

The introduction of the symbolic language in secondary school: experimental analysis of a-didactic situation by Vigotskij semiotic tools.

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Abstract The purpose of the paper is to show and to analyze the role of the teacher in a didactic situation and to try to sketch and to make clear such role, and thus, the intervention of the teacher in a a-didactic situation by an experimental analysis that uses semiotic tools. One of the theoretic reference is the theory of situation of G. Brousseau. In our double faced activity of teacher-researcher we think it is important to take pupil's spontaneous conceptions, and pupil's natural languages (words, gestures, attitudes, culture,..) as a starting point and to focus on the phase of symbolic translation. Such phase, in the didactic practice, may tie together natural language to mathematical language and thus to Mathematics. We start from an empiric example of didactic situation used to favor a cognitive continuity in the passage from arithmetic language to symbolic language.

Our experimentation regards the three classes of a secondary school of Palermo: one first class, one second class and one third class. The age of students is of 10, 11, 12 and 13. By the use of a particular game ("*Guess the number*"), we tried to analyze the best conditions so that the communication of the considered mathematical contents can be understood and lived at best within the objective limits of the communication between men.

Studying the limits of this parallel, that are located in the motivation, a key element for who is learning, we tried to sketch a particular function of the teacher during the a-didactic phase on the basis of "attitudes" that are instrumental to the educational goal. The word "attitude" here means the whole of all those gestures, those words and those fictitious strategies that a teacher can consciously use in order to introduce some objects inside an atmosphere which involves the pupils from an emotional point of view. This "**sly-strategy**" interventions, if they are opportunely studied, they seem to result functional and instrumental to the teacher's didactic intervention which was declared and explicit during the didactic phase. Thanks to the double, didactic and a-didactic, intervention of the teacher, in the analysis of the various problems, the students find themselves working in a field of meanings which is increasingly richer and various, thus they assimilate a language which is more and more conscious and artful. Three fundamental didactic interventions are individuated: the teacher motives translation; the teacher primes the translation; the teacher makes explicit the fact that she is translating from the natural language to the mathematical language. Considering such interventions we try to do a detailed experimental analysis of the teacher "attitude" by Vigotskij semiotic tools to show how some interventions seem to contribute to the development and to the understanding of some specific concepts of the symbolic language.

Symbols became instruments of semiotic mediation, some particular signs, with particular specific meanings, that the teacher brings into the classroom as external objects to be studied starting from their relations with a "sign" that lives and, have a meaning, in natural language.

Introduction

In our double activity of teacher-researcher we think it is important take the pupils' spontaneous conceptions and their natural language (words, gestures, attitudes, culture..) as a starting point for the development of the didactic activity. So it is important to pay attention to the phase of symbolic translation that in the didactic activity ties together the natural language to the mathematical language.

We are agree to the definition of D.Barberi (2002) to define the **language as an ambient** more than an instrument of communication: we inhabit the first while we use the second in relationship our communicative aims.

The languages are different aspects of the global ambient of the communication and consequently they are strongly connected, interlaced, in continuous reciprocal interaction.

The language is an ambient in which we stay and inside we think. Many times as a temporary solution we try to explain in a language what we think in another language. So every languages end to be crossed by several languages.

Ideas develop inside a language. So the language in which we think in that moment will not have only the characteristic of a instrumental used to transmit the ideas but it will be the ambient in which we stay when we form them. In this way the characteristics of the language in which we are thinking influence our thoughts.

So it is important analyze the best conditions by which the communication of the considered mathematical contents can be understood and lived at best within the objective limits of the communication between men.

In our experimentation, to the analysis of this point of views, we try to define a particular function of teacher during the a-didactic phase. During this phase the teacher motivates from a didactic institutional contract can use a "sly attitude" that are instrumental to the educational goal.

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The word “attitude” here means the whole of all those gestures, those words and those fictitious strategies that a teacher can consciously use in order to introduce some objects of semiotic mediation. By our experimentation analysis follows that this “sly” interventions if they are opportunely studied seems to result instrumental to the teacher’s educational goal declared and explicated during the didactic phase.

Our empiric example is the game *Guess the number*. In this game this instruments seem to encourage the development of the following specific aims of the symbolic language:

- valorization and importance of the sign “=”,
- reflection about the importance and the potentiality of the concept of “formula”;
- reflection about the symbolic manipulation as a instrument that let :
 1. the passage from an articulate expression to a easy expression;
 2. to resolve many problems in less time.
- valorization of the phase of translation into mathematical symbols and then the passage from a natural language to a mathematical language;
- the importance of the use symbols in the process of abstract;
- valorization of the use of maths to resolve problems,
- reflection about the use of parenthesis;
- reflection about the operation of multiplication.

If we want to create a scheme of our teacher-researcher activity we can say that it is characterized by the following organization:

-initial aim: solve a problem;

-condition the moves the modellization: we are unable to solve it;

-phase of discussion: the teacher listens the pupils’ observations and ideas;

-phase of translation: the teacher translate in mathematical symbols the problem showed with a natural language and then he starts to read again and to reflects about;

-it creates a mathematical model and it read again the problem in this model: in this way it is in evidence the possibility of utilizing some mathematical instruments to solve the problem;

-it solves the problem inside the model

-it came back to comment the solution of the real problem: it uses the relationship between the model and the realty to interpret in realty the mathematical result.

In particular we consider the following four didactic interventions very important: after creating the problematic situation using a natural language the teacher:

-gave the reason to the word for word translation of the formulation of the problem from natural to mathematical language.

-to prime the translation

-explicit the fact that he is translating from a natural to mathematical language and in this way he makes a difference from a natural world to a mathematical world,

-induces the metacognitive reflection of the problem trying to favour the generalization.

In our experimentation in particular it is important the basis-attitude of the teacher during the phases of confront and of discussion between pupils:

1. the teacher must not interfering during the phase of communication between the pupils whether he listens an interesting sentence or a false declaration. It is the situation created that must be a source of validation and of a chose of a strategies or another.
2. he must persuade pupils of his neutrality but not of his indifference. He must stimulate their investment of energy and their eagerness to succeed and he must oversee that the laws are respected.

Description and comments about the experimentation of the game.

Our experimentation regards the three classes of a secondary school of Palermo: I°E, II°E and III°F. The age of students is of 10, 11, 12 and 13. The classes were organized in groups: the I°E was divided in groups of 4-5 elements by the teacher while the other two classes were organized in group of two elements (a school bench is a group).

Presentation and phases of the games

After he had divided pupils in groups the teacher explains that the object of the game is to guess how the teacher divines the numbers thought by the pupils. So the teacher gives the following information:

1° PHASE:

- every group is provided with a sheet of paper and a pen. Then the teacher asks to think a number and to keep it in mind without saying it neither the teacher nor other group.

2° PHASE:

- the teacher asks to all groups to operate on their number according to the instructions written on the blackboard and to keep the result in mind (1^o instructions of the game).

3^o PHASE:

- the teacher asks to the first group to say the result and he writes it on the blackboard under the first instructions of the game. After he asks the result to the other groups and he writes them near precedent.

4^o PHASE:

- a this point the teacher writes quickly under every results the number thought by all groups and he asks them how he guess it.

5^o PHASE:

- the teacher listens and lets the all groups communicate between them trying to arrive first of all to the solution of the problem (15 min).
- ended the phase of communication every group explain its idea or its strategy of resolution. So the teacher writes their ideas on the blackboard inducing the pupils to comment their validity.

6^o PHASE:

- before writing the second instructions of the game the teacher becomes a phase of translation in symbols of the first instructions. He gives the reason for his decision for a problem of empty space on the blackboard!

7^o PHASE:

- a this point under the first instructions, translated in mathematical symbols and with a mathematical logic, the teacher writes the second instructions of the game and asks to think another number and to keep it in mind. So he asks to all groups to operate on their number according to the instructions and then to say the result.
- Under every result the teacher writes quickly the number thought by all groups and he asks them how he guess it.
- Follow the phase of communication (15 min) and then the transcription on the backboard of the eventual new strategies and/or or the falsification of the foregoing statement.

8^o PHASE:

- before writing the third instructions of the game the teacher becomes a phase of translate in symbols of the second instructions giving the reason for his decision for a problem of empty space on the blackboard according to the preceding phase! He writes it under the first.

9^o PHASE:

- the teacher gives the third instruction to the pupils but in this phase all groups play against the teacher. This once the teacher thinks more numbers (4 or 5) and he communicates the results.
- Follow a phase of reflection (15 min) in which pupils try to guess the teacher's numbers. It win who guess the numbers giving a strategy of resolution first of all.

10^o PHASE:

- In the final phase the teacher provides to compare the eventually various strategies and after a phase of discussion he reveals his strategy.

11^o PHASE:

- At this point he asks to pupils to create some instructions and so to play among them.
- Ended the game the teacher gives a set of questions.

Results and comments: the importance of sly instruments in our experimentation

-After had written the first instructions the sly motivation of necessity of empty space induces the translation in mathematical symbols and in a concise way.

In this translation the teacher pays attention to write what the pupils suggest to do and the various mathematical instrument they know. During this phase the teacher induces them to critical reflect about the use of parenthesis, the use of the sign “ \Rightarrow ”, the operation of multiplication and the distributive propriety.

-the teacher leaving voluntarily the results and their original numbers written under induces the possibilities of a confront between the results and then the possibilities of the existence of a strategies in common.

-the teacher writes the second instructions and here after the phase of game giving the reason for his decision for a problem of empty space on the blackboard according to the preceding phase starts a translation in mathematical symbols.

Also this time the teacher leaves voluntarily the results and their original numbers written under to induce a confront and to prepare the diagram that in the final phase will allow him to resume and to generalize the experience.

-the general scheme on the blackboard is the following (see box below):

-when the teacher suggests to write synthetically the text to occupy less space on the blackboard he induces the use of parenthesis and then to eliminate they. So the pupils suggest all operations to do. In

1° instructions :	{ [(n * 5 + 6) * 4] + 9 } * 5 = n * 100 + 165
	665 1065 1265 965 265
	5 9 11 8 1
2° instructions :	[(n + 3) * 5 + 7] * 4 = n * 20 + 88
	108 168 468 248 188
	1 4 19 8 5
3° instructions :	1. Multiply the number thought by 3.
	2. Add 4 to the result;
	3. Multiply this sum by 5;
	4. Add 4 to the result;
	5. Multiply this sum by 4.
	236 416 596 96

this way the pupils reflect on:

1. the grammar of parenthesis;
2. the use of a unknown;
3. the distributive propriety and the operation of multiplication.

In particular with the second instructions the teacher shows the use of the symbol “=” as:

1. an equality between two quantities written into two different ways;
2. as an element that ties together a difficult expression and an easy expression.

-leaving the result on the blackboard with the original numbers guessed the teacher induces an very well time of reflection of the pupils because in this way he seems to encourage the confront between the results and to eliminate the wrong strategies. Moreover, the understand by the confront the general validity of their strategies.

-when the teacher suggests underhand to play against him he induces an exchange of roles and so of responsibility. In fact who knows the number works with a arithmetic logic while who doesn't know the number works with an algebraic logic because he use a symbol to identify a unknown.

-another sly instrument to reflect is the chosen of the number though by the teacher. It is import the teacher think a negative number, for example, or a zero because in this way he seems to induce a very well comprehension of the nature of the unknown to anticipate the classic pupils' answers when they resolve an equation: “.there isn't solution because the solution is a negative number.. or.. why we must transform $-x = 3$ into $x = -3$?. in this case the solution is 3...”.

-after the last phase of validation the teacher recalls their attention to the scheme on the blackboard and to their reflections written on it during the game. So the teacher converge to his didactic object and the scheme on the blackboard results a power semiotic instrument which if very well utilized seems to induce a critical reflection about the use of mathematical instrument and the generalization of the problems using a ‘formula.

-At the last the teacher asks to pupils to create some instructions and to play among them. So the pupils discover the importance and the exigency to calculate their formula to resolve their game that will permit to win and to astonish.

With the first instructions pupils suggest these possible strategies:

1. in relationship to the numbers of the figures that constitute our result we think:
if they are three it subtracts 1 from the first figure;
if they are four it subtracts 1 from the first two figures ($265 \Rightarrow 2-1=1$; $1065 \Rightarrow 10-1=9$)
2. when the groups say their results the teacher does the inverse calculation following the instructions and beginning by the last instruction.

and with the second instructions also in relationship to the things written on the blackboard:

3. in relationship to the numbers of the figures that constitute our result we think:
if they are three it subtracts the third from the first two figures and the result is divided to 2. (II°E - III°F)
4. the teacher uses the result $165+n*100$ in the first case and the $n*20+88$ in the second case and he works on the results with inverse calculation. ($365-165 = 200$, $200/100 = 2$) (III°F).

With the fourth instructions pupils play against the teacher and their strategies to guess the numbers are the following:

1. they work on the results with inverse calculation following the instructions and beginning by the last instruction.
2. somebody tries to calculate an own formula to resolve the problem exploiting the precedent examples. (In this phase they have any difficulties in the translation of the instructions in mathematical symbols whether for problems of calculation or to apply the distributive propriety).
3. they make attempts thinking a number and then they follow the instructions. (I°E - II°E)

In the last instructions pupils enjoy to invent a instructions. In this way the teacher's sly-demand induces a valorization of.

- the semantic value of symbolic translation with the use of specific mathematical instrument
- the importance to calculate the right formula to resolve the problem

During the phases of validation

Before introducing the first and the second instructions and after the third instructions the pupil reflects alone to the validity of their strategies. In this phase the teacher is a notary. So the results are:

- the 2. is wrong for a problem of time: the teacher is not an electronic machine and he needs of a time to make calculations and this is against to the rapid time of teacher's answer (almost instantaneous).
- The 1. is no valid in the second instructions and so it is no acceptable.
- The 3. is no valid in the first and in the third instructions and so it is no acceptable.
- nothing about the 4 statement.

The problem of the time

In the first individual phase of discussion (or in the second phase) the teacher pays attention to show his time of answer (rather rapid) so to show the differences between the arithmetic time with the algebraic time to calculate a result of an expression.

Conclusions: The role of "sly" interventions and of affect in Mathematical Problem Solving

By our experimental analysis it seems that it is possible to try to sketch a particular function of the teacher during the a-didactic phase on the basis of "attitudes" that are instrumental to the educational goal. The sly interventions seem to result functional and instrumental to the teacher's didactic intervention which was declared and explicit during the didactic phase. In this way symbols became instruments of semiotic mediation, some particular signs, with particular specific meanings, that the teacher brings into the classroom as external objects to be studied starting from their relations with a "sign" that lives and, have a meaning, in natural language.

In fact, the particular attitude of the teacher or the use of particular instrument seems to encourage to remember a propriety and/or to understand better any specific concepts of symbolic language.

This particular interventions become a part of the interaction pupil-teacher and they take part in that emotional factors that play a fundamental part in Mathematical problem solving.(Douglas B.McLeod).

This interventions are sly in the sense that in a particular moment during the game-lesson the teacher provides to use them and then to recall these instruments making them functional and instrumental to his didactic aim.

In this way the teacher help students to analyse how consciousness operates and how they can manage their own mental resources. Kilpatrick (1985) refers to this as *metacognition*.

The teacher makes the objects-instruments inside an atmosphere that involves the pupil from an emotion point of view and these affective factors may have a substantial effect on the metacognitive process of problem solvers (Silver, 1982).

It is important to observe that the sly-interventions during the a-didactic phase are different from the didactic interventions planed for institutional didactic contract. They are studied with a sly and functional logic in relationship to the maturity of the classroom and to the argument to tell about.

During the phase of action it is the situation that must seem to suggest them and they must recall the emotion of the classroom.

In this game, for example, the sly intervention of the teacher seems to result a instrument of activation of proximal area in the development of the comprehension of particular algebraic concepts.

By the sly-interventions pupils lives an 'emotional condition' different from that in which they must think and they must give an answer to the teacher. Initially pupils believe their didactic task is only to guess the teacher's strategy so after they find themselves to live a news mathematical experiences. In this way they will found themselves in a new ambient and with news iconic experiences they learn a new language: the symbolic language that extends their precedent language.

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