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Concept mapping and context in mathematics education

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This paper presents some findings from a broad study that used concept mapping as a tool to explore first-year university students' understanding of key concepts in the South African secondary mathematics curriculum. A concept map constructed by two students is presented and examined in this paper in order to illustrate the kind of connections students made between concepts and to reflect on the nature of experiences these students had while learning mathematics in school.

Introduction

A concept map is a visual tool for representing knowledge relationships. In a concept map (see Mwakapenda & Adler, 2002, p. 62, for example), lines are drawn between pairs of concepts to denote relationships between concepts. Linking words on the lines indicate how pairs of concepts are related. In this way, propositions indicating particular relationships between concepts can be discerned. Concept mapping has frequently been used as a pedagogical tool to help students "learn more meaningfully" and form a "conceptual understanding of the subject" (Novak, 1990, p. 943). Concept mapping has the potential to make a knowledge discipline more "conceptually transparent" (Novak, 1998, p. 162), and to "convey ideas that are not easily put into words" (Raymond, 1997, p. 1). Concept meanings are constructed by determining relationships between concepts. "The network of propositions interlinking a group of concepts tells us much about the meaning of the concept from the perspective of the map makers" (Roth & Roychoudhury, 1992, p. 357). In concept mapping, interrelationships between concepts are an "essential property of knowledge" (Ruiz-Primo & Shavelson, 1996, p. 592). The flexibility of concept mapping makes it a useful tool for investigating a wide range of aspects associated with student learning in mathematics. Raymond (1997) reports that there has been little reference to the qualitative use of concept mapping in mathematics education research. There is a need to explore the enabling or constraining aspects of concept mapping as a tool in mathematics education research.
The study set out to answer the question: What do the concept maps and follow-up interviews on these tell us about students' understanding of specific mathematical concepts and their experiences of school mathematics? A concept map drawn by two students is examined here.

Design of the study

The study involved first-year students from the 2001 University of the Witwatersrand intake belonging to three groups[1]. The study involved three quite differently positioned groups of mathematics students both in terms of previous school performance and their current university enrolment. Participation in the research was purely voluntary. The research had planned to involve 30 students (10 from each group). 22 students (3 from the Mathematics major course, 9 from the Foundation mathematics group and 10 from the College of Science group) volunteered to participate in the research. After being introduced to concept mapping and the processes involved in constructing concept maps, students were asked to construct a concept map to show how the following concepts were related: ratio, parallel, function, tangent, infinity, perpendicular, inverse, zero, equation, limit, absolute value, similar, gradient, angle, variable, bisector. These 16 concepts were selected because they cut across the algebraic, numerical, graphical and geometric settings of secondary mathematics in South African schools. The aim here was to see whether and how students could and would link concepts across topics that are usually fragmented in the curriculum. These concepts occurred

[1] Group 1: students with at least a 60% pass on the Higher Grade Matric (Grade 12) mathematics examination and who had enrolled in Mathematics major. Group 2: students with at least a 60% pass on the Standard Grade Matric mathematics examination and were enrolled in the College of Science (an access college). Group 3: students who did not obtain a 60% pass on the Standard Grade Matric mathematics examination who had enrolled in the Foundation mathematics course.

frequently across the mathematics texts and assessment items and were considered to be key concepts in the mathematical content students would have studied at the secondary school level. Although familiarity with the concepts could be assumed, connecting them in the form of a concept map was not a familiar task, hence the elaborate introduction to them on the nature of the task (Mwakapenda, 2001).

Reflective interviews were conducted with students because of two reasons. First, students' concept maps did not include any linking words or phrases to indicate relationships between concepts. This occurred in spite of having stressed the importance of inserting linking words to denote concept relationships in the introduction to concept mapping. These maps "left us wanting to know more" (Wilcox, 1998, p. 466), to explore what these students knew about these concepts and their links and how they came to know these. Secondly, Allchin (2002) has argued that concept maps are "inherently selective. They can only represent *selectively*, based on the mapmaker's purpose" (p. 146, emphasis in original). While a map is a model of reality, one needs to understand the map's "context" (Harley, 1988) in order to appropriately interpret how it represents that reality. The map externalises only a part of an individual's thoughts (Roth & Roychoudhury, 1992). Reflective interviews were therefore critical in order to probe students' thinking: to give an opportunity for students to explain and elaborate on the meanings in the links, and to ask them to provide examples to illustrate these links.

Data analysis: In many concept-mapping studies, the analysis of concept maps is predominantly quantitative and proceeds by scoring various aspects of student maps such as the presence of hierarchy levels, propositions, links and cross-links, and specific examples provided to illustrate links (Ruiz-Primo & Shavelson, 1996). While scores may indicate the extent to which a student is able to make connections between concepts in a subject, "any map scoring procedure reduces some of the richness and detail of information contained in a concept map" (Novak & Musonda, 1991, p. 127). In this study, some rudimentary counting was used only to show the degree of interlinking displayed in the maps. This involved counting the number of linked concept pairs (White & Gunstone, 1992), the number of linking phrases used, the number of concepts used or omitted from the 16 concepts given in the task, and what and how many extra concepts or terms students included apart from those in the list. The maps were examined to determine whether students regarded certain concepts as central in developing links among concepts. The central concepts that students used were identified. The meanings students associated with these concepts were examined. Students' descriptions of their maps were then analysed to examine the adequacy of connections made. As well as providing insights into the meanings and nature of links between concepts, the analysis also raised questions about students' understanding of specific concepts.

Findings

Detailed findings of this broad research has been presented elsewhere (Mwakapenda, in press; Mwakapenda & Adler, 2002). In this paper, the focus is on the data from two students, S1 and S2 (both from the College of Science group), who jointly produced a concept map to show how the concepts given in the task were related. This focus is intended to illustrate in some detail what we can learn about students' understanding of specific concepts by examining their concept map.

Figure 1 below shows the concept map drawn by S1 and S2.

Figure 1 shows that S1 and S2 used 13 out of 16 concepts given in the task. The map consists of two main branches under the term mathematics, the left branch beginning with angle, and the right with equation. Structurally, the map suggests that S1 and S2 saw the given concepts as falling into two categories: the geometric and algebraic. No substantive links can be seen between the ideas in these categories apart from an indication that all these are ideas in mathematics (i.e., "mathematics can be [in] geometry form, e.g. 'angle'" or "mathematics can be in algebra form, e.g. 'equation'" (see linking phrases at the top of Fig.1)).

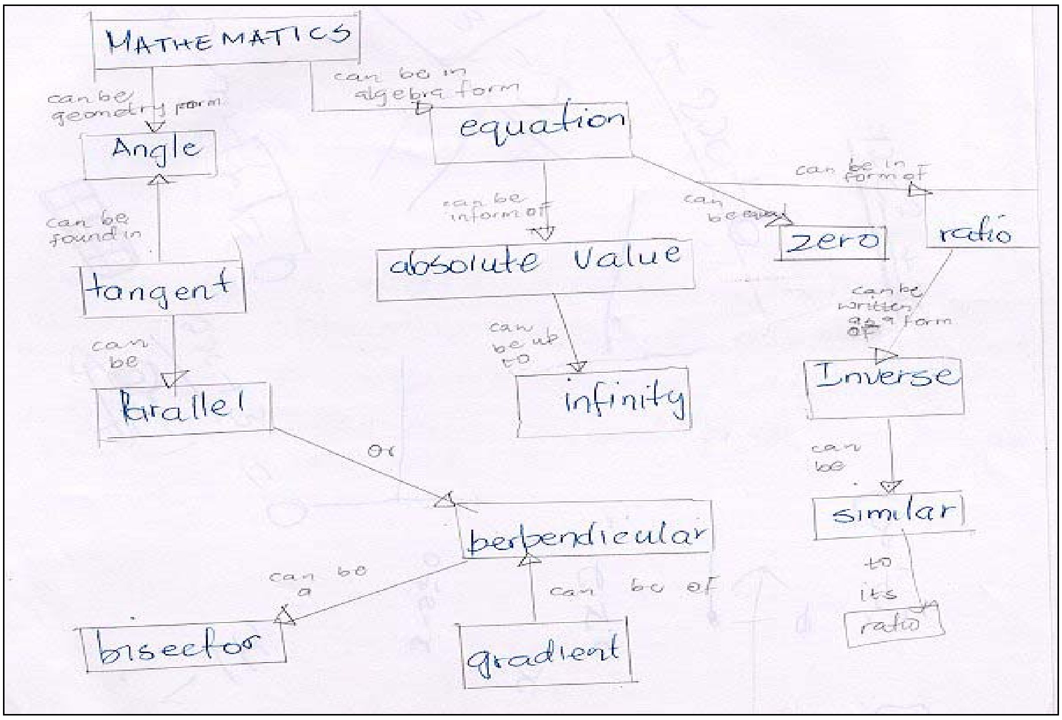

Figure 1: S1 and S2's concept map

What can be said about the *linking phrases* that have been used in the map? S1 and S2 have frequently used the term "can be" to express links between concepts, for example, "angle *can be* found in tangent", "tangent *can be* parallel" and "equation *can be* in form of absolute value". A number of questions could be raised here. What does the term "can be" as used by S1 and S2 mean? What does it mean to say, for example, that "angle *can be* found in tangent"? What kind of meaning does this statement have in mathematics? In what way could this statement be mathematically adequate? How could we know more about the type of meanings that S1 and S2 intended for this statement? In order to respond to some of these questions, I examine S1 and S2's elaboration of some of the links on their concept map: links between geometric concepts involving angle, tangent, parallel and perpendicular, and links between algebraic concepts.

Links between angle and tangent

S1 and S2 said the following when asked to describe their links (WM = Interviewer).

S1: In high school we are told that maths is divided into two, geometry and algebra. So we thought in geometry we can find angles. We can find angles. OK. Angles can be found in geometry. That's from maths anyway. And we thought of tangents. Tangents like the theorem in Grade 11, theorem 9 talking about tangent. Yah. So we thought of, it was, they were both tangent and angles... Tangents can be found in angles. That's what we did here (pointing at the map).

WM: Would you like to show an example here? You say tangent can be found in angles.

S1: Yah but it is not possible. We just thought that we can connect even though we did not know how to connect them. But they can connect that's why we did that, that tangent can be found in angles. And we thought of what's the, the Grade 11 theorem, theorem 9. Yah it was talking about angles and tangent. So we thought angles and tangent can be found in the same place, so we just connected here. We were not sure. We just connected them because that theorem was talking about tangent and angles.

S2: From this theorem, we are not sure whether there was something that is connecting angles and tangent. But we know that these two they come together somehow. (S1 agrees). So we are not sure whether they work together like they connect physically or there is something between them that connects the two.

In the above excerpt, we see that S1 and S2 were able to recall a specific mathematical topic (theorem 9 in this case) which dealt with tangents, and when this topic was taught. However, there is a degree of uncertainty in their knowledge of concept links in this topic. "From this theorem, we are not sure whether there was something that is connecting angles and tangent. But we know that these two they come together somehow". S1 and S2 also seem to see mathematics as a kind of big "container". This can be seen from statements such as "angles can be found in geometry" and "tangents can be found in angles". Their comments reflect a compartmentalised view of mathematics, a view that seems to originate from what S1 and S2 were "told" about mathematics in school.

Links between tangent, parallel and perpendicular

S1: Just like tangent, again we thought of theorem, theorem… I don't remember whether it was 8. Yah tangent can be parallel. We said tangent can be parallel or can be perpendicular. Yah they can be parallel or they can be perpendicular… We said a tangent can be a perpendicular line. This one we were not really sure about it. We just thought of including it.

S2: Yah because when we saw this task, it was like we were not sure which words were going with which.

S1: Yah we were not sure.

S2: But we know that gradient and perpendicular fall under geometry. So they have to be connected somehow.

S1: Yah. We know they are in geometry and they are really close, I mean.

S1 and S2 were able to "recall" a theorem that involved the concepts tangent, parallel and perpendicular. However, these students were not able to provide explicit details about the theorem. What is clear, however, is that S1 and S2 were certain that these concepts were "connected somehow" since they dealt with these in their school geometry. They were, however, not able to describe the nature of the links between these concepts.

The above descriptions indicate that students' abilities to make links between concepts, though limited, seem to be supported by their recollection of specific learning experiences in school mathematics: what they were told; the topics they learned, and when this occurred. There is limited success in recalling detailed information about mathematical topics. Without access to this detailed information, students seem unable to make adequate *conceptual* connections between tangent and angle, for example. We can see that S1 and S2 could not remember the theorem relating tangent and angle. They could not even provide examples to illustrate this link. However, S1's comment: "if we didn't do maths at high school, then we couldn't even have any clue" indicates that the links these students made, though limited, drew on their experiences in learning school mathematics. A question to be raised here is: what kind of learning experiences did these students have while in school? To what extent do these experiences help students to engage effectively in mathematical practices that involve talking about concepts and making links between them?

Links between equation and other "algebraic" concepts

S1 and S2 said the following in order to elaborate on the links between equation, absolute value, zero, infinity, ratio, similar and inverse.

S1: Algebra can be in form of equations. Like x squared minus 2x plus 6 ($x^2 - 2x + 6$), it is an equation (S2 agrees)… Again this one is very complicated. We said an equation can be in the form of absolute value.

WM: What did you mean here?

S1: Here we said like a mod of 2x like we this is equal to zero. It is in form of equation but in an absolute value. And then we said absolute value can be up to infinity. I mean we don't know where it'll end. So we just said they can be up to infinity.

S2: Like when you say x plus 2 is greater or equal to zero and x minus 2 is greater or equal to zero. So x is, no, this side is greater greater than 2 and then x. Ooh, what am I doing? x is greater or equal to minus 2 (S1 helps). x is less or equal to minus 2 and then x is greater or equal to 2. x is here. So x is less than 2.

S1: And we said like a equation can be equal to zero. Obviously here there can be $x^2 -2x+6=0$. This is an equation. And an equation can be in the form of a ratio like $(2x +3)/(2x-1) = 0$. This is an equation and it is in a form of a ratio. And we said again a ratio can be written as a, like in terms of an inverse. Like you can an inverse of what can I say? An inverse of let me see x over 1. It's inverse will be 1 over x. Yah. A ratio can be written in the form of an inverse. Here we said an inverse can be similar to its ratio. They are similar but they are not equal. I mean x is here and x is here. 1 is here and 1 is here.

The above excerpt indicates that S1 and S2 were able to describe links between the concepts equation, absolute value, zero, infinity, ratio, similar and inverse. In contrast with the "geometric" concepts presented earlier, S1 and S2 were able to provide some examples of mathematical situations that involved algebraic concepts. We can also see that S1 and S2 were relatively less uncertain about the links they made between algebraic concepts than they were with geometrical concepts. However, a number of questions can be raised concerning the degree of adequacy of the links shown here. In what way can we say that "*x* squared minus 2*x* plus 6" is an equation? What does it mean to say: "an equation can be equal to zero"? What does the statement "an absolute value can be up to infinity" mean? What does the use of the term "can be" imply here? Does it indicate some level of "uncertainty" in S1 and S2's descriptions of concept links or does this indicate that there are other possible ways in which the concepts may be linked?

Unlinked concepts

S1 and S2 did not include the concepts "variable", "limit" and "function" in their map. Reflective interviews indicated that S1 and S2 had difficulty connecting the concept "variable" with other concepts because they did not seem to have understood this particular concept. The lack of understanding of this concept could be attributed to two factors: ineffective teaching/learning ("we did understand it really"), and lack of teaching ("they didn't teach us"). While S1 and S2 learnt about the concept of "limit", they could not see how this concept linked with other concepts. S1 and S2's comments provided some insight into the type and level of mathematical knowledge that these students had access to in school. S2 indicated that she wasn't taught about "variable" because her teacher thought that this concept was for "Higher Grade".

Emerging issues

Concept mapping has been described as being instrumental in helping learners to "organise and reflect on their conceptual understanding" (Roth & Roychoudhury (1992, p. 532). Malone and Dekkers (1984) have proposed that concept maps are "windows to the minds of the students we teach" (p. 231). More generally, maps provide insight into "fundamental questions about how humans see and depict the natural world" (Turnbull, 1993, p. 1). What questions does the above analysis raise about S1 and S2's understanding of mathematical concepts presented here and how this understanding is acquired? The concept map constructed by S1 and S2 is relatively simple, largely linear and has virtually no cross-links. There are no cross-links between concepts in the algebraic and geometric categories (see Fig. 1). Also, S1 and S2 did not include all the given concepts in their concept map. To what extent does this suggest that these students have a poor understanding of mathematical concepts presented here? How effective has concept mapping been as a tool to explore this understanding?

A key assumption in concept mapping is that concepts are linked with other concepts rather than being seen as entities on their own. Apart from being linked to other concepts, they are linked to broader contexts such as theorems associated with the learning of school mathematics. S1 and S2 seem to have found it easier to recall mathematical topics or situations in which particular concepts were learned than to describe how such concepts are connected. Rather than describe substantive connections, these students merely gave what Novak and Gowin (1984) have called "instances" designated by given concepts. It appears here that students were more able to display *contextual* links than to express understanding of *conceptual* links. Does this analysis suggest that concept mapping was less successful at enabling S1 and S2 to recall substantive ways in which concepts are connected but enabled S1 and S2 to *remember contexts* in which concepts were learned?

Why were these particular students unable to adequately describe conceptual links between concepts? When does reference to context rather than subject content become an issue? It appears here that reference to context may become more predominant for those students whose knowledge about mathematics generally and mathematical connections, in particular, are not adequately developed[2]. When these students attempt to make connections between concepts, they may find it easier to recruit their knowledge of the situations, which structured their learning of these concepts. "We just connected them because that theorem was talking about tangent and angles" (S1). The lack of fluency in describing links between concepts suggests that there may be specific ways of *talking* about and *expressing* mathematics which these students seem not to have adequately developed while in school, assuming that talking about and expressing mathematics was a feature of school mathematics practices that these students may have experienced. Arising here is the question of how the effectiveness of concept mapping relates to students' expertise in expressing mathematics and understanding of mathematics as a language. The fact that these students showed uncertainty about the links they made suggests that they may have "understood" these mathematical concepts only when learning and using these concepts in school and not much longer afterwards. This result is not unusual given the culture of many classrooms whereby mathematics is experienced as an inert subject, taught as a disconnected set of facts and algorithms rather than as a "living body" (Rogerson, 1986) of knowledge concepts that are related to each other and to other knowledge disciplines.

Although being relatively "incomplete" (Westbrook, 1998, p. 90) and inadequate, the connections and mathematical meanings made by S1 and S2 need to be considered important as they could contribute towards understanding the context of mathematics education. Such connections may be deeply rooted in diverse experiences in which learning takes place. Some of these experiences may lead students to have limited access to mathematical knowledge and hence constrain students' abilities to engage

[2] S1 and S2 were enrolled in the College of Science, an access college at Wits University whose aim is to build up students' understanding and of knowledge subject matter so that they are able to engage more confidently upon subsequent enrolment in the mainstream courses.

effectively in mathematical practice. For example, when asked to explain why they did not use certain concepts in their map, S1 said:

> *We didn't do inequalities in Grade 11. So when we go to Grade 12, you have to fill out those forms say you want to do maths at Higher Grade 12. They just come in to class and say, "Who wants to do maths in higher grade?" And when you raise up your hand they just say, "Aha, do you know inequalities?"... If you say no, they just say, "Standard Grade is for you. Fill up Standard Grade".*

The critical role of teachers is explicit in the above comment and suggests that explanations for inadequate connections by learners may be located in mathematics teaching. In this connection, Zwaneveld (2000) has argued that mathematics teaching "pays little attention to the structure of the mathematical concepts presented". As a consequence, the "network or relationships between these concepts does not become a part of [students'] mathematical knowledge and skills, and is consequently not fully available for purposes of reasoning, proving, mathematising and solving problems" (p. 393). Roth and Roychoudhury (1992) have argued that even though instruction may attempt to teach for connections, "textbooks and teachers can never provide all possible connections. Besides, no matter how many formulations there are and how explicit they are, students will always have to construct their own ways of expressing the relationship between pairs of concepts" (p. 547). S1's comment shows that these explanations need to be broadened to include contextual and political aspects such as the nature of the curriculum to be taught, to whom this is taught, and an understanding of this curriculum in terms of what knowledge need to be made available to learners, when this is to occur and why. When a curriculum is organised in such a way that it does not explicitly show potential connections between concepts, it is highly unlikely that teachers could steer learning to show connections. These findings suggest that there is need to allow students to reflect on aspects concerning key concepts in a discipline and how these concepts may be related. Concept mapping provides entry into reflecting on these connections. When making sense of students' concept maps, there is need to reflect on the context in which the map was produced.

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