Could metaphorical discourse be useful for analysing and transforming individuals' relationship with mathematics?

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Summary. *Several studies today deal with the influence of beliefs, attitudes and emotions on the learning of mathematics: a process which is no longer considered exclusively cognitive.*
The main results of didactical research in this field are summarized, before a short description of the most recent contributions from Neuroscience about the connection between cognition and emotion. The general features of metaphorical discourse are then described, particularly underlining how it can be considered an efficient transformation mediator of our cognitive reality. New goals in the use of metaphor will be therefore evident, in addition to those already reported in mathematics education literature. Considering that metaphorical discourse, in the wider meaning of the term, does not concern only our rational mind, it could allow a deeper and more global involvement of the individual, what is necessary to a meaningful knowledge construction.

1. Introduction

For a long time the learning of mathematics was seen mainly as a cognitive problem: researchers have investigated on the specificity of mathematical terms and concepts, on the different phases of conceptual construction, on the influence of specific representations, on the learning process and so on.

The result of such researches were several theoretic models for the description of conceptual construction and resolution strategies. Then, with the consideration of the learning process as a social negotiation of meaning inside the classroom, the scenario widened and in the last decade several studies regarding the influence of beliefs, attitudes and emotions on mathematics learning have been developed.

Rosetta Zan's text on problems and beliefs (1998), offers a wide range of such researches, linking her studies to the likewise international inquiry. Beliefs, or better, the systems of beliefs, influence the perception, and thus the learning, of a certain subject. They also lead the control processes that characterize problem solution activities. Not only the individuals' beliefs towards mathematics as a discipline have been investigated but also those wider beliefs concerning the features of intelligence, in addition to those that each individual builds on itself, particularly with regards to the sense of self-efficacy.

Furinghetti e Pehkonen (2000) describe in such a general way the function of beliefs: "(a) beliefs form a background system regulating our perception, thinking and actions; and therefore, (b) beliefs act as indicators for teaching and learning. Moreover, (c) beliefs can be seen as an inertial force that may work against change, and as a consequence, (d) beliefs have a forecasting character." (p. 8-9).

Before a short description of the most recent contributions from Neuroscience about the connection between cognition and emotion, this work will describe the main results of the research on attitudes and beliefs of mathematics teachers, who surely have a great influence on their students' attitudes and beliefs. The general features of metaphoric discourse will be then described, particularly underlining how it can be considered an efficient transformation mediator of an individual's cognitive reality.

2. Mathematics' teachers emotions, beliefs and attitudes

With reference to mathematics' education the interest in studies about attitudes and more generally about affective factors, has greatly developed since the '70s, with researches originally regarding the students alone.

The subjects taken into consideration have been numerous and have analysed, for example, the existence of a relation between the output and the attitude, the variables which affect the attitude, the possibility to control these same variables, the evolution, during the school experience, of students' attitudes towards mathematics' and particularly the reasons of the deterioration which occurs in students between the beginning of elementary school and the end of the high school.

As teachers' influence on students' attitudes, beliefs and emotions is unquestionable, the need to widen the research field soon occurred and since the end of the '70s several researches regarding teachers developed, considering problems similar to those analysed with relation to the students.

A recent contribution by N.A. Malara and R. Zan (2002) offers us the present picture of the situation. Several inquiries were lead into teachers' beliefs about mathematics as a discipline, about the teaching-learning process and also about the students' themselves.

As it occurs in the students' case, also teachers' emotions hugely affect their decisions, widely influencing the whole teaching-learning process. The features of teaching environment, like the need for a given time for a new subject, governmental programs prescriptions, the number of students per class, are perceived and interpreted by teachers according to their goals and beliefs thus creating emotions which affect their decisions.

Time, for instance, is often a source of anxiety but there are also deeper reasons for this: for example the difficulty of the teacher in managing an uneasy class situation as an open discussion. In these circumstances "teachers can feel 'fears', such as fear of criticism, fear of hostility, fear of loss of control". (N.A. Malara, R. Zan, 2002)

It is evident that this is all linked to the teachers' ideas about the "good teacher" features, often seen as the one who constantly has to display his knowledge and never give his students' any suspect about his preparation. This vision, in the end, is linked to a vision of mathematics as a subject without place for doubts, with no kind of uncertainties (R. Zan, 2001).

Considering the answers of a questionnaire directed to a hundred high school teachers, all graduated in mathematics, we have noticed that the idea that one of the main tasks of a teacher is to follow (on blackboard) the entire program, is really widespread. This, of course, does not consider the creation of a class situation of involvement for every student. So, the idea of teaching as a simple knowledge transfer, the idea of the student as a "tabula rasa" and of the teacher as a knowledge transmitter seems still profoundly rooted in many teachers. We however have to say that many of them, feeling the need for a change, try approaches different from the traditional frontal lesson and claim to be open to more constructive methods (A. Pesci, 2001). We have already underlined the contradiction between what is declared and what is really practiced in class.

What is strongly brought to the surface, from didactical literature, is the urgency for deeper researches into these subjects and for more precise interventions, with anthropologic approaches; that is to say conducted in teachers' natural environment, for example with new modes, like the use of tales and metaphors, which could simplify the emergence and a new elaboration of their 'tacit knowledge'. (N. A. Malara, R. Zan, *ib.*)

The starting point of an experience with a group of teachers (A. Pesci, 2003) was precisely a path through metaphors, starting from the ones that each of them chose to describe his relation with mathematics, according to his past experience as a student and his present one as a teacher and towards the reach of consciousness of such relation and its evolution.

It was a proposal of reflection put into practice with methods facilitating the emergence and the transformation of emotions: autobiographical memories, verbal and non-verbal communication modalities (gestures, actions, sounds, objects...), performative improvisation.

The goal of the present contribution is not the description of such an experience but the explanation of the deep reasons of its origin, particularly those referring to metaphorical discourse use. It will be evident that we thought of this use in a new mode and the main aim here is to propose such reflections, so that they can be shared, deepened and developed even by other researchers, in the direction of an improvement of individuals' relation with mathematics.

3. Cognition and emotion: some contributions from Neurosciences

The interest on studies about metacognition, which in the field of mathematics education has developed mainly in the research on problem solving, has provided several results.

Particular attention is paid on two categories of metacognitive behaviour, different but related: the knowledge of one's own cognitive richness and thought strategies and also the control of one's own processes or self-regulation (A. H. Schoenfeld, 1987, R. Zan, 2002). Didactical interventions following these studies were intended to develop or to rescue students' metacognitive skills; however the need to widen the theoretic field soon occurred, also considering emotional and affective aspects in the learning process: "It seems clear that the cold cognitive aspects of learning are only part of a much larger system that influences development; indeed, the purely cognitive aspects may be less primary than we like to think they are" (A. Brown, J. Bransford, R. Ferrara, J. Campione, 1983, quoted by R. Zan, 2002, p. 147).

With reference to the terms *belief* and *emotion* it is necessary to underline that we do not have any interpretation uniformity in the existing literature. On the first of the two, for example, an article by Furinghetti and Pehkonen (1999) presents a serious analysis of the different meanings which are usually given to the term and stresses the need for a deeper theoretic research. Maybe the problem concerning the term *emotion*, often linked to that of consciousness, is even more complex.

It could be interesting, as far as the present work is concerned, to underline some essential results of neuroscience research, even if with no excessive detail.
A generally shared fact is that the brain, the organ in which is represented the whole knowledge of the world, both outside and inside, of an individual, can be considered as built on systems resulting from an evolution, the main aim of which is to help the individual to survive.(A.R. Damasio, 2000, J. LeDoux, 1998).
Emotion and cognition are two particular, closely connected sub-systems of the brain.
What sets the possibility, for human beings, to build knowledge is the existence of consciousness, which allows to know that we have an emotion.
Emotions affect the functions of several brain circuits: the variety of emotional reactions is responsible of the deep changes in the landscape both of the body and the brain. (A.R. Damasio, 2000, p. 70).
For what is concerning the types of emotions, the distinction between the six *primary* or *universal emotions* (joy, sadness, fear, rage, surprise and disgust) and the *secondary* or *social emotions* as embarrassment, jealousy, blame and pride, is well spread.
To these A.R. Damasio adds what he calls the *background emotions*, like wellbeing and malaise, calm or tension. (*ib*., p. 69). The importance of background emotions, differing from the others even in how they are induced (the former from an internal source while the latter from an external one) and in how they manifest in individuals (the former with more internal reactions while the latter with muscular-skeletal and visceral reactions) is remarkable, having a more global control function than the others. For example, background emotions are considered directly responsible of what is commonly referred to as *mood* of a person: a particular background emotion can last in time and determine a mood. (*ib*., pp. 412-413). In conclusion emotions are the basis for our identity and our personal construction of knowledge, in a deep connection with our autobiographical history. (*ib.*, Chapter 6)
Another interesting result is the fact that the interconnections between emotion and cognition, which affect several brain sub-systems like those connected to memory, remain mainly on an unconscious level, that is they are not part of an individual's consciousness. Le Doux distinguishes, at this concern, an explicit memory about emotions, of which the individual is conscious and an implicit emotional memory, which operates unconsciously but can deeply affect the behaviour of the individual in specific situations.
This observation seems particularly meaningful with reference to the fact that research methodologies inquiring into beliefs and attitudes generally use questionnaires or interviews, which are both based on individuals' memories. Thus, it can occur that the collected data are not responding to the actual behaviour of the same individuals' in specific situations; for example, really uncomfortable circumstances could provoke automatic responses, not stored into the explicit memory but present on an unconscious level, thus not being controlled on a cognitive level (LeDoux, in W. Schloglmann, 2002).
The described picture surely outlines how difficult is the research on beliefs and attitudes but at the same time it also illustrates how it is impossible to leave out of consideration the emotional aspects if we want to inquire into or intervene on knowledge construction processes.
Neurological studies also put in evidence the specializations of the two cerebral hemispheres: the left hemisphere is specialized in logical, analytical and rational verbal language, the right one in imagistic, metaphorical, symbolic, non verbal language.
When we need to re-organize our own attitudes and emotions to access our resources, it seems necessary to reach our right hemisphere, and metaphor is a privileged modality. To facilitate the reach of this hemisphere we need to reduce left hemisphere's activity, without using direct logical reasoning but rather a symbolic discourse, such as the metaphorical one. (Damasio, 2000, p. 27, p. 171, Watzlawick, 1984).

4. What is metaphor

If we consider a dictionary we can find, for the term "metaphor", the following definition:
"Substitution of a proper term with a figurative one after a symbolic transposition of images: *the spikes are waving* (as if they were a sea); *the sea is howling* (as if it were an animal); ..." (translation from the Italian Language Dictionary G. Devoto – G. C. Oli, Vol. II, 1980, p. 101).
In his book *Metaphor*, Terence Hawkes, from Cardiff University (UK), gives the following definition, more general than the former because it refers not only to a single term but to a wider linguistical construction:

"the term refers to a particular set of linguistic processes whereby aspects of one objects are 'carried over' or transferred to another object, so that the second object is spoken of as if it were the first. ...Ex.: "the brain is a computer", "the human body is a machine", "man is a wolf". (T.Hawkes, 1972, p.1).
He also observes that even analogy and simile imply a transfer of aspects between objects but usually they use the terms "as" or "like" to underline the transfer. In other words the transfer is explicit in analogies and similes, implicit in metaphors.
It is surely interesting to remember Aristotle's definition too:
"metaphor is the transfer to an object of the name of another's: this transfer occurs from the genus to the species, from species to species or through analogy." (Aristotle, *De Poetica*, 21, 1457 *b*, Opere, Laterza, Bari, 1973).
It is curious the fact that the philosopher, referring to the skill in the use of metaphors, underlines the importance and the complexity of it: " the greatest thing by far is to be master of metaphor. It is the one thing that cannot be learnt from others; it is also a sign of genius, since a good metaphor implies an intuitive perception of the similarity in dissimilars" (Aristotle, *ib*.).
If, like Colin Murray Turbaine, from Rochester University (NY), suggests in his book *The Myth of Metaphor*, we assume that what Aristotle's calls "name" could signify, more generally, a sign or a set of signs, we can also assume that a metaphor must not necessarily be expressed through words and thus conclude that "the model, the parable, the tale, the allegory and the myth are all metaphor subclasses." (Turbayne, 1970, in P. Barker, 1987, p.15)
Philip Barker, sharing this widening of meaning, pushes further and adds: "In the same way, diagrams on the blackboard, coloured blocks that kids use in representing battles or the raised eyebrow of an actor can all be considered metaphorical expressions" (P. Barker, 1987, p. 15).
More will be said on the use of metaphor, specifically in scientific thought, in paragraph 6, when we will consider mathematics education case.

5. Metaphoric communication functioning and efficacy

Metaphor functions when we apply to a principal object (for example a man) a system of common places typical of a secondary subject (for example a wolf).
What is important, for the efficacy of a metaphor, is that this commonplaces are evoked with immediacy and for this purpose a certain familiarity with this secondary object or a sharing of this commonplaces is needed.
To stress how metaphor has always been considered essential in human communication it is sufficient to remember its use in universal works like the Bible, the Gospels or Greek mythology and even in more recent novels (George Orwell's 'Animals farm',...) or in children tales.
With relation to the causes that make metaphoric communication more efficient than other more direct modes, Philip Baker lists a number of reasons (P. Barker, 1987, pp. 23-27), among which these are the most important:

- a well elaborated and explained story or metaphor easily catches the imagination and can suggest actions, choices or new point of views for a personal situation;
- because it exposes a certain subject indirectly and it has a veiled meaning, it tends to be less menacing than direct statements and individuals do not oppose them rationally;
- the meaning expressed by a metaphor can be varied and the individual can perceive it in the most comfortable way;
- it favours a good relation between persons, especially if the expression mode is interesting and funny;
- it is a communication mode which individuals could adopt to successfully communicate with others.

The basic hypothesis on which is built the use of metaphorical communication is the following:
"a person can consciously perceive a metaphor literally, while unconsciously the symbolic meaning is perceived." (*ib.,* p. 15)
What is often needed "is not a better logical understanding of one's own situation, but rather a different emotional attitude and different interpretation modes of the outside world." (*ib.*, p.26) .
Metaphorical communication can be experienced as a chance to easily assume various point of views and to have wider vision perspectives, for a better reorganization of one's own personal resources.

6. The research on metaphor use in mathematics education

Several researches, from the 80's to nowadays, underline the decisive role of metaphor in the meaning construction process of mathematical concepts. Metaphor is not a simple linguistic support for a better visualization or understanding of a concept but rather an instrument of thought, a primary part in mathematical reasoning.

The use of a metaphor needs

- the ability in perceiving analogies and differences in different situations
- the ability to apply to a new context the features typical of a more familiar one.

(R. Boyd, T. S. Kuhn, 1983)

If we succeed in both these operations the access to scientific thought is easier: "the use of metaphor is one of many devices available to the scientific community to accomplish the task of accommodation of language to the casual structure of the world." (R.Boyd, 1979).

A similar thought is evidenced by Thomas S. Kuhn, author of *The Structure of Scientific Revolutions* (1970): "Metaphor plays an essential role in establishing a link between scientific language and the world. Those links are not, however, given once and for all. Theory change, in particular, is accompanied by a change in some of the relevant metaphors and in the corresponding parts of the network of similarities through which terms attach to nature." (T. S. Kuhn, 1970)

It is interesting to remember that the important role of metaphoric thought was not always acknowledged, with relation to mathematical concepts construction. The Bourbakist movement of the 60's was totally against the use of images, starting from the assumption that every representation, even that of geometrical figures, was unreliable and that one could be deceived by the visual apparatus: there was not, therefore, any place for the analogical use of real life situations. These strict assumptions have fortunately been abandoned so that today, even because of the wider and wider use of computer images, arts and sciences are considered without separation and their reciprocal contamination is kept as of major importance.

The research in didactics of mathematics evidenced several conceptual situations in which the use of metaphorical thought could be particularly fruitful and individuated specific metaphors among which the following are the simplest:

- numbers as objects collections (or steps on a path)
- zero as an empty box (or starting point or separation point)
- addition as putting objects together (or taking steps of a given length in a certain direction)
- multiplication as a repeated addition (or the number of points in a rectangular scheme)
- equation as a balanced couple of collections, with a same weight
- the two equivalence principles of equations as physical operations on the two scales of a balance
- function as a machine which "takes" a number, "works" on it and produces another number.

Works by Pimm (1981), Sierpinska (1994), Lakoff and Nuñez (1997 e 2000), Boero, Bazzini, Garuti (2001), Arzarello, Robutti (2001) and Bazzini (2002) are just some of the didactical research studies examples focused on the use of metaphor as an instrument of thought in mathematical concept construction.

An interesting debate on the opportunity of the use of metaphorical thought in mathematical concept construction is presently going on. More precisely, what is being discussed are some metaphor types already practiced and even the motivations for the choice of a certain metaphor instead of another with reference to specific concepts.

An important contribution in this discussion has for instance been given by the logic mathematician Gabriele Lolli, from Turin University. In one of his works (*La Metafora in Matematica*, 2002), he declares that in the explanation of the concept of functions limits the use of a metaphor taken from the game of gambling would be fitter (as illustrated in R.Courant and H.Robbins, 1950 and adopted for example on the high school text *Elementi di analisi matematica* by G. Prodi and E. Magenes, 1984) than the metaphor of movement: " Weierstrass definition with ε and δ is not a formalist mania not saying anything more than what we can say by words; the concept is defined in association to a process which is not a movement; other metaphors are more useful, for example that of gambling..." (G.Lolli, *ib*.).

The use of metaphor is not into discussion but rather the opportunity of a certain type of metaphor: it is all about using, in different cases, the more effective and productive one, well knowing however "that all metaphors of a mathematical idea are inadequate and that there are points in which they lack something of the mathematical concept; and this is good and right because, if there were a metaphor fitting every mathematically useful aspect of a concept, we would have no need for the mathematical concept itself: a common word would be enough, with no need for an artificial symbol which is instead essential. It is in the very nature of a mathematical concept the feature of not perfectly fitting a single metaphor but of being common to a whole family of concurrent metaphors." (*ib*.)

Several contributions, until nowadays, have however evidenced the role of metaphor in the educative process as a help in mathematical meaning and sense construction; they also share the conviction of the need for a higher use of metaphoric thoughts in mathematical education.
A recent collection of interventions into this subject, which we recommend, is that of those discussed in the workshop "Role of metaphors and images in learning and teaching Mathematics" during the meeting CERME 3 (2003, proceedings in press).

7. New goals in metaphorical discourse use

In the end we think we should add something on the efficacy of metaphorical modality in mathematical education. To talk about mathematics to students through symbolic images, actions, gestures, not typical of the world of mathematics but rather of everyday life could promote communication on such a subject. Indeed mathematics, because of personal beliefs and stories, could rationally bring emotional blocks, thus impeding comprehension of the simplest ideas and strategies.
Metaphorical discourse could thus have a further value, in addition to those already depicted, and propose itself as a mode which could indirectly reach, without encountering the eventual block of the "rational mind", the natural "mathematical spirit" living inside any individual as a thinking being.
On the general problem of the relation between a person (teacher, student or other) and mathematical discipline, specific studies and experience tell us how this relation often is (or has been) painful and source of anxieties. It would then be useful a specific reflection on this relation and once more metaphorical mode could facilitate such afterthought, loosing blocks that might have been built in each personal school or extra-school history.
It seems evident that this could be very interesting for teachers, who are essential actors on the school scene and thus responsible, often unconsciously, of conflictual relations between their students and mathematics: the proposed reflection could take them to promising results, both on the relational and disciplinary level. (The project "*Il palcoscenico in classe*", described in Pesci A., 2003, was born and developed with such a goal).
It is also evident that such a work on teachers could have important effects in their classrooms, both for the consciousness of the responsibility in their students' relation with mathematics and the competence that they could achieve in the use of metaphorical thought. Once we recognize verbal disciplinary discourse as insufficient, because is linked only to the rational mind, we deduce that in building knowledge we need a deeper and more global involvement, which is for example possible through metaphorical discourse, in the wider meaning of the term.

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