

## Attempts to Improve the Problem Solving Abilities of Practicing Teachers

Agnes Tuska

California State University, Fresno, agnest@csufresno.edu

**ABSTRACT:** Practicing teachers of grade 5-12 mathematics in the United States of America are expected to live up to various, and sometimes contradicting expectations. One of the key expectations is to make mathematics accessible to all students.

This paper analyzes the quality and accessibility of some mathematical problems and investigations that are used in American classrooms, and discusses a few difficulties that teachers face when they try to engage students in problem solving activities. Particular examples are drawn from classroom observations and results of statewide testing in California, and from a variety of professional development workshops that were recently developed to improve the problem solving abilities of practicing teachers in Central California.

In addition, the paper compares some of the problem posing and problem solving practices in the American educational system with the corresponding practices in Japan and Hungary.

### Expectations towards American mathematics teachers

Teachers of mathematics in the United States of America are expected to provide standards-based simultaneous instruction in foundational skills, reasoning, and applications. A good mathematics teacher should do well on all of the following 5 strands: “(1) Conceptual understanding of the core knowledge required in the practice of teaching; (2) fluency in carrying out basic instructional routines; (3) strategic competence in planning effective instruction and solving problems that arise during instruction; (4) adaptive reasoning in justifying and explaining one’s instructional practices and in reflecting on those practices so as to improve them; and a (5) productive disposition toward mathematics, teaching, learning, and the improvement of practice” (National Research Council, 2001, p. 380).

Effective classroom instruction is expected to be reflected in higher overall student achievement and narrower achievement gap among traditionally low and high performing groups on nationally normed standardized assessments and state assessments that are used for accountability purposes. However, the focus on the various tests is different, and sometimes not aligned with the taught curriculum. Teachers are many times forced by their principals to “teach to the test” instead of following the curriculum. When test results are used for accountability purposes, it is also beneficial for the schools to keep many students in remedial classes so that the students’ mathematical knowledge would be measured by easier tests that are below their grade levels. Sometimes, there is even disagreement in the intended curriculum. The discrepancy between the standards set by the National Council of Teachers of Mathematics (NCTM, 2000) and the State of California (California Department of Education, 2000) is a specific problem in California.

### Difficulties engaging students in problem solving

The German poet and essayist Hans Magnus Enzensberger gave an interesting analysis for the exclusion of mathematics from the cultural sphere by the public (Enzensberger, 1999). He compared the public’s attitude towards mathematics with intellectual achievements in other fields, such as music, paintings, or literature, and posed the question: “How does it happen that mathematics has remained as it were a blind spot in our culture – alien territory, in which only the elite, the initiate few have managed to entrench themselves” (p. 13)? His conclusion was that mathematics instruction may play the key role in the public’s attitude towards mathematics, claiming that it “seems a pedagogical *idée fixe* that children are incapable of abstract thinking.” Teachers are “forced to operate at the end of a bureaucratic tether that describes quite brutal curricula and scholastic goals...Nevertheless, it must be said that there are teachers who resist the obsolete routines they are saddled with, and who manage to introduce their students to the

beauties, the riches, and the challenges of mathematics. Their successes speak for themselves”. (p 39)

The public’s mathematical illiteracy was addressed by many authors, for example by Paulos, in his book, entitled *Innumeracy*. However, there is no quick-fix to the problem. There is a long list of mathematics education reform and anti-reform movements, most of which are related to Enzensberger’s point. The dilemma is that all attempts of emphasis on abstract thinking may fail if the teachers are not capable of teaching in that spirit, and all curricula that cuts back on abstract thinking is doomed to produce only drilled number-crunchers, not creative thinkers and problem solvers, who would be able to understand and admire mathematics. (Tuska, 2001)

Brown and Cooney (1985) found that high school students often believe that “mathematics consists of rules and definitions to be memorized and used to solve assigned exercises” (p. 35). Schoenfeld (1988) stated that students, who are capable of performing symbolic operations in the classroom context, demonstrating “mastery” of certain subject matter, often fail to map the results of the symbolic operations they have performed to the system that has been described symbolically. These students fail to connect their formal symbol manipulation procedures with the “real-world” objects represented by the symbol, which constitutes a dramatic failure of instruction. (p. 150)

The results of a recently piloted California High School Exit Examination illustrate this point clearly. The purpose of this multiple choice test is to measure minimum competency required for receiving a high school diploma. Among the 11 representative sample questions in the area of measurement and geometry, the highest percentage of students that answered a question correctly was 82%, the lowest 10%. The problem with the highest percentage of correct answers was about selecting a figure that is congruent to a given figure. The problem with the lowest percentage of correct answers turned out to be the following:

A cereal manufacturer needs a box that can hold twice as much cereal as the box shown below. (Note: There was a picture of a rectangular prism given with “cereal” written on it.)

Which of the following changes will result in the desired box? ( $V = lwh$ )

- A Double the height only.
- B Double both the length and width.
- C Double both the length and height.
- D Double the length, width, and height.

The problem seems to be a straightforward application of the concept of volume and proportional reasoning. The fact that only 10% of the students could answer the question correctly is very alarming.

A variety of researchers argued that in order to learn the subject, whether it is math or any other subject, students need more than just knowledge of abstract concepts and self-constrained examples. Students need to be exposed to the use of a domain’s conceptual tools in authentic activity and to teachers acting as practitioners and using these tools in wrestling with problems of the world.

#### Examples from classroom observations

Unfortunately, the curriculum reaching the students in the United States of America is often not a conceptually challenging, problem-solving approach to mathematics. For example, the TIMSS Videotape Classroom Study (National Research Council, 1999) found that, in international comparison and in absolute value, American instruction of 8<sup>th</sup> grade mathematics lacks mathematical reasoning and emphasis on problem solving.

This happens in spite of declared national and state standards for K-12 mathematics education that emphasize mathematical reasoning (National Council of Teachers of

Mathematics, 2000; California State Board of Education, 2000). The analysis of the videotaped lessons revealed that the percentages of mathematics tasks that students decided how to solve rather than used a teacher-prescribed method was 40% in Japan, 27% in Germany, and only 9% in the U.S. Also, the percentages of 8<sup>th</sup>-grade mathematics lessons with instances of mathematical (deductive) thinking were 61% in Japan, 20% in Germany, and 0% in the U.S. Furthermore, American instructional practices included much higher percentages of using procedures or giving results only, versus making connections or stating concepts, than in any of the other six participating countries (National Center for Education Statistics, 2003).

#### Professional development workshops for teachers

Teachers are not the highest valued professionals around the world. Yet, American mathematics educators may be particularly strongly hit by the policy makers' distrust of their professionalism. Textbook publishers do their best to make their products "teacher proof": the teacher's role in their eyes is reduced to a worksheet distributor, who collects and checks the answers with the help of the teacher guides and solution manuals.

The typical mathematics textbooks are very thick, colorful, and contain plenty of worked-out exercises, guiding the students through the algorithms. Since most teachers feel obligated to follow the textbook, they need all the instructional time to rush through at least some of the exercises, and they can't find the time for reaching conceptual understanding. Textbook publishing is a profitable business in the U.S. The publishers do their best to "make teachers' lives easy" by providing ready-to-use instructional materials. Unfortunately, this tradition has cut off many teachers from the intellectual challenge of designing their own materials and understanding the "conceptual punch lines" of the instructional materials.

Kennedy (1998) reviewed studies of inservice programs that aimed to enhance mathematics and science teaching. She focused exclusively on studies that examined effects of programs on student learning. She found that "programs whose content focused mainly on teachers' behaviors demonstrated smaller influences on student learning than did programs whose content focused on teachers' knowledge of the subject, on the curriculum, or on how students learn the subject. Moreover, the knowledge that these more successful programs provided tended not to be purely about the subject matter – that is, they were not courses in mathematics – but instead were about how students learn that subject matter" (p. 17). Her conclusion was that by concentrating on how students learn the subject matter, "teachers leave these programs with very specific ideas about what the subject matter they will teach consists of, what students should be learning about that subject matter, and how to tell whether students are learning or not" (ibid.). She also emphasized that in order to understand how students understand a particular content the teachers also have to understand the content itself, so teachers' subject matter understanding is likely to be a by-product of these programs.

In Central California, the San Joaquin Valley Mathematics Project provided professional development activities and conducted teacher leadership institutes for more than a decade. The main goal of the project is to empower teachers for decision making tasks and to increase the pool of high quality professional development providers/peer coaches. Even among the highly motivated teachers who sign up for the activities, many are so insecure about their mathematical abilities that prefer to talk about mathematics instead of doing any mathematics.

In accordance with Kennedy's findings, the model that seems to work best in these institutes and workshops is to concentrate on students' learning difficulties and on brainstorming about the improvement of instructional effectiveness instead of direct content enhancement for the teachers. The discussion of students' error patterns or mathematical questions from the classroom is very popular (Ashlock, 1998; Crouse & Sloyer, 1987a, 1987 b), as well as the analysis of the fundamental concepts of mathematics in the curriculum, including best practices

to show the meaning and usefulness of these concepts in real-life situations (Ma, 1999). Since in California many students have language difficulties, teachers are particularly interested in strategies that work for English language learners (Herrell, 2000).

During the institutes, mathematical reasoning and problem solving is introduced as a necessity for the continuation of these discussions. For example, the content focus of the 2002 Summer Teacher Leadership Institute was visualization in mathematics because visuals, drawings, and manipulatives seemed to help communicating mathematics with the students who had language difficulties. The 13 participating teachers of grade 6-12 mathematics made remarkable progress during the two weeks. One item that was common on the pre-test and post-test asked teachers to write down how they would lead their students to develop a formula for the area of a trapezoid. On the pre-test, the typical answer was that the teacher wrote down a formula, and explained the names of the letters in the formula. By the post-test time, teachers could show at least three different ways for developing the formula, using a partition of the trapezoid, or some transformations of it that justified the steps. The sad news was only that all of these were news to them.

#### Problem posing and problem solving practices

Some countries have long-standing problem posing and problem solving traditions. For example, the “Közepiskolai Matematikai és Fizikai Lapok” (Roland Eotvos Physical Society, 2002), Hungary’s eminent mathematics and physics journal for high school students is in existence since 1894. It was founded and maintained by generations of highly motivated and highly qualified secondary teachers and university professors. A large portion of current mathematics teachers and professionals in mathematical fields was regular reader and contributor to this journal. The Eotvos Mathematics Competition (Mathematical Association of America, 2001), which is one of the oldest high school-college mathematics competitions in the world, with a tradition dating back to 1894, is another example of the cultivation of problem solving approaches in the Hungarian mathematics education. Almost every mathematics class in Hungary uses problem books in addition to the textbook. There is a tradition of TV programs to prepare students for the High School Exit Examination, and to explain the solutions to the examination problems in the evening of the examination day. There were also several televised mathematics competitions that had a large group of viewers and fans.

In the U.S., the mathematics category is considered to be dreadful in any competitive TV game program. Mathematical talent is rarely nurtured, at most accepted as a “nerd thing”. A good example to the differences between the mentality of the two countries is that when the Hungarian George Polya realized the need for some high quality literature on problem solving in the U.S., he was advised to give the title “How to Solve It?” to his book, because, as he was told, Americans wanted recipes, not mental challenges. Of course, there is a wide variety of excellent mathematical investigations, expository works, journals, and problem books available in English (see, for example, Gardner, 1989; Zeitz, 1999). Yet, these materials rarely have an impact on the mathematical education of the public. One, crucial step towards improving the problem solving abilities of the students would be to make their teachers more involved in the arts and crafts of problem posing and problem solving.

#### Conclusion

In order to improve the problem solving abilities of practicing teachers, the educational policy makers, textbook publishers, and mathematics educators must act coherently. The expectations, the rewarding system, the textbooks, and the professional development opportunities should all promote meaningful and challenging mathematical investigations and problems. Teachers should become active participants in the development and cultivation of the problem posing and problem solving culture. Mathematical reasoning should be an integral part

of the instruction. Without the teachers' love and joy of abstract thinking and reasoning, all new generations of students will remain ignorant towards mathematics and incompetent in using mathematics to tackle the ever more abstract and complex challenges in their lives.

In the United States of America, mathematics educators must work very hard to establish a problem solving culture that values reasoning because of existing cultural biases and counter productive forces in the educational system. Yet, there are examples of rapid improvements in the problem solving and reasoning abilities of teachers. Practicing teachers can enhance their abilities through carefully designed professional development workshops, and can give a headstart to the new generation of prospective teachers. Students who have the opportunity to face multi-step, engaging problems from early age, will be better prepared for the teaching profession, and will be more successful to instill the love for mathematics into their students.

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