentrated on the aspects of quantification and spatial form. However in modern mathematics the abstract forms, structural aspects and functional connections are more important. Just as in mathematics in our world of today the *structuring of fields of experiences* is a necessary task to master life. Therefore mathematics education can supply us a *special way of thinking for better understanding of our world and better ability to act*. Just mathematics built a wide field for experiences of connections and functional relationships. Moreover alive mathematics demand not only linear, algorithmic thinking (as – unfortunately – it is often to be seen in school) but also creativity, problem-solving, more-dimensional thinking, looking out for connections and structures as well as finding and holding out till the end while solving problems[2]. I think this to learn especially today is a very important task for everybody because of the lot of things we get presented bit by bit.

Now I would like to focus on the question how we can assess general abilities like ordering, structuring and analogising in school. For that I will start with help of examples of mathematics instruction to explain <u>what we have to understand by general aims like "ordering", "structuring" and "analogising".</u>

Already in *grade 1 and 2* the children must order sets in respect to their number of elements. By structuring such sets they build the concept of natural numbers as cardinal numbers as well as the order of them. Simple structuring one also can already train in grade 1 for instance

[1] For more details see e. g. Graumann 2002a (In: Proceedings of the last conference of The Mathematics Education into the 21st Century Project) or Graumann 1993.
[2] Examples concerning geometry you can find e. g. in Graumann 2001 (In: Proceedings of the last but one conference of The Mathematics Education into the 21st Century Project) or Graumann 1989.

with constructing the concepts of even and odd numbers. Moreover the representation with decimal structure of numbers between ten and twenty or later on ten and hundred leads us to structuring our world. Also the finding of connections between the sums 3+6 and 3+7 or 3+6 and 4+5 trains the ability of structuring. To train the strategy of analogising for example we have to handle with the sums 3+6 and 13+6 or later on with 3+6 and 53+6 or 3+6 and 30+60. In geometry teaching the children can order solids (e. g. in form of packaging) and find qualities like sharp, round, etc. or the number of vertices.
In *grade 3 and 4* we then can find a lot of tasks which can stimulate the children for ordering, structuring and analogising. For example the finding out of connections between different series of multiplication (which is already a standard in mathematics education) leads us to structuring the world of the natural numbers. But also working on problems with palindroms (reflection numbers) or sequences of numbers trains the abilities of ordering and structuring. In geometry teaching it is normal to discuss symmetry and patterns, an area where we can train abilities of ordering and structuring very well. A popular topic for training the ability of building systems and structures is the work with polyominos[3]. These figures built of congruent squares which are connected on sides can be ordered in different ways but first of all one has to develop systems in order to find all polyominos for a given number of squares. Also we can train the ability of analogising by comparing polyominos with different numbers of squares or by generalizing the investigations with polyominos in space or with other figures (instead of squares) like triangles, rhombs, regular hexagons or even cubes.
In *grade 5 and 6* a good problem field for ordering and structuring is the theory of divisibility. For instance at the beginning the children have to find the set of all divisors of a given number. For well known numbers like 36 or 44 they normally find them with help of their knowledge from working with these numbers some years before. But if you ask for *all* divisors of numbers like 136 or 144 then you have to develop a systematic method. So the children learn the necessity of a systematic, how you can find a systematic, that there is sometimes more than one systematic and that it is important to hold one systematic through the whole problem. These are typical procedures in mathematics later on but also in everyday life the ability of systematisation and structuring can help a lot for understanding and mastering special topics respectively problems. In geometry teaching for example in grade 5 and 6 we learn different types of triangles and have to order them in the so-called house of triangles with help of the relation "is special case of". A nice combinatorical task where we also can train the abilities of ordering, structuring and analogising is the problem of finding all triangles with lengths of sides 1 or 2 or 3 (respectively 3 or 4 or 5 etc.)[4]. With this problem the students have to develop a systematic but also they can find new understandings like the theorem of the inequality $a + b > c$ for the three lengths of the sides and that a triangle with the lengths of sides $k \cdot a, k \cdot b, k \cdot c$ has the same shape as (is similar to) a triangle with the lengths of sides a, b, c.
In *grade 7 and 8* the engagement with different types of quadrilaterals as well as regular polygons[5] deliver possibilities of ordering, structuring and analogising. The same is right for the dealing with formulas in algebra. Furthermore working on word problems or geometry in everyday life[6] produces a lot of opportunities to train abilities like ordering, structuring and analogising.
In the *higher grades* of course there are many opportunities to train the named abilities. But it is not the right place here to spread out all possible themes for that. Moreover I think until now it became clear want is meant with the abilities of ordering, structuring and analogising.

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[3] See e. g. Graumann 2002b.
[4] For more details see Graumann 2001.
[5] See e. g. Graumann 2001.
[6] See e. g. Graumann 1985 or 1987.

I rather will come back now to our question of the assessment of general aims. At first we can realize that the parts of the named aims which are more technical we can assess in normal tests. So e. g. we can let order the first-graders the numbers 7, 5, 8, 3 and let find them all "neighbour"sums of 4+7 (namely the sums 5+7, 4+8, 3+7, 4+6). Or in grade 4 the children have to put down in a test all shapes of polyominos built of four equilateral triangles. Or in grade 6 the students have to find all divisors of 244 in two different ways and write down the method they have worked with. Though this assessments are important they are not the heart of the assessment of general aims.
In respect to the assessment of most aspects of general aims one has to realize that such aims can not be assessed with a written test. Therefore to get a feed back about abilities like ordering, structuring and analogising the teacher has to watch the remarks and the behaviour of the students while working on a problem. This is possible if the teacher not only presents mathematics and mathematical methods but also from time to time gives problems the students have to work on by themselves. The teacher then can go to everyone or every group and watch only or ask what they are doing and thinking or give hints and discuss the problems and ideas with the students. A very good way of assessing general abilities is to let the students present their work and way of thinking about the given problem to other students and discuss their work as well as their thoughts. By this they also can train the ability of communication and presenting ideas, a general aim which often is trained in mathematics education very seldom. Another way of assessing general aims can be done by discussing with the students a problem from a meta-cognitive standpoint where for example it is talked about strategies directly. Of course this is possible only in higher grades.

If we now ask whether the assessment of general aims like ordering, structuring and analogising is a decidable or an undecidable task so we can say it is a decidable task if we are not only thinking of assessments by tests and assessments that can summarized in a mark or percentage or rank. And we also have to consider that such an assessment is not often precise. But with vague assessments we always have to do (as e. g. already Robert Mager said in the 1960th) and which is normal for most assessments in other subjects. Knowing that there is no other way to reach the general aims - for which education in school is organized originally – also mathematics teachers have to become familiar with such soft methods of assesment. But with considering the named boundaries of assessment we can conclude that the assessment of general aims like ordering, structuring and analogising is possible and therefore a decidable task.

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