

## **Mathematical principles of graphic systems**

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The new mathematical branch was created in the sixties of the last century – computer graphics. A lot of systems based on its basis were developed and formed the fundament of CAD (Computer Aided Design). Nowadays, mechanical engineering, electronics, architecture and many other branches take advantage of computer graphics.

The development of computer graphics implied necessity of deep theoretical knowledge and led to foundation of so called computer geometry.

Unfortunately nowadays, the computer graphics and its mathematical theory are getting separated very quickly. The theoretical basic is concentrated on mathematical problems like digital topology, the practical application are more interested in creation of computer based algorithms in order to satisfy demands of graphical programs input. The mathematical fundament of computer oriented methods is missing even in specialized computer literature. The literature about computer graphics usually based on vague terms and not exceptionally on incorrect ones. The vague basis of computer graphics was considered to be mathematics even eight or ten years ago. The computer graphics itself was “defined” as processing of graphical information. If the same method would be used in “classic” geometry the line would be defined as the set of points on the line.

The majority of authors nowadays feel this problem but the solution is usually only limited to statement that it is not easy to define these basic terms of computer graphics. As an example we can use introduction chapter of large book about computer graphics in which we can find this explanation of an image and its representation. „If we try to find the definition of an image we meet some problems. They are caused by the fact that the human vision is not a exact measurement system and it cant quantified“. Finally we can say that the majority of computer graphics literature is based on vague terms with absence of correct mathematical fundament.

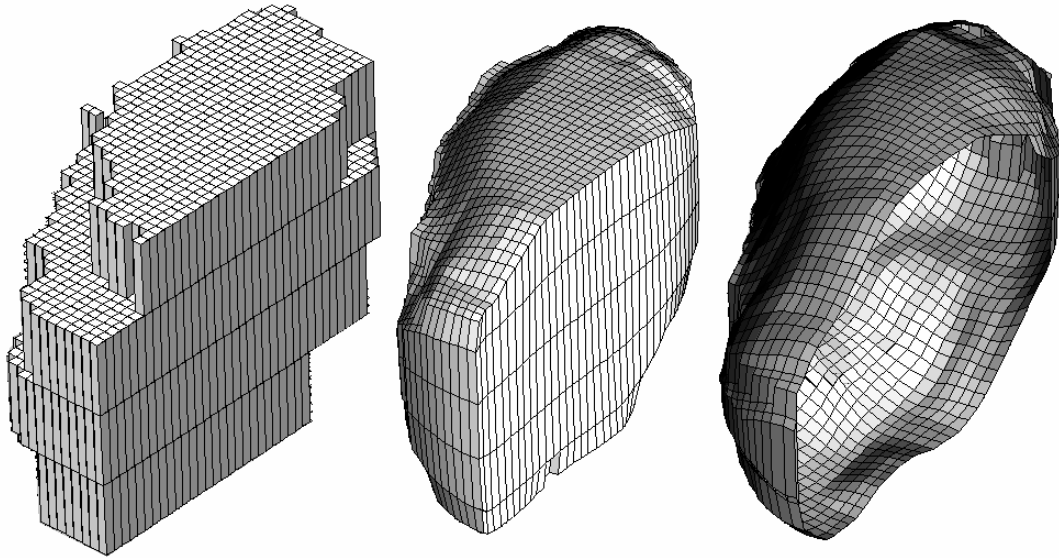
Even if the problem itself is difficult it must be solved. The absence of theoretical fundament is often a limiting factor of development of computer graphics. Many computer programs using the computer graphics with rather professional outlook with astronomical prices often contains fatal errors and are based on mixture of elementary arithmetics and something which may be called „mathematical alchemy“.

There is nothing mysterious on effective computer graphics algorithms. Correct graphical inputs of programs may be reached often by means of simple methods which may be explained often on secondary schools and all that can be done on correct mathematical basis. In the following text we will concentrate on several examples.

### **1. Object filters and surface detection**

Border surface detection is very important problem in computer graphics and data visualization. Today common used methods are based on vector principle and are very complicated. They calculate intersections of "viewing rays" with processed objects. They solve particular events only and are not very consistent. Very problematical terminology is used ("manifold solid", "ring edges" "crease angle", "winged-edge") inclusive of quite incorrect terms ("solid normal", "vector normal to solid", e.t.c). Filter apparatus is very efficient for this problem.

In the left hand part of Fig. 1. we can see the voxel reconstruction of a protozoon Paramecium organella part. This reconstruction is quite raw in itself but it is useable as a ground for more accurately technique.



**Fig. 1. Object moulding by using the 3 - D object filters**

Fig.1. illustrates the using of 3 - D filters which are motivated by 2 - D convolution filters. Left figure is not filtered 3 - D voxel model, the middle one is filtered by means of 3 - D convolution low-pass filter. The right ones is based on boundary gradient detection method.

The using of 3 - D filters to 3 - D data reconstruction is more effectively than nowadays used methods (marching cubes, marching triangles, dividing cubes etc.) and presented comparable results.

## 2. Transparent surface modelling

The opacity plays very important role in some tasks of computer graphics. Displaying of infinitely thin surfaces there is no change in ray direction and the opacity can be constructed by means of mixing the color of surface and color of background. These techniques are usually not used in commercial software even if they are able to increase the display quality of the graphic output.

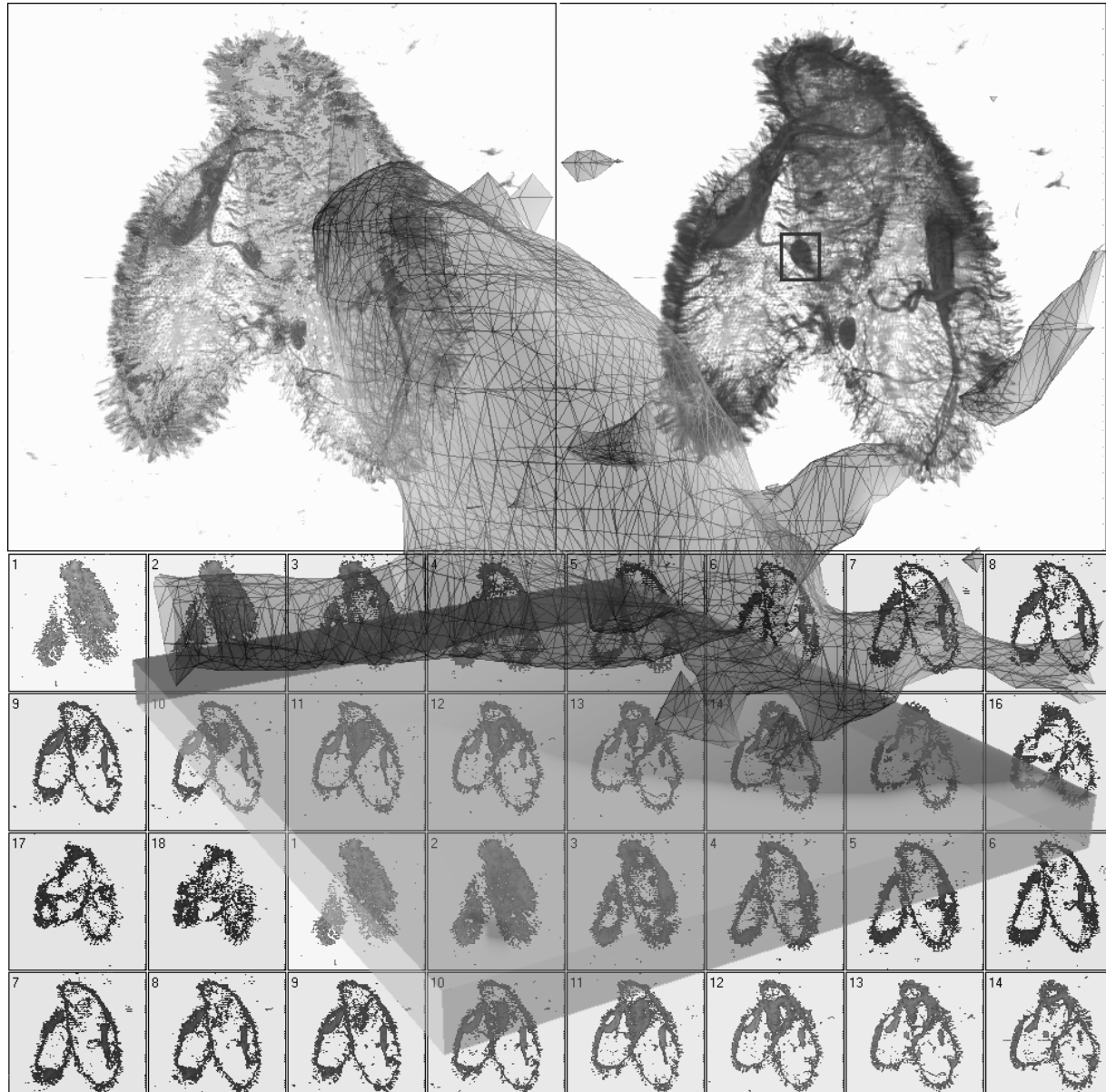
Clebsch cubic surface you may see on the following figure. It is defined by formula in following limits:

$$81 \times (x^3 + y^3 + z^3) - 189 \times (x^2y + x^2z + y^2x + y^2z + z^2x + z^2y) + 54xyz + 126 \times (xy + xz + yz) - 9(x^2 + y^2 + z^2) - 9(x + y + z) + 1 = 0$$

$$x, y, z \in (-0.85; 0.85).$$

the surface on the left is constructed by means of Maple 7 software the right one by explained algorithm.

**Fig 2: Object reconstruction by using of 3 - D filters (object as transparent surface)**

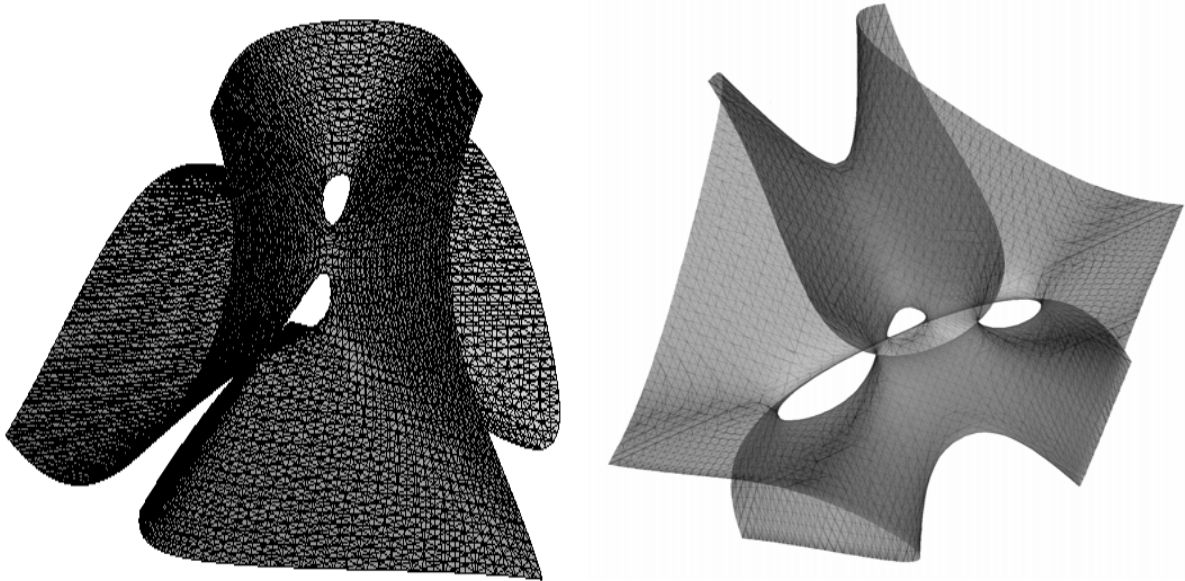


### 3. Optical phenomena modelling

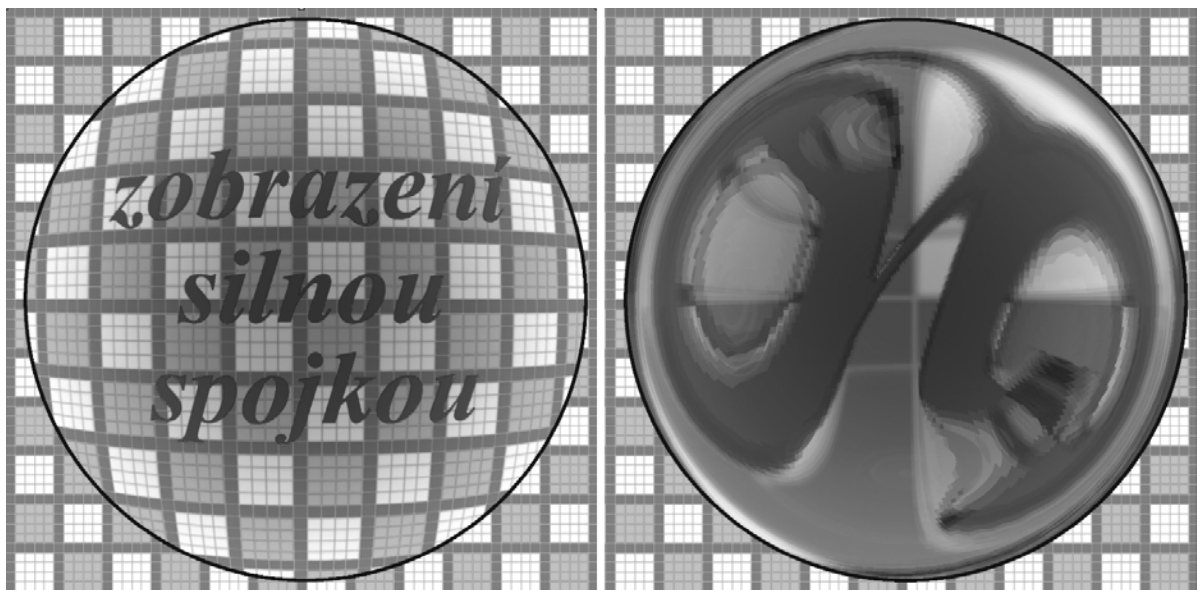
By means of mathematical methods we can model not only passing the light through the infinity thin layers but through real bodies too. As an example let us show mapping by means of positive lens which is formed one by sphere surface of diameter  $R$  and one plane surface. The result may be found using ray tracing taking in account of geometrical optics. The properties of the lens may be studied including in physics known distortions.

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**Fig. 3. Clebsch cubic surface constructed by Maple 7.0. software (on the left) and by the new mathematical methods (on the right.)**



**Fig. 4.: Optical phenomena modelling**

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