The Mathematics Education into the 21st Century Project
Proceedings of the International Conference
The Decidable and the Undecidable in Mathematics Education
Brno, Czech Republic, September 2003

Application of mathematical concepts across disciplines: an example

Stamatis Voulgaris* & Anastasia Evangelidou **
*National and Kapodestrian University of Athens, Department of Primary Education, Greece
** Primary School Teacher, Cyprus Ministry of Education, Cyprus

The concept of volume conservation and its measurement cannot be thought in isolation but in relation to several associated aspects such as visualisation of pictured arrays via practising with construction of separated and solid blocks.
It was observed from previous research with pupils of 10-12 years old that there are specific skills necessary for the children to develop before they are able to fully understand and use the multiplication formula. These skills proceed in a hierarchical fashion.
Starting from evidence that show this hierarchical sequence we present suggestions for an interdisciplinary approach to teaching volume in late primary school, rather than strictly within the frame of the mathematics discipline (e.g. Geography, Art, Science, Language). This approach seems to be a good way of facilitating the incorporation of the mathematical knowledge and its relatedness to real world.

1. Theoretical background

Piaget (1960) has researched different aspects of conservation (conservation of substance, quantity, weight and volume). To Piaget the concept of conservation demands the development of the schemas of multiple relationality and atomism. The first refers to the realisation that in an object under transformation if one dimension increases another decreases. The second one refers to the understanding that matter is composed of tiny units, which interchange their location when the whole undergoes a transformation. Furthermore, the child must have developed the concepts of density and compression before he/she is able to conserve volume. Therefore, according to Piaget there is an order in which each kind of conservation is mastered: conservation of quantity and substance are mastered first, conservation of weight follows and conservation of volume is not mastered until about eleven to twelve years of age.

Piaget distinguished two main aspects of volume conservation. Conservation of interior volume (which is the volume defined by the boundary surface of an object) and conservation of occupied volume (which is defined in relation to the object's surroundings in space). Lunzer (1960) has identified a third aspect of conservation, conservation of displacement volume (that is the equivalence of the quantities of water displaced by equal but dissimilar volumes). According to Piaget, it is not until eleven to twelve years of age when the child becomes capable of realising volume in relation to the surrounding spatial medium and is able to associate volume with the three dimensions and therefore, conserve volume in all its aspects.

Post-piagetian theories such as the SOLO-Taxonomy theory (Biggs and Collis, 1982; Collis and Watson, 1991; Biggs and Collis, 1991) focus on the quality of a response given to certain task rather than the cognitive development thus allowing for external factors such as language, specific learning experiences and motivation to mediate so as to produce the observed levels of response. Further studies of Campbell, Watson and Collis (1992) examined “intra-modal” development - the nature of the development within one mode of functioning- and it was concluded that developmental sequences of a specific concept are hierarchical and can be analysed as a sequence of very specific smaller skills. Similar evidence was produced by Battista and Clements (1996) in their study of cognitive operations such as co-ordination, integration and structuring that appear to be required for students to conceptualise and enumerate cubes in three dimensional rectangular arrays.

Overall structuralist research has demonstrated that the development of volume understanding follows a specific step-by-step sequence where children move from an appreciation of the external visible aspects of the object to its internal structural organisation in terms of units of measurement. An appreciation of different aspects of volume conservation (conservation of interior, occupied and displaced volume) has been shown to correspond to different levels of competence in volume measurement.

On the basis of the above theories, research has been carried out with 90 late primary school children in Cyprus (age 10-12 years old)(Voulgaris, S. & Evangelidou A, 2002, 2003). The research items included a structured interview as well as a written test. The children were asked to perform on tasks on volume measurement and conservation. Among our main results it was found that different conceptions of volume are mastered through successful completion of content specific activities of increasing order of sophistication. Furthermore simple learning tasks, that require individual skills (unistructural responses) have to precede more complex tasks, that require individual skills in sequence (multistructural responses), which in turn will have to be followed by tasks, that require co-ordination of individual skills (relational responses).

It was also concluded that, there are specific skills necessary for children to develop before we can expect meaningful use of the multiplication formula for the measurement of volume of a rectangular construction. First children need to practice with concrete tasks of increasing structural complexity through which they can acquire personally constructed views of the organisation of the three dimensional rectangular arrays made of individual cubes before engaging with pictorial representations of divided or undivided rectangular solids. Second children have to master conservation and guided through transformation tasks come to a realisation of volume in terms of its metrical continuity doing away with distraction imposed by shape or positioning of objects. Third children must become proficient with the numerical operations of multiplication and division.

According to the constructivists views (von Glasersfeld, 1995; Steffe, 1991) meaningful learning occurs as students take accommodations or adaptions to their current cognitive structures as a result of their reflection on experiences. The cognising subject actively constructs knowledge not passively receives it from the environment. Coming to know is described as an adaptive process that organises one's experimental worlds; it is not to discover an independent pre-existing world outside the mind of the knower. It was also suggested (Battista, 2001; Battista 1992) that to genuinely understand, appreciate, and use properly based conceptual systems in geometry, students should actively participate in developing and working with the system, not in memorising facts that others have established for them. True understanding arises as student progress through phases of action (physical and mental manipulation), abstraction (the process by which actions become mentally solified so that students can reflect and act on them), and reflection (conscious analysis of one's thinking).

Furthermore, in line with constructivists' views, learning is best accomplished when information is presented in meaningful, connected patterns. This includes interdisciplinary study that links multiple curricula areas. An interdisciplinary approach gives the students the opportunity to actively engage in their own learning as they make sense of a theme (e.g. measurement and conservation of volume) across disciplines and with the world outside the classroom. Such an approach involves generating meaning across different subject areas and facilitating learning experiences and the development of new knowledge and skills while leading to an understanding of conceptual relationships (Jacobs, 1989; Shoemaker, 1989).

The approach suggested in this presentation makes use of relevant concepts in other disciplines and connects the concept of volume with the real world. It is an alternative approach to the one that uses the multiplication formula as the starting point for teaching volume of a rectangular construction. The students instead of rotely using the formula could acquire personally constructed views of the three dimensional rectangular arrays and link them with reality.

2. The example

The concept of volume, its measurement in terms of unit cubes and the use of the multiplication formula and conservation of volume are met in curricula at about the fifth grade. In Cyprus and Greece teaching most of the time follows the course of the textbook. For the fifth grate the material included for the concept of volume is in the last volume (i.e. is normally taught at the end of the school year). It includes exercise in the calculation of volume of constructions made out of unit cubes and non separated solids, calculation of the capacity of different containers and the calculation of volume of different objects of irregular shape, based on Archimedes principle. Pictures of rectangular arrays of cubes are also frequently used to represent the thousands throughout the numerical activities included in the books.

In the following example the proposed activities are encountered in other disciplines across the school curriculum for the fifth grade. This can be an alternative to the traditional introduction of a mathematical concept strictly within the frame of the mathematics discipline.

<u>Introductory activities- Conservation of interior volume and measurement</u>

The initial activity can by provided by asking the children to engage into a real live problem. The children can be organized in project type groups to plan the building of a new school with a given number of classrooms. A (fixed) piece of paper may represent the field for the building and a (fixed) number of cubes can be used to represent the classrooms. Other spaces can be included (for example playgrounds, yard, garden) as the designing group wishes. The models of the different groups can be presented and discussed. Observations and conclusions can be drawn from the different shapes

constructed with the given number of cubes. The advantages and disadvantages of having for example a taller block of classrooms (to save space for the playground) can be analysed.
This can be an alternative and naturally presented version of the "Piagetian task of building a new house on an island of smaller size". While the children build their construction or compare it with the construction of other groups they are progressing across conservation and measurement of volume.
The concept of mean population density predominantly encountered in Geography can provide a very good way of broadening the understanding of the concept of density as realised in Physics and Mathematics. It may also provide a useful tool of exploring the student's conception of multiple relationality and atomism, which according to Piaget are critical for children's understanding of volume conservation.
Following, the discussion at the end of the initial task, pupils can be engaged in the observation of a table presenting population, land and water areas for a map region of towns, counties or countries and continents. Measures of different aspects (including the mean population density) can be verified and children can compare them across different map regions. Different reasons referring to why a country or town has a larger or a smaller mean population density can be proposed.

<u>Measurement of rectangular volume and conservation of occupied volume</u>

The "Piajetian task" can be combined with an art-type activity where project groups can create the model of a rectangular shaped house with the use of coloured cubes. The volume can remain fixed (e.g. 120 cubes) and the discussion can reveal different shapes of the house as well as preferable colours for walls, roof, windows etc. in connection with the properties of the colours. A discussion can follow on modern architecture, shapes of modern buildings, outside decoration and materials used.
Apart from being fun and creative, this activity may help the students proceed into conceptualisation of the internal parts of a construction made out of unit cubes gradually learning to avoid distractions imposed by shape and colour of the construction. The multiplication formula for the calculation of rectangular volume is not formally introduced but should naturally arise as the children compare their constructions.

<u>Measurement of volume and conservation of displacement volume</u>

The text appearing in the appendix is an English translation of Greek text included in the language book in the fifth grade. This text beyond providing the basis for the language lesson of the day can be an excellent ground for developing an interdisciplinary approach to conservation of displacement volume.
After the initial examination of the text pupils could engage in small group project exploration of different aspects of the text focusing in language (reorganizing the narrative text structure, producing creative extended or summary versions of the original text or introducing dialogues), mathematics (pupils can generalise on Archimedes' principle by calculating the volume of differently shaped objects or compare the result when objects of the same volume but different shape are inserted into the water), physics (designing and presenting the actual experiment using simple everyday materials), history (examine the life and work of Archimedes or the development of his principle through the ages and its application in modern day life), geography (examination of geographical locations associated with Archimedes' inventions, possible connection and extension to ecological issues such as the greenhouse effect and global warming and their possible consequences as observed in the melting of the polar ice and the global rising of ocean waters), arts (creating visual representations of Archimedes' principle, painting or cartoon versions of the story in the text and role play acting of the text in class).

3. Conclusions and implications

Since the development of volume understanding has been shown to follow a sequence where children move from an appreciation of the external visible aspects of the object to its internal structural organisation in terms of units of measurement, simple activities that use concrete material (unit cubes) should be introduced before children are asked to calculate volume with the use of the multiplication formula. The above-proposed activities use concrete material and without giving the label that "this is a maths lesson" proceed to the last stage of volume understanding which is the conservation of displaced volume and the principle of Archimedes. The transition in children's understanding is more likely to be facilitated as these activities connect the "scientific principles" with other disciplines and real life situations. There are many more activities known to belong strictly to a discipline, which can

be combined to facilitate children's progress through the understanding of volume and its conservation.
Such an approach is certainly a positive step to the direction of reversing the already established view in many pupils minds that "maths are strange or difficult to understand or of no apparent value" by facilitating their personally constructed knowledge and generalisations of mathematics principles across schools subjects and in everyday reality.

Bibliography

Battista, M. T. & Clements, D. H.: 1996, 'Student's understanding of three-dimensional rectangular arrays of cubes', *Journal for Research in Mathematics Education*, 27, 258-292.

Battista, M. T.: 2001, 'A Research-based perspective on teaching school geometry', In J. Brophy (ed.), Vol. 8: Advances in Research on Teaching series, *Subject-specific instructional methods and activities*, JAI Press, Elsevier Science, New York.

Biggs, J. B. and Collis, K. F.: 1982, '*Evaluating the quality of learning: The SOLO Taxonomy',* Academic Press, New York.

Biggs, J. B. and Collis, K. F.: 1991, 'Multimodal learning and the quality of intelligent behaviour', In H. Rowe (ed.), *Intelligence: Reconceptualisation and measurement,* Hillsdale, NJ: Lawrence Erlbaum.

Campbell, K. J. Watson, J. M. and Collis, K. F.: 1992, 'Volume Measurement and Intellectual Development', *Journal of Structural Learning*, 11 (3), 279-298.

Clements, D. H. & Battista, M.: 1992, 'Geometry and spatial reasoning', In D. Grouws (ed.), *Handbook of Research on Mathematics Teaching and Learning*, (pp. 420-464), New York, NCTM/ Micmillan.

Collis, K. F., and Watson, J. M.: 1991, 'A mapping procedure for analysing the structure of mathematics responses', *Journal of Structural Learning*, 11, 65-87.

Jacobs, H.H.: 1989, 'The interdisciplinary concept model: A step-by-step approach for developing integrated units of study', In H.H. Jacobs (ed.), *Interdisciplinary Curriculum*, Alexandria VA: ASCD.

Lunzer, E. A.: 1960, 'Some points of Piagetian theory in the light of experimental criticism', *Child Psychology and Psychiatry*, 1, 191-202.

Piaget, J., Inhelder, B. and Szeminska, A.: 1960, '*The child's conception of geometry',* Routledge and Kegan Paul, London.

Shoemaker, B: 1989, *Integrative Education: A curriculum for the Twenty-First Century,* Oregon School Study Council 33/2.

Steffe, P.L. 1991, 'The constructivist teaching experiment: Illustrations and implication', In Von Glaserfeld E., *Radical constructivism in mathematics education* (pp 177-179). London: Klower Pumblishers

Voulgaris S.,& Evangelidou, A.: 2002, 'Understanding of three dimensional arrays of cubes – Children in transition', In A. Rogerson (ed.), *Proceedings of the International Conference: The humanistic renaissance in Mathematics Education*, (pp.371-376), Palermo, Italy.

Voulgaris S.,& Evangelidou, A.: 2003, 'Volume conception in late primary school children in Cyprus', In Gagatsis, A. & Papastavridis, S.(eds), *Proceedings of the 3rd Mediterranean Conference on Mathematics Education*, (pp.485-493), Greece.

Von Glaselfeld, E.: 1995, '*Radical constructivism: A way of knowing and learning*', Falmer Press, London.

Appendix

Archimedes in his bath

Archimedes was born in 287 BC and was a family relative of Ieron the tyrant of Syracuses in Sicily.

One day Ieron summoned Archimedes and requested the solution to a difficult problem. Ieron had ordered a crown made of solid gold to a Syracusian goldsmith. Was the goldsmith an honest man though? Archimedes had to verify whether the crown was made of solid gold or whether it was hollow inside or even whether it was made of an alloy of gold and silver without destroying or reshaping the original crown.

The solution to the problem appeared impossible at first. Archimedes, however, was not a man likely to give up when faced with a difficulty. He was endlessly thinking about the problem until one moment the solution came unexpectedly.

While taking a bath one day, sinking his body slowly inside the water he was suddenly enlightened. Legend has it that he leaped out of the bath and went naked on the streets shouting: "Eureka! Eureka!"

What has he come up with? He had found the solution to the problem, by observing that the surface of the water in the bath was rising as his body was sinking in it. That happened because the problem was constantly in his mind and that moment he thought that if the crown was immersed in the water it would also cause the water surface to rise. Regardless of whether the crown was made of gold or silver or led, whether it was solid or hollow, the crown would have to occupy some space, it would have some volume and if it was immersed in the water, then the water unable to be compressed would have to rise inside the container at a higher level that could be easily marked.

Putting the crown aside and repeating the experiment with pieces of pure gold he attempted to bring the water surface to the level it reached when the crown was immersed into it. Both the crown and the pieces of pure gold have the same volume. The crown that is supposedly made of pure gold and the actual pieces of pure gold must have the same weight. If the crown is hollow, it follows that it has to weigh less than the pieces of gold with the same volume as the crown. The crown should also weigh less if its made of a gold and silver alloy because silver is two times lighter than gold. The actual experiment proved that the crown did not contain the correct amount of gold.

The script where Archimedes describes his inventions was lost for 1700 years and was found in 1540.