The Decidable and the Undecidable in Mathematics Education "Going Beyond The Paradox"

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This conference is held as a tribute to Kurt Gödel, who introduced the theory of "undecidability". His work has contributed significantly to the change of the methodology of science towards "complexity". As a result "uncertainty" characterizes almost all human activities and scientific phenomena. This creates a paradox concerning the "decidable" in mathematics education. To go beyond such a paradox, we might consider the relative "decidable" and "undecidable" in the area.

The present paper deals with some "decidable" and "undecidable" aspects of mathematics education – in the above sense, and attempts to study their roots in the light of complexity. In that sense, the paper discusses some issues in the area, such as; integration, non-formal teaching of the subject, employing multiple intelligences, and "humanizing" of mathematics education. Actually, these trends represent challenges to the current teaching of the subject, but the forthcoming and future trends in the area. Luckily enough, they constitute the base of our project.

Introduction

This conference is held as a tribute to Kurt Gödel[(1)], who introduced the theory of "Incompleteness" (or undecidability). This theorem is one of the most important proven in the last century, ranking with Einstein's Theory of Relativity and Heisenberg's uncertainty principle [(2)]. It ended a hundred years of attempts to establish axioms to put the whole of mathematics on an axiomatic basis [(3)], as Gödel, in this theorem, proved fundamental results about axiomatic systems showing that in any axiomatic mathematical system there are propositions that cannot be proved or disproved within the axioms of the system. In particular the consistency of the axioms cannot be proved [(4)].

In addition to the appearance of the General System Theory, cybernetics and theories which are studying behaviour of systems (eg catastrophe theory and chaos theory), some other major points of departure has led to the methodology of the contemporary science, ie complexity [(5)], among which is the theory of "undecidability" of Gödel has a distinguished place [(6)].

The Paradox

It seems that attempting to study the "decidable" in mathematics education itself carries a paradox that is we must be "certain" about such decidable matters in an era of uncertainty of science. So, we should think in terms of the relative "decidable" and "undecidable" in mathematics education, whether concerning time, place (culture) or some other considerations. Going beyond such a paradox, however, must be looked at in the light of - at least – the following possibilities:

1- There could exist other paradoxes in dealing with the decidable and the undecidable in mathematics education not have been discussed in the present paper.

2- There must be some dialectical relationship among the decidable and the undecidable in mathematics education. So, the whole situation is changeable.

3- There are many different cultural (or local) ways to deal with, ie to implement, the decidable and the undecidable in mathematics education in the framework of the governring principle of "unity and diversity".

The Decidable and the Undecidable in Mathematics Education

To start with some criteria should be established to judge what are the decidable and the undecidable in mathematics education. It seems that literature can help much in this concern, particularly those calls for the future, which have not been yet implemented can be considered as the "decidable". Further judgment can be obtained from the position of such calls from the nature of contemporary science. As for the "undecidable" they can be considered as the controversial issues regarding the implementation of the decidable aspects, keeping in mind the cultural effects, whether in making choices, ways of dealing with or the timing of introducing change.

Within the above framework, the natural approach for identifying the decidable aspects in mathematics education are the paradigm shifts in both mathematics and mathematics programmes (7). We can sum up these paradigm shifts in "humanizing mathematics and mathematics education", being concentrated on serving man, with a high consideration of human needs, problems and aspirations, and recent developments in both human life and thought (8). So, the core of the decidable in mathematics education is that its subject would be problem solving, of course actual or real problems - not artificial ones. This have many important educational implications. Some of them are; curriculum integration (9), the intensive use of mathematical modeling, paying great attention to the cultural context as well as to developing creativity (10). Naturally, problem solving, in the above mentioned sense, requires non-formal teaching of the subject and employing multiple intelligences.

It is obvious that the whole of the suggested "decidable" in mathematics education copes with the contemporary science, being highlighted the unity of knowledge, uncertainty … and so on.

Needless to say, the "decidable" aspects in mathematics education, as mentioned above, represent challenges to the current teaching of the subject, but - at the meantime - the forthcoming and future trends in the area. However, there are still some undecidable aspects in mathematics education, such as; the extend of the use of advanced technology in learning - in general and particularly in mathematics education, the degree of depending on self-education against the "ordinary" teaching as well as many issues related to experimentation, the timing of introducing radical changes …etc.

Conclusions

In conclusion, it is worth mentioning that our project The Mathematics Education into the Twenty-First Century Project, as an innovative project, supports the decidable aspects in mathematics education, paying all efforts to put them into practice in the appropriate cultural context of different cultures. As well, it attempts putting the undecidable aspects in mathematics education in the arena of dialogue, though it may has some stand from them.

Notes

(1) Kurt Gödel was born on 28 April 1906 in Brünn, Austria – Hungary (now Brno, Czech Republic) and died on 17 January 1978 in Princeton, New Jersey, USA.
http://www.groups.dcs.st-and ac.uk/~history/Mathematicians/ Gödel.html

(2) See: http://www.miskatonic.org/godel.html

(3) One major attempt had been by Bertrand Russell with Principia Mathematica (1915 – 13). Another was Hilbert's formalism which was dealt a severe blow by Gödel's results. http, Op cit.

(4) Ibid.

(5) Morin pointed out that:
" … Very briefly, once the idea of the world as a trival mechanical puppet obeying the sovereign order of the laws of nature, in which chance and disorder are mere illusions that will be dissipated through greater understanding, is abandoned, once the second principle of thermodynamics, which a principle of disorder, agitation, collision, and dispersion, is generally accepted, it follows that disorder appears in the universe and complexity is first seen as a problem of irreducibility of disorder. But complexity does not boil down to being merely a problem of disorder; it reappeared at the beginning of the century in quantum physics, as the principle of uncertainty …". Morin, p p. 63-64.
Note the contribution of Einstein, Gödel, Heisenberg, Popper and many others to the appearance of complexity.

(6) As a result, it is no more that thought is controlled by logic, but rather logic is controlled by thought.
See : Mina (October 2000).

(7) The paradigm shift of mathematics can be described as from seeing mathematics as the study of formal systems to seeing mathematics as a living body (Ormell; Rogerson, p. 611). This shift has been reflected in primary school mathematics programmes "from seeing mathematics as a large collection of concepts and skills to be mastered in some strict partial order to seeing mathematics as something people do" (Romberg, p. 3655), and in secondary school mathematics programmes from the "formal" teaching of mathematics to introducing mathematics as a human activity in order to provide a basic preparation of learners for the full participation as functional members of society (Travers, p. 3661).

(8) See: Mina (2002), p. 267.

(9) As actual or real problems are, be nature, integrated, where it is difficult – almost impossible – to deal with their components as related to different separate disciplines.

(10) Developing creativity, with its pre-requisites such as "criticism" and multiple right answers or solutions …, can be means to combat extremity (Thus to combat terrorism).

References

(1) Mina, Fayez M. (October 2000). "The Methodology of Complexity and Prospective Analysis", **Egypt 2020 Pamphlets**, 4. Cairo: Anglo Egyptian Bookshop. (In Arabic).

(2) (2002). "The Role of the Systemic Approach in the Humanistic Renaissance in Mathematics Education". In: Alan Rogerson, **Proceedings of the International Conference of the Mathematics Education into the 21st Century Project on "The Humanistic Renaissance in Mathematics Education** (pp. 267-268), September 20-25, 2002, Palermo, Italy.

(3) Morin, Edgar (1984). "On the Definition of Complexity". In: Aida, S. et al, **The Science and Praxis of Complexity** (pp. 62-68). Tokyo: The United Nations University.

(4) Ormell, C. (Ed.) (1992). **New Thinking about the Nature of Mathematics.** Norwich: MAG-EDU, University of East Anglia.

(5) Rogerson, A. (1986). "The Mathematics in Society Project: A New Conception of Mathematics", **Int. J. of Educ. Sci. Technology**, 17 (5), 611-616.

(6) Romberg, T.A. (1994). "Mathematics: Primary School Programs". In: Torsten Husén and T. Neville Postlethwaite (Eds.), the International **Encyclopedia of Education,** Second edition (pp. 3655-3661). Oxford: Pergamon Press.

(7) Travers, K. (1994). "Mathematics: Secondary School Programs". In: Husén and Postlethwaite (Eds.), Ibid (pp. 3661-3668).

(8) http://www.groups.dcs.st-and ac.uk/~history/mathematicians/Gödel. html

(9) http://www.miskatonic.org/godel.html