The Mathematics Education into the 21st Century Project
Proceedings of the International Conference
The Decidable and the Undecidable in Mathematics Education
Brno, Czech Republic, September 2003

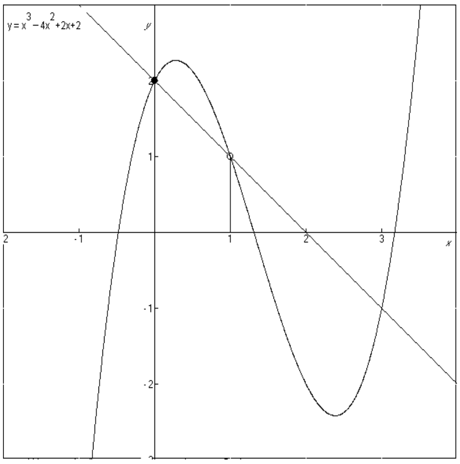
Applying the *Graphic Calculus* for Differentiation in Secondary School.

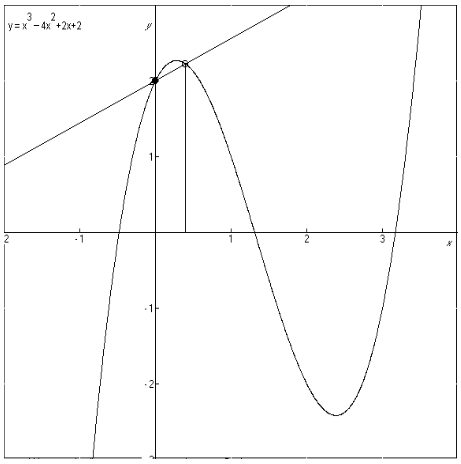
Anna Rybak, University of Bialystok, Poland

In my work with students aged 16-17 I noticed that the concepts of calculus were rather difficult and abstract for them. At the same time we know how useful for a tool investigating properties of functions calculus is. So how can we help our students to learn about calculus and its applications? In my opinion the program *Graphic Calculus* is a very good tool for facilitating teaching and learning calculus, the option *GRADIENT* is especially useful in this respect.
When and how can it help us and our students?

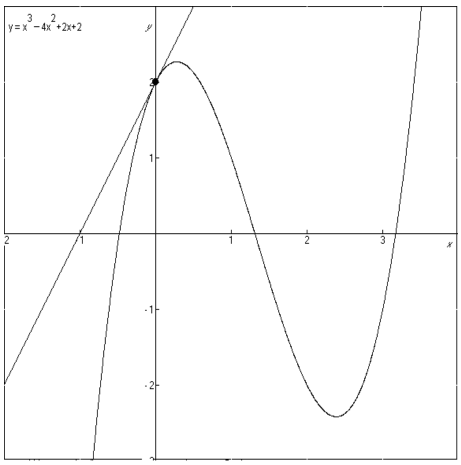
1. At the beginning of our work on calculus using program *GC* can help students to understand the idea of difference quotient and its geometric interpretation

   Example 1.
   Let's have a look at function $y(x) = x^3 - 4x^2 + 2x + 2,\ x \in R$ and plot its graph. We know that value of difference quotient (gradient) is the slope of the chord through points $(x, y(x))$ and $(x + \Delta x, y(x + \Delta x))$. If $\Delta x \to 0$ then the chord draws near (approximates) tangent at point $(x, y(x))$. We can observe this process on the screen:





Δx	*Δy*	*Δy/Δx*
0,01	0,019601	1,9601
0,009	0,0176767	1,9641
0,008	0,0157445	1,9681
0,007	0,0138043	1,972
0,006	0,0118562	1,976
0,005	0,0099001	1,98
0,004	0,0079361	1,984
0,003	0,005964	1,988
0,002	0,003984	1,992
0,001	0,001996	**1,996**

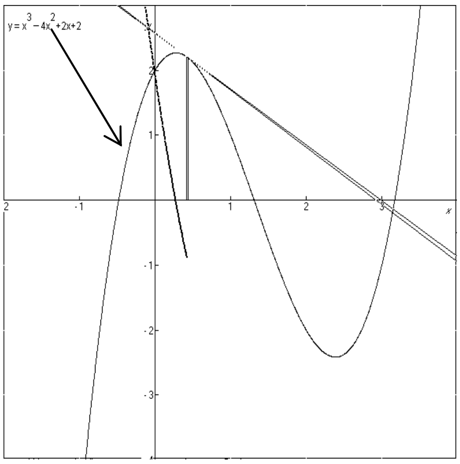


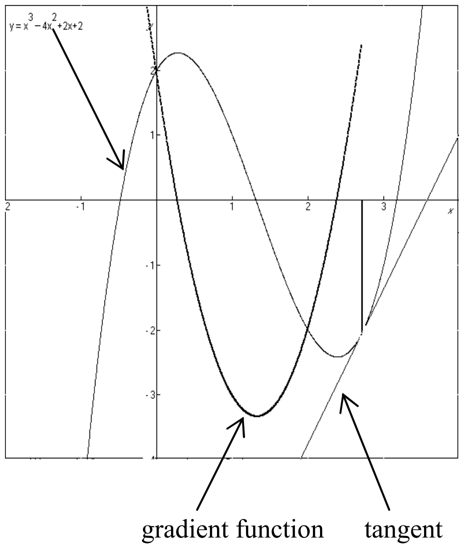
At first we choose x=0 and Δx=1. In the table we can observe values of slopes of chords for lessening Δx. After these observations we can put forward the hypothesis:

The limit of difference quotients (derivative of y at x) is the slope of the tangent to the graph of the function *y(x)* at point *(x, y(x))*.

2. Using sub-option *Gradient Function* we can observe the process of creating the graph of gradient function point by point. We choose Δx and the program is plotting the graph of gradient function using the formula $\dfrac{y(x+\Delta x)-y(x)}{\Delta x}$.

   Example 2.
   The function is: $y(x) = x^3 - 4x^2 + 2x + 2, \quad x \in R, \quad \Delta x$=0,01.

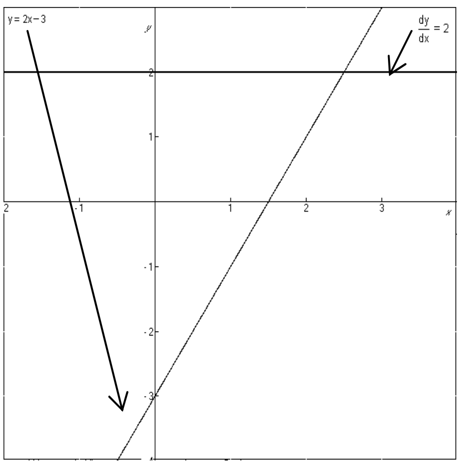


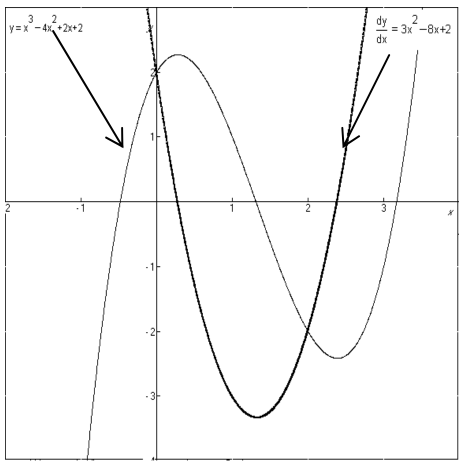


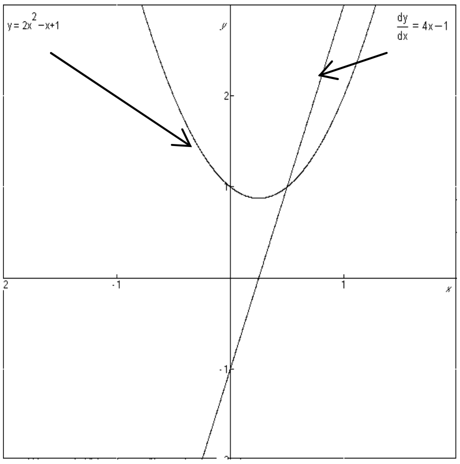
   We should be careful: if we get too big Δx, our curve will not be correct, which means it will be different from the graph of derivative function.

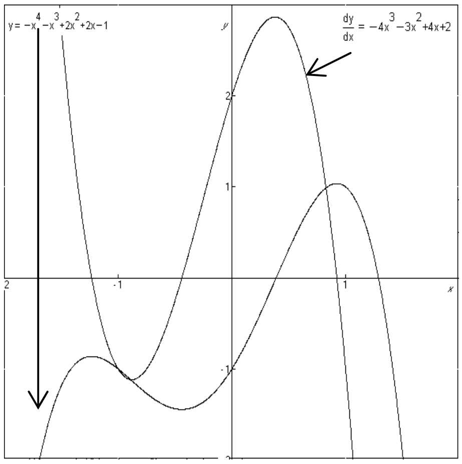
3. If we know the formula of the derivative function, we can input it and the program (sub-option *Derivative*) will plot its graph in the same plane as the graph of the function. We can then observe the connections between the properties of the function and the properties of the derivative of this function.

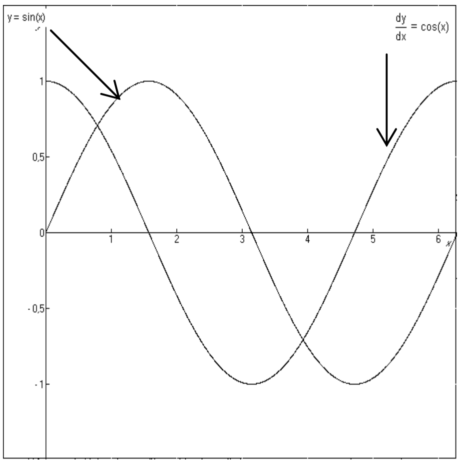
   Example 3.
   In the pictures below we can see the graphs of different functions and their derivatives:

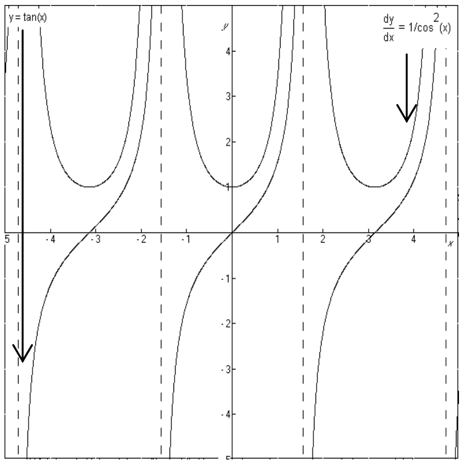


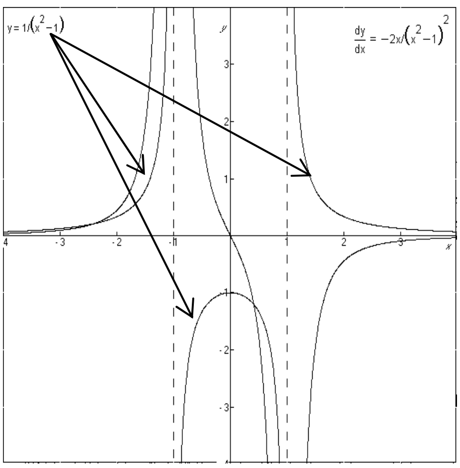


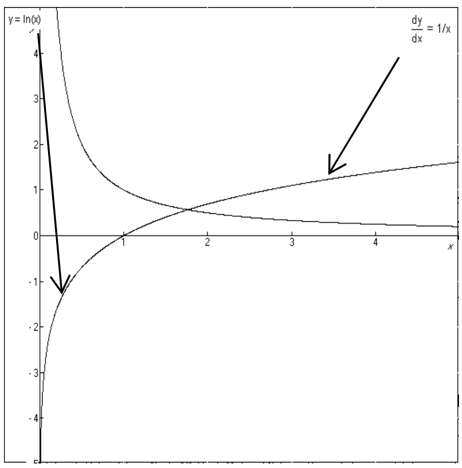












After the analysis of the graphs of functions and the graphs of their derivatives we can notice that:

- the derivative of polynomial of degree *n* is the polynomial of degree *n-1*,
- the function is increasing in intervals in which the derivative is positive,
- the function is decreasing in intervals in which the derivative is negative,
- in points in which the function reaches extreme values, the value of derivative is 0,
- in these points the derivative of the function changes its sign.

Our conclusions are only hypotheses of course; we have to prove them using algebraic methods.

4. We can easily solve problems concerning the physical interpretation of derivative (using graphic methods).

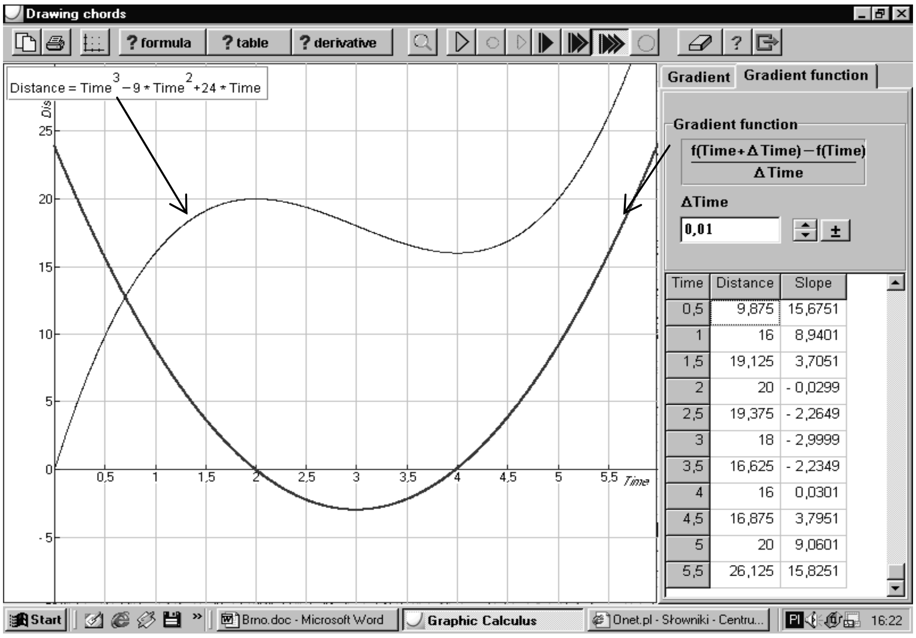
We have to remember that the derivative of the function expresses the rate of change of function's values. If the function expresses any physical value (for example distance or speed), we can conclude that:
- speed is a derivative of distance (distance changes in time),
- acceleration is a derivative of speed (speed changes in time).

Example 4.
A child's toy – a remotely-controlled small car is moving along a straight line and in time *t* seconds travels *s* meters according to the formula:

$$s(t) = t^3 - 9t^2 + 24t$$

A child is playing with the car in time of 5 seconds. What is the instantaneous velocity of the car while *t*=1,5 s? In which moment does the velocity of the car equal 0? What does the child observe then?

Solution.
The velocity is a derivative of the distance, so let's plot graphs these functions:



We can read from the table that when t=1,5 s velocity is equal to about 3,7 meters per second. Velocity equals 0 two times: when t = 2 s and when t = 4 s.
When 2<t <4 distance is decreasing – the car is moving back – velocity is "negative". That means that when t = 2 the car stops and begins to move backward, when t = 4 the car stops again and begins to move forward.

The program *Graphic Calculus* is a very good tool that helps to teach and learn mathematics. My students like to work with it very much, because they can understand better and faster a lot of difficult concepts.