

Assessment of a Comprehensive, School-wide Intervention: The CAMMP Experience

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We introduced the Comprehensively Applied Manipulative Mathematics Program (CAMMP) as a summer enrichment camp for elementary students in 1991. Emphasizing fun and manipulative-based mathematics, we introduced students to arithmetic as ways of arranging objects. An early version of the program was named in 1994 the “most innovative and creative” summer camp in North America. As the model developed, the CAMMP program was extended to year-round use by classroom teachers and then to a vehicle for teaching teachers how to teach mathematics. These extensions of CAMMP enabled teachers to become more effective in teaching North Carolina’s comprehensive *Standard Course of Study* for mathematics to students at different ability levels.

Essential Components of CAMMP

Teachers who have had CAMMP training learn an approach to mathematics education we call “Guided Constructivism.” In this approach teachers and students use concrete, representational, transitional, or symbolic manipulatives. Students move up the hierarchy of manipulatives (from concrete to symbolic) as they demonstrate consistent success. In ten years of the program we have never had a student prefer to move “backward” to a lower manipulative level when he/she understood the more general, more abstract manipulative. We have had teachers move students back a level when they were moved upward prematurely.

The CAMMP approach reflects a constructivist orientation that integrates the five learning processes promulgated by NCTM (2000): 1) problem solving, 2) reasoning and proof, 3) connections, 4) communication, and 5) representation. Other essential components of the CAMMP approach center on teacher behaviors and include the following:

- Math objectives taken from North Carolina’s *Standard Course of Study*
- Small, developmentally appropriate instructional groups
- Developmentally appropriate math manipulatives (concrete, representational, transitional, symbolic)
- Math software introduced first at the representation level and continued upward through manipulative levels
- Problem solving context for teaching and learning mathematics
- Calculators and computer software incorporated into math instruction at the representational level)
- Estimation (reasonableness) and checking solution for accuracy
- Student assessment checklist to track growth and adjust instruction
- Pacing guides for sequencing and timing mathematics instruction throughout the academic year

The Thomasboro Elementary School Experience

In 1999, Thomasboro Elementary School in Charlotte, N.C., finished dead last for its math scores among all elementary schools in the state – for the second year in a row. This urban school draws predominantly poor (83% free or reduced school lunch), minority (93%) students. Midway through the 1999-2000 school year, teachers at this school began their CAMMP training through after-school workshops and grade-level team meetings. School-wide training consisted of approximately 30 hours of teacher inservice.

In the 1½ years of implementation (since mid-year 1999), the CAMMP approach has produced:

- an 11% gain among 3rd graders, from 38 to 49 percent on grade level, compared to a 3% gain in schools with similar demographics;
- a 29% gain among 4th graders, from 50 to 79 percent on grade level, compared to a 12% gain in demographically similar schools; and
- a 38% gain among 5th graders, from 45 to 83 percent on grade level, compared to a 14% gain in demographically similar schools.

This data was taken from the public domain and does not include author-collected data or scores of individual students or classrooms. Still, the state’s data shows that Thomasboro has increased student math achievement faster than comparable schools in its demographically similar “feeder area.”

Given the effectiveness of manipulative-based math instruction for Thomasboro elementary students, we wondered whether or not such an approach might have any direct pedagogical benefit for preservice elementary teachers, beyond learning new strategies for teaching. Put differently, given their

previously successful rise through middle school, high school, and college mathematics courses, was there anything about arithmetic these individuals might learn from using manipulatives?

STUDY 1

METHOD

Participants Participants were 53 undergraduate volunteers (68% of 78 students) enrolled in three sections of a child development course taught by the co-authors. Historically, students in this course have had greater difficulty understanding constructivist theory than they have had with more traditional endogenous or exogenous theories. Consequently, both instructors place a relatively greater emphasis on constructivism and its application to classroom pedagogy.

Treatment To illustrate how constructivism can be applied to the elementary classroom, the course instructors incorporated abbreviated lessons with concrete and representational manipulatives adapted from their mathematics education class. The lessons lasted 5 1/3 hours of a 45-hour class. The content of these classes consisted of first building a concrete model, then posing a problem, and finally asking students to use their manipulative models to solve the problem. Students were asked to compare solutions, and the instructor demonstrated with overheads or whiteboard how he would solve the same problem.

Participants actively solved whole number addition and multiplication problems with Cuisenaire rods and base-10 blocks, and then they observed the instructor's use of a place value chart, expanded notation, and partial sums/products. For subtraction and division of whole numbers and for all four operations with fractions, students used only Cuisenaire rods and observed no higher-level manipulations. At no time did the instructors answer student questions about correct solutions. Instead, such questions were redirected to the class, to arrangements of manipulatives, or to white board examples of problem solving. Without telling students about their misconceptions, instructors managed class discussions with questions like:

Do we end up with more, less, or the same amount that we started out with?

How much is on the left side of the "=" mark? How much on the right side?

Can you ever get more/less than you started out with?

Is the equation balanced? What is a balanced equation?

Is it important to balance an equation? Why?

What does the answer mean? (particularly important with division of fractions)

The general format for presentation & instruction was as follows:

Build a Cuisenaire model (whole numbers and fractions)

Present a problem

Have students solve problem and share solutions and manipulations

Model solution for students with emphasis on the action of the problem

Answer or redirect student questions

Pose one or two practice problems

Compare and evaluate different solutions

Repeat process working up the hierarchy only for whole number addition and multiplication (i.e., regrouping with base-10 blocks, Place Value Chart, Expanded Notation, and Partial Sums/Products)

Measurements

Four dependent variables were measured. Two were obtained from an eighteen-item, multiple-choice Math Survey. The other two were free-response performance items.

The multiple-choice Math Survey was constructed to assess both misconceptions and knowledge. Misconceptions were derived from the Tirosh & Graeber (1989, 1990a, 1990b) studies of teachers' misconceptions. We incorporated items that tested false beliefs about operations on whole numbers and fractions, the meaning of the "=" sign, algorithmic procedures, positional meanings in open number sentences, conceptual understanding of computing algorithms, and meanings of mathematics terminology.

Each item in the survey contained four choices: a misconception, a correct answer, and two distracters. A misconception score was obtained by summing the number of times a misconception was selected for the 18 items. Similarly, a knowledge score was obtained by summing the number of times a correct response was selected for the 18 items. These scales were not independent in that for any item, selection of a myth response precluded a correct response and vice-versa. Math Surveys typically took 15 to 20 minutes to complete and were machine-scored.

Following the Math Survey, students completed an additional sheet showing a division of fractions exercise (i.e., $1 \frac{1}{2} \div \frac{3}{4} = ?$). They were asked to first show the actual computation of a correct solution (variable 3) and then to draw a representation (variable 4) of their solution to show what it means. Key to an appropriate representation was a picture or graphic that clearly conveyed the idea of two $\frac{3}{4}$ - pieces of pizza (not two whole pizzas).

STUDY 1 RESULTS

Two dependent *t*-tests were conducted to examine differences between pretest and posttest in *misconception* scores and *knowledge* scores. The means, standard deviations, and sample sizes for these variables are reported in the Table 1. There was a significant decrease between the pretest and posttest for misconception scores ($t=26.05$, $p<.001$) and a corresponding increase for knowledge scores ($t=22.46$, $p<.001$). These differences were very large for both misconceptions ($g=4.66$) and knowledge ($g = 4.40$).

	Mean	Standard Deviation	Sample
Misconceptions			
Pretest	10.00	1.40	50
Posttest	2.32	1.90	50
Knowledge			
Pretest	5.04	1.60	50
Posttest	14.00	2.47	50

Table 1

Discussion

The five-hour treatment, in the context of a child development class, clearly had an impact on reversing arithmetic misconceptions and improving arithmetic knowledge. Misconceptions first identified by Tirosh and Graeber were reduced significantly, and understanding of basic arithmetic concepts were improved significantly. Additionally, significantly more students provided appropriate depictions for our division of fractions problem (2 sets of $\frac{3}{4}$ s, instead of two wholes).

The results presented here surprising. They suggest that the method of instruction is a more powerful pedagogical device than the instructional content. Put differently, Study 1 results indicate the powerful impact of hands-on manipulatives even for adults who have already successfully completed far more advanced levels of mathematics. The most immediate question here is, can these results be replicated with an independent sample?

STUDY 2

METHOD

Study 2 was designed to replicate the findings of Study 1 with an independent sample of preservice elementary students. However, lessons learned in Study 1 led to two specific revisions. First, the Math Survey was shortened to 16 items, and editorial changes were made to clarify language precision. All items retained their original structure: a misconception response, a correct response, and two distracters. Second, in the free response section, a division of whole number computation and picture were added to the original division of fraction problem. Both computations and solutions were scored in Study 2 just as they had been in Study 1.

Participants

Participants were 39 elementary education majors (64% of 61 students) enrolled in three sections of the same child development course described in Study 1. Again, four sessions (5 $\frac{1}{3}$ hours) were utilized in the 30 one-hour and 20-minute class periods.

Treatment

The treatment in Study 2 followed the same timing, duration, sequence, and content as for Study 1.

STUDY 2 RESULTS

Two dependent *t*-tests were conducted to examine the differences between pre- and posttest arithmetic knowledge and misconceptions. Of the 44 participants, 39 completed all the pretests and posttests and were included in the analyses. The means and standard deviations are reported in Table 2. There was a statistically significant increase between the pretest and posttest knowledge scores ($t=14.6$, $p<.001$) and a statistically significant decrease between pretest and posttest misconceptions ($t=11.5$, $p<.001$). Both the increase in knowledge ($g=2.7$) and the decrease in misconceptions ($g=2.1$) were large.

Study 2 Pretest and Posttest Scores for
 Arithmetic Misconceptions and Knowledge

	Mean	Standard Deviation	Sample
Misconceptions			
Pretest	7.21	2.10	39
Posttest	2.87	1.81	39
Knowledge			
Pretest	2.74	1.85	39
Posttest	8.64	2.43	39

Table 2

DISCUSSION

It is reasonable to presume that unless treated directly, elementary teachers will carry misconceptions and misunderstandings with them and transmit them to their own students. In that context, the results reported here are important for three reasons.

First, while widely recognized in the literature (see e.g., Graeber, Tirosh, & Glover, 1989; Tirosh & Graeber, 1989, 1990a, 1990b), teacher misconceptions have not been adequately addressed in their math education preparation. The results of Study 1, replicated in Study 2, indicate that elementary education majors exhibit a substantial number and range of misconceptions about arithmetic. It is ironic that these misconceptions have survived through middle school, high school, and college-level mathematics courses. Equally important is the finding that, even though most of our subjects could correctly compute a division of fractions solution, the vast majority could not provide an adequate representation of the solution's meaning.

Second, one explanation for the relative absence of work in this area could be the labor-intensive nature of one-on-one interviews used to implement conflict teaching (e.g., Tirosh & Graeber, 1990a). The results reported in Study 1 and replicated in Study 2 indicate that remediating misconceptions can be efficiently and effectively undertaken with short-duration, large-class instruction using hands-on manipulatives. With this in mind, it should be relatively easy to implement a five hour module like the one used here in college-level mathematics education courses.

Third, we do not know how arithmetic misconceptions are born, but the results of the two studies reported here clearly suggest that misconceptions can be effectively reversed by using hands-on manipulatives to re-construct meanings. In the treatments reported here, hands-on manipulatives were directly connected to symbolic procedures for whole number addition and multiplication. Yet, misconceptions related to subtraction, division, fractions, and equality were also reduced and replaced with more accurate knowledge. Participants in both Study 1 and Study 2 came with the ability to use the *invert and multiply* algorithm (multiply by the reciprocal) to compute a correct answer to $1\frac{1}{2} \div \frac{3}{4} = 2$. What changed as a result of the treatment is their understanding of the solution's meaning, and this was accomplished with only 1.6 hours of treatment with hands-on manipulatives with fractions. In short, prior to the treatment elementary education majors could compute fractions without understanding. After the brief treatment, they could not only compute correctly, they also could understand what their solution meant.

In the larger sense, the efficacy of using hands-on manipulatives to promote reconstruction of arithmetic concepts and principles can have a profound and far-reaching impact in the elementary school. Instead of carrying misconceptions and misunderstandings into their classrooms, teachers could utilize instructional tactics that prevent both the transmission of misinformation as well as precluding in their students the mis-construction of false beliefs to begin with.