The Mathematics Education into the 21st Century Project
Proceedings of the International Conference
The Decidable and the Undecidable in Mathematics Education
Brno, Czech Republic, September 2003

The Treatment of Mathematical Communication in Mainstream Algebra Texts

David K. Pugalee, Barbara Bissell, Corey Lock, Patricia Douville
University of North Carolina – Charlotte, NC 28262, United States of America

ABSTRACT. Communication is an essential part of teaching and learning mathematics. The National Council of Teachers of Mathematics [NCTM] standards emphasizes that communication helps build meaning and permanence for ideas. Teachers depend on curricular materials to provide students with opportunities to engage in mathematical communication. While traditional textbook publishers claim to support the standards movement, texts may not provide rich communication items that engage students in expanding their mathematical thinking. This study compares two popular Algebra I texts available in the United States. Chapters addressing six key mathematical concepts (equations/functions, graphing linear functions, solving equations and inequalities, systems of equations and inequalities, quadratic/exponential functions, and quadratic function) were sampled for the number of questions that required students to engage in communication of ideas beyond providing a numerical answer. The analysis showed that the two texts differed widely in the total number of such questions. In addition, qualitative analysis found that there was notable variation in the extent to which these questions required an extended response.

Communication is an essential element in the teaching and learning of mathematics (NCTM, 2000; NCTM, 1989). These standards documents underscore the importance of communication as one of the five process standards emphasizing the role of writing, speaking, and listening in developing mathematical understanding. The *Principles and Standards for School Mathematics* (NCTM, 2000) asserts "Students gain insights into their thinking when they present their methods for solving problems, when they justify their reasoning to others, or when they formulate a question." The Communication Standard for pre-kindergarten through grade 12 consists of four goals for students:

- Organize and consolidate their mathematical thinking through communication;
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- Analyze and evaluate the mathematical thinking and strategies of others;
- Use the language of mathematics to express mathematical ideas precisely (NCTM, 2000).

Given the importance of texts as the primary instructional resource of teachers, it is imperative that such curricular materials support the implementation of communication as a process standard. Teachers are likely to teach content if it is included in their curricular materials and the instructional approaches suggested by the materials is a primary influence on teachers' decisions about instructional strategies (Reys et al, 2003).

Publishers of textbooks make claims that their materials align to the NCTM Standards. Though these materials are aligned to mathematical content, the process standards may not receive explicit attention. Recent data indicates that only 10-28% of K-12 textbooks engage students in explaining strategies using multiple representations with 30% of K-8 and 55% of 9-12 texts ***never*** asking students to write reflections (Horizon Research Center, Inc., 2000).

This study will address these concerns by closely examining several commonly used Algebra I textbooks to determine the extent to which the communication standard is incorporated into the student exercises.

Methods of Analysis

In order to analyze textbooks specifically related to their treatment of the communication standard, a rating scale was developed that allowed for differentiating the types of exercises found in texts. The scale was modeled after evaluation criteria used in Project 2061 (American Association for the Advancement of Science [AAAS], 2000) which included "encouraging students to explain their reasoning" as one of the instructional criteria for the category "promoting student thinking about mathematics" in their analysis of Algebra I texts. Other sources that guided the development of the assessment tool were *The Principals and Standards of School Mathematics* (NCTM, 2000) and work done by Pugalee (1999, 2001).

Table1. Rubric for Rating Textbooks on Communication

Level	Criteria	Benchmark
1	One word answer Factual phrase Simple description Define	Q. About how many gallons will it take...? A. about 15 gallons.
	Explain	

2	Report Describe steps Tell procedure Present and explain strategies Error analysis	Q. How did you find the perimeter of the rectangle? A. First I added the 2 lengths. Then I added the 2 widths. Then I added both answers.
3	Evaluate Analyze Site evidence Conjecture Reflection Clarify and justify Make connections Compare and contrast	Q. A rectangle has length and width with the ratio of 3 to 4, and an area of 300 square units. What are the length and width? A. Detailed discussion of methods and thinking process of students. (See NCTM, 2000, p. 269.

For the purposes of this preliminary investigation, two Algebra I textbooks were selected for analysis. The chapters analyzed contained the following key concepts that were identified by a panel of three mathematics educators as central content concepts for Algebra I:

- Equations/functions
- Graphing linear functions
- Solving equations and inequalities
- Systems of equations and inequalities
- Quadratic/exponential functions
- Quadratic function

First, chapters were identified that corresponded to these key concepts. Next, the number of questions in each chapter was compared to the number of free-response questions and a percentage was calculated. Those questions that were worded in such a manner that the response required more than a numerical answer were then analyzed using the 3-point rubric, 3 requiring more extensive communication (and reflection) on the part of the student. Each text, therefore, was scored both on the percentage of free-response questions and the level of the communication required in those questions.

The following tables report the number of questions in the target chapters, including the number of free response questions and the percentage of the total questions they represent.

Problems using a unit word in the answer such as "miles" or "peaches" were not counted as free-response; prompted responses such as "true or false" and "yes or no" were not counted. In this textbook, most free-response questions were usually prompted with the following terms:

- Open ended
- Writing
- Critical thinking
- Error analysis

"Open ended" and "critical thinking" did not always prompt an extended response of the nature that would result in extended communication beyond supplying an answer.

Table 2. Ratings of Algebra I (Prentice Hall, 2003)

Chapter/ Topic	Total Number of Questions	Total Number Free Responses	Percentage of Free Responses	Percent Rated 1	Percent Rated 2	Percent Rated 3
Chapter 2 Solving Equations	532	18	3.3	7.1	64	28.5
Chapter 3 Solving Inequal.	589	50	8.4	36	58	6
Chapter 6 Linear Equations & Graphs	511	45	8.8	13.3	82.2	4.4

Chapter 7 Systems of Equations/ Inequal.	358	39	10.8	33	56.4	10
Chapter 8 Exponents Expon. functions	339	10	2.9	10	60	30
Chapter 10 Quadratic Equations/ Functions	716	45	6.2	8	77	13

Table 3. Ratings of Algebra I (McDougal Littell, 1998)

Chapter/ Topic	Total Number of Questions	Total Number Free Responses	Percentage of Free Responses	Percent Rated 1	Percent Rated 2	Percent Rated 3
Chapter 3 Solving Linear Equations	365	48	13	45.8	43.7	10.4
Chapter 4 Graphing Linear Equations	439	41	9	39	43.9	17
Chapter 6 Solving and Graphing Linear Equations	296	22	7.4	31.8	68	0
Chapter 7 Solving Systems of Linear Equations	342	41	11.9	41.4	58.5	0
Chapter 8 Powers and Exponents	354	25	7	64	28	8
Chapter 9 Quadratic Equations	337	43	12.7	51.1	37.2	11.6

This text included special features that are not reflected in the data presented in the above table. Such features included:

- Communicating in Mathematics contained several discussion questions just before written work.
- Some chapters have Enrichment Activities in the teachers edition, usually involving communication
- Directions to teachers to extend certain problems, eg. “Encourage students to write...”

Questions for these features were not included because they were presented outside the regular exercises for those chapters. Features in the teachers’ edition were not included in any of the analysis of texts, though the researchers recognize the importance of providing such pedagogical tools to guide instruction.

Conclusion

Research shows that student reflection is a critical component in retaining and applying mathematical concepts and skills (AAAS, 2000; Pugalee, 2001). Standards emphasize the role of communication as a tool for promoting and developing students’ abilities to reflect on important mathematical concepts (NCTM, 2000). Given the power of communication to develop and extend student understanding, the results of this study are discouraging. In one chapter only 2.9% of the questions incorporated some level of communication. That text, Prentice hall, had no more than 11% of questions requiring communication in any chapter. The McDougal Littell text had a range from 7% to 13%.

When considering those questions that did call for communication, there is some positive trend toward problems that required more extensive communication using the scale developed for this study. In the Prentice-Hall text, free response questions at level 3 were greater than those at level 1 in all six chapters, and level 3 questions were greater than level 1 questions in three of the chapters. In the McDougal Littell text, free response questions were greater than those at level 1 in four of the six chapters, and the percentage of level 3 questions was never greater than level 1 questions with 2 of the six chapters containing no questions rated at level 3.

Still, there are few examples of in-depth discourse, as described in the *Principals and Standards of School Mathematics* (NCTM, 2000). The findings of this study are consistent with those of AAAS (2000) that found great variability among texts rated on the category "Promoting Student Thinking about Mathematics". This category included criteria encouraging students to explain their reasoning, guiding interpretation and reasoning, and encouraging students to think about what they've learned. Likewise, Irvin (1993) found in an analysis of middle grades math texts used in Texas that the majority of writing exercises called for modeling, explaining or defining with few asking for reflection, argument, or conjecture.

After analysis of middle school textbooks, Schmalz (1990; 1994) concluded that if the newly adopted NCTM Standards were to be implemented, the textbooks would have to be used differently. Procedures in those texts could not be followed too closely, and considerable licensure would be required. Additionally there would need to be time allowed for alternative activities. The reader is reminded that several factors affect the nature and degree of mathematical communication and this study focuses only on the external structure of exercises and the level of communication that was likely was on the wording of the problem or exercise. First, there is the task itself. According to NCTM, "Students need to work with mathematical tasks that are worthwhile topics of discussion. Tasks that are procedural in nature requiring students to have well-developed algorithmic approaches are not the best problems to promote discourse (NCTM, 2000). Additionally, the teacher may or may not have a pedagogical understanding of how to plan and implement communication into the mathematics classroom. Textbooks need to lead teachers in this critical area (Reys et al, 2003). This study raises questions about the extent to which mainstream textbooks promote the type of communication envisioned in standards for teaching and learning mathematics. Further study of such texts and how teachers use them is necessary in order to change both the nature of the curricular materials and the instructional repertoire of the teachers who use them. Changes in both are essential if students are to be given opportunities to develop mathematical power.

References

American Association for the Advancement of Science. *Project 2061 Textbook Evaluations.* Retrieved 3/18/2003. **http://www.project2061.org/tools/textbook/matheval/default.htm**.

Horizon Research Center, Inc. (2000). The 2000 National Survey of Science and Mathematics Education. Horizon Research, Inc., Chapel Hill, NC. retrieved 4/16/2003. http://www.horizon-research.com

Irvin, B.B., (1993). A content analysis of the writing assignments contained in the four Basal textbook series adopted by the state of Texas. University of North Texas.

NCTM. (2000). Principles and standards for school mathematics. Reston, VA: author

NCTM. (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: author

Pugalee, D. K., (2002). Beyond numbers: communicating in math class. *ENC Focus,* 9(2), 29-32.

Pugalee, D. K., (2001). Writing, mathematics, and meta-cognition: looking for connections through students' work in mathematical problem solving. *SchoolScience and Mathematics, 101(5),* 236-245..

Reys, R. E., Reys, B. J., Lapan, R., Holliday, G., & Wasman, D. (2003). Assessing the impact of standards-based middle grades mathematics curriculum materials on student achievement. *Journal for Research in Mathematics Education*, 34(1), 74-95.

Schmalz, R., (1994). The mathematics textbook: how can it serve the NCTM Standards? *Arith. Teacherr, 41*(6), 330-32.

Schmalz, R., (1990). The mathematics textbook: how can it serve students? *Arithmetic Teacher, 38*(1), 14-16.