Using PC and TI-92 in teaching Binomial and Normal Distribution in the final Grades of Austrian Grammar Schools

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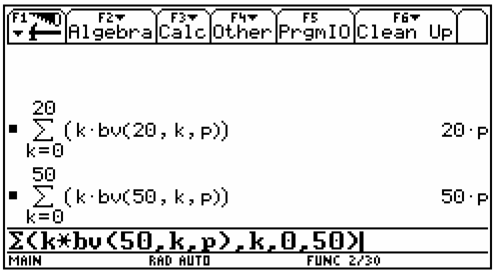
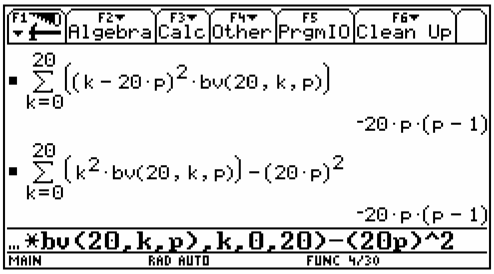
After the last reforms of the curricula in mathematics (1993, 2000), **statistics** and **the use of the computer** have been fixed in mathematical instruction for ten to fourteen year-old students in Austria. **Probability & statistics** have become a compulsory part in the last grades of all schools (AHS/BHS) preparing students for university level. Unfortunately probability & statistics and the use of the computer have only recently become compulsory for teacher students at university. This is the reason why a young mathematics teacher might still not have a satisfactory knowledge of it. In order to improve this situation the **ACDCA** (Austrian Center for the Didactics of Computer Algebra) has been founded. ACDCA organises conferences and meetings and makes publications with the intention of offering a framework for university teachers, teacher trainers and school teachers, to exchange their experiences and to do research projects (homepage: *http://www.acdca.ac.at*) The following experiences have been made in the last CAS projects (1999/2002). All students of the research class have used TI-DERIVE or PC-DERIVE in the mathematics lessons and at home. The use of a CAS calculator is a very great advantage and offers new possibilities of solving problems especially in probability & statistics.

1. With **CAS calculators** the students can quickly **manage summations** and **understand** more easily and more independently algebraical **proofs**, e.g. the derivation of the formula of the expectation and the variance of a binomial distribution, in the algebra window.



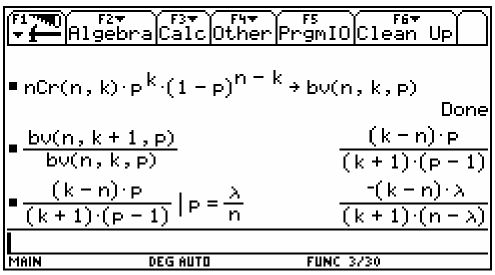
Expectation $\mu = 20p, 50p \rightarrow \mu = n.p$



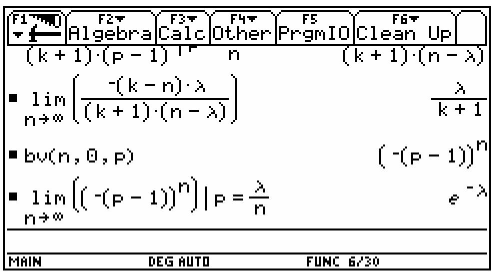
Variance $\sigma^2 = 20.p.(1\text{-}p)$

2. With **CAS calculators** the students can quickly manage the **limit of sequences.**

Therefore it is possible to show the students a way to find the formula of the poisson distribution by themselves. The binomial distribution can be approximated with the poisson distribution, if the number n of the trials is very large and the probability of the experiments is very little and if it is granted that **n . p → μ** with μ as a real and finite number. This approximation is very good for $0 < p < 0.1$. In 1975 A. ENGEL suggested a new way to derive the formula, which is easy to calculate with the TI-92+.

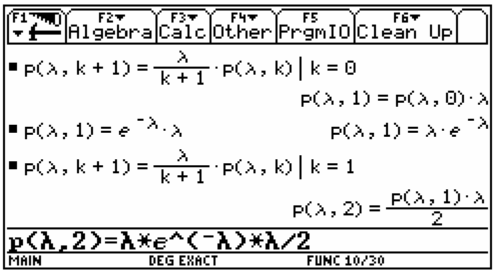


bv(k+1)/bv(k) with p = μ/n

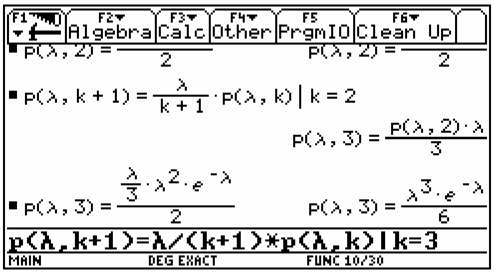


bv(k+1)/bv(k) → p(μ,k+1)/p(μ,k) = μ/(k+1)

formula of recursion:	$\dfrac{p(\mu,k+1)}{p(\mu,k)} = \dfrac{\mu}{k+1}$ **with** $p(\mu,0) = e^{-\mu}$



$k=1 \rightarrow p(\mu,1)=\mu.e^{-\mu}$, $k=2 \rightarrow p(\mu,2)=e^{-\mu}.\ \mu^2/2$



$k=3 \rightarrow p(\mu,3) = e^{-\mu}.\ \mu^3/6$

Given k = 1, 2, 3, ...you can easily find the formula of the poisson distribution.

$pv(\mu, k) = \dfrac{\mu^k}{k!} \cdot e^{-\mu}$	with **$\mu = n\ .\ p$ as expectation and as variance σ^2**

3. The solution of a problem with the TI-92+.

4 % of all the air passengers who have reserved their seats usually do not turn up. *The airlines know this and so they sell 75 tickets for 73 available seats.* *What is the probability that all passengers find a seat ? (KIRSCHENHOFER/ARNOLD, 1978/79)*

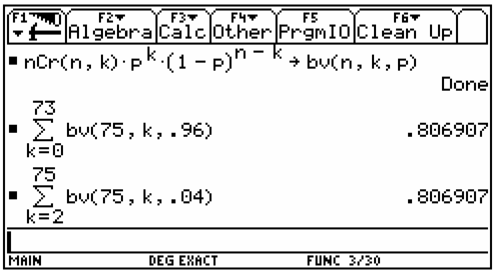
The probability of a passenger collecting the ticket is 0.96, of his not turning up is 0.04.

Two variants of calculation are possible: All passengers get a seat, if 73 passengers turn up, or if at least two of the 75 passengers do not turn up.

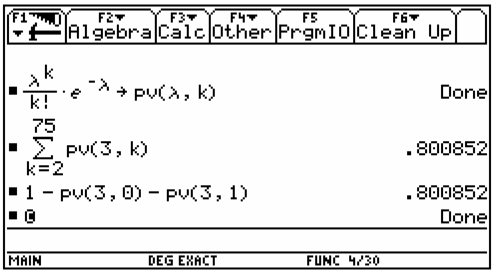
Calculation of μ:

n = 75 | *for **p = 0.96** is **μ** = 75 ∗ 0.96 = 72* | *for **p = 0.04** is **μ** = 75 ∗ 0.04 = 3*

Solution as binomial distribution bv(n,k,p)



Solution as poisson distribution pv(μ, k)



The calculation with the TI-92+ is now possible with both distributions in different variants.

The passengers turning up get a seat with a probability of about 80%.

Result of binomial distribution 80.7% Result of poisson distribution 80.1%

4. With **CAS-calculators** you can quickly manage **a sequence of figures**.

If a table for the binomial distribution e.g. n=20 and p=0.5 (**k = 0,1,2, ..., 20)** is created, a point diagram, a histogram and at last a probability density polygon can quickly be plotted.

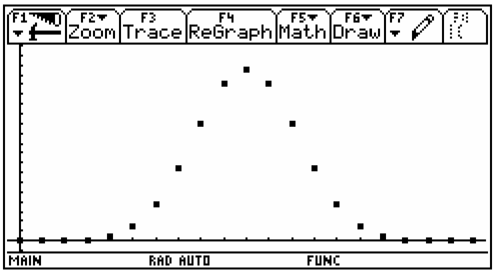


Fig.1: Point diagram (square)

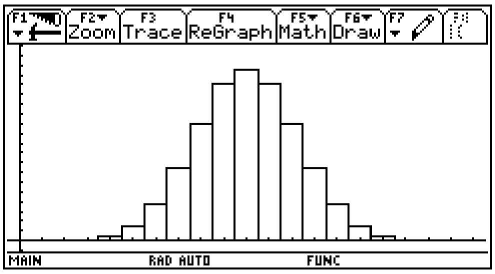


Fig.2: histogram, width of the bars = 1

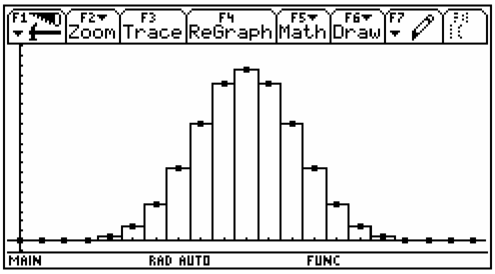


Fig.3: Points as midpoints of the bars

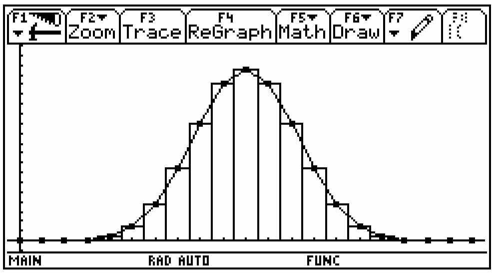


Fig.4: Points connected (xy-line)

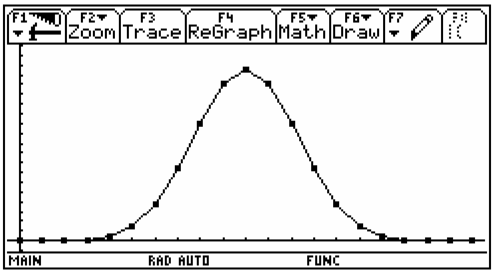


Fig.5: probability **polygon**

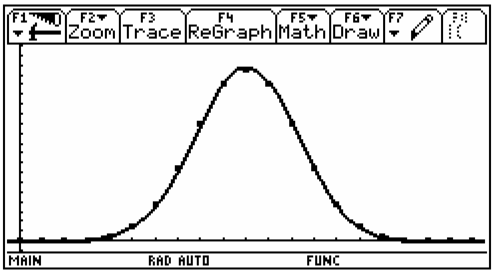
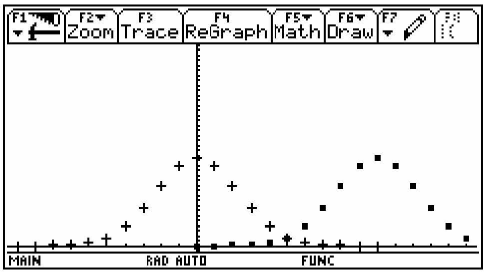


Fig.6: Does a function exist?→ **nv(x,μ, σ)**

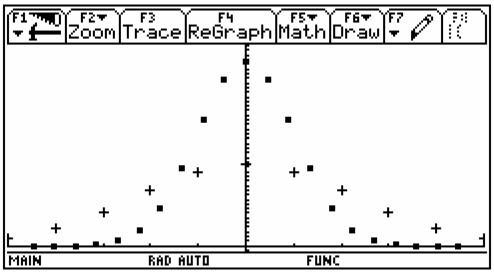
With this sequence of figures it is possible **to get an imaginable transition from the discrete distribution to the continuous distribution** including the important correction. In order to replace the polygon to the fitted curve of the probability density function **nv(x,μ,σ) = 1/(√(2π)σ) . e** $^{-0.5((x-\mu)/\sigma)^2}$ in figure 6, it is necessary to look for the standardized function with μ=0 und σ=1.

5. With **CAS-calculators** you can quickly **give an impression of the standardization**.

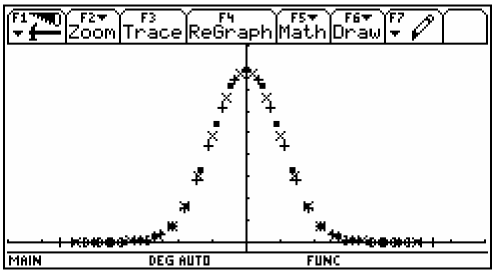
If there is no time to derive the function nv(x,μ, σ), **nv(x,0,1) = 1/(√(2π)) . e** $^{-0.5x^2}$, the standardized function, can be entered by the students on their own screens and they can see that the curve is running through almost all the points.



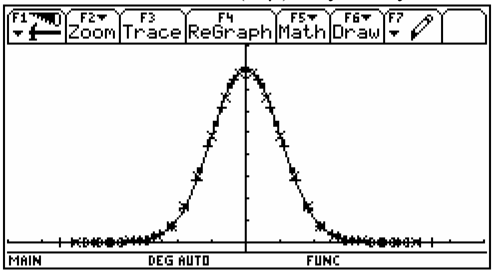
Translation with (x-μ)



Transformation: x → (x-μ)/σ, y →σ.y



standardized points: n=20, 25, 30; p=½



The graph of nv(x,0,1) fits !

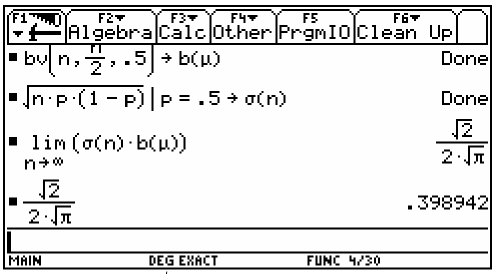
6. With **CAS-calculators** you can **derive the function nv(x,0,1).**

If there is time to derive the function nv(x,0,1), it is possible to go the way of standardization with all students stepwise (*WURNIG, 2001, 2002*). Regarding the figure with the standardized points, many students want to know how you can derive a function so that the graph will closely approximate the points. In order to achieve this, you have to find a function f with the following qualities:

- f(x) ≥ 0 for all x and f(0) = d > 0 (y-intercept);
- the graph of f has to be „bell shaped" and symmetrical to the y-axe.
- the area between the bell-shaped curve and the x-axe has to be 1.

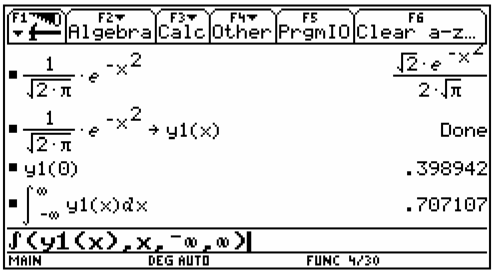
Calculation of **f(0) = σ(n).b(μ)**
p.e.: n=40, p=0.5 → μ=40*0.5=20,
σ(40)=√(40*0.25) → σ(40).b(20) ≈ **0.396**

Does a limit of σ(n).b(μ) exist for n → ∞?



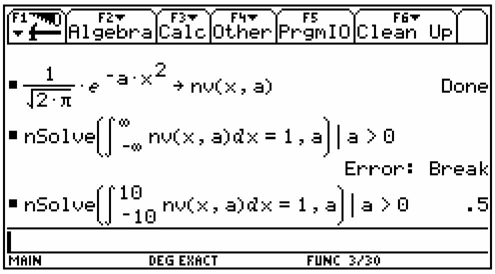
d = 1/√(2π) = ≈ 0.398942

y1(x) = **(1/√(2π)) . e$^{(-x^2)}$ is a bell-shaped curve**, which is symmetrical to the y-axe. But the area between the curve and the x-axe from -∞ to +∞ is 0.7 and not 1.

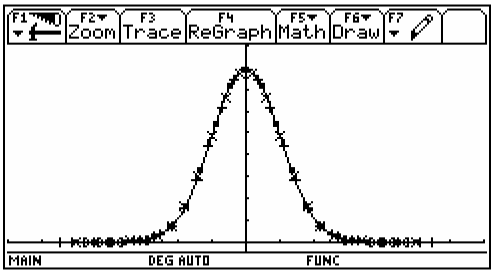


the area is calculated with 0.7 (≠ 1)

Therefore the exponent of the Function y1(x) has to be corrected by the factor a. The newly defined function nv(x,a) = (1/√(2π)) . e$^{(-a.x^2)}$ has to be integrated by the bounds -∞ and +∞ . If the CAS-calculator cannot solve the entered equation, the bounds have to be changed to -10 and +10. This offers the result: ***a = 0.5***



Calculation of a: a = 0.5



The graph of nv(x,0,1) fits the points.

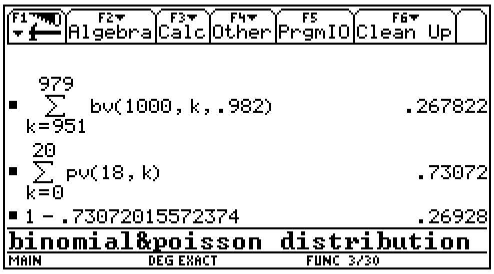
$nv(x,0,1) = (1/\sqrt{2\pi}) \cdot e^{(-0.5x^2)}$

7. With **CAS calculators** the students can quickly **integrate functions.**

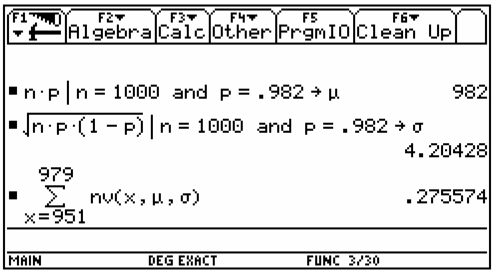
Problems of probability can thus be solved **without tables**. This increases the number of different ways of solving problems and stochastics loses its isolated position. Probability becomes a part of calculus.

The death rate of newborns, this means the probability of babies to die during their first year of life, is 1,8% in a country. What is the probability that of 1000 newborns chosen at random more than 950 and less than 980 will live to see their first birthday ? (*REICHEL, 2002)*

Normally this problem is solved with the binomial distribution (p = 0.982). But you can also solve it with the poisson distribution, if you calculate the probability that of 1000 newborn babies less than 21 will not live to see the first birthday. After doing this you have to find the contrary probability.

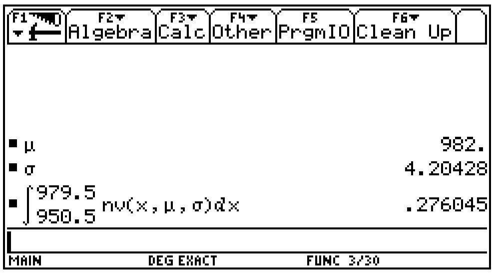


PV: $\mu = n \cdot p = 1000 \cdot 0{,}018 = 18$

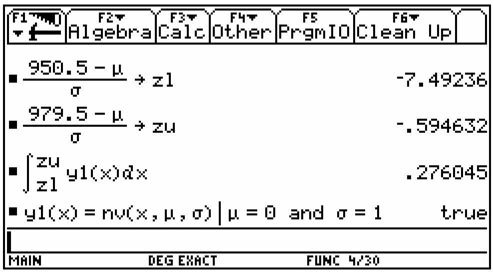


NV: summation with nv(x,μ,σ)

The binomial distribution can also be approximated by the normal distribution. But with DERIVE you can choose between two ways: you can interpret nv(x,μ,σ) as a closely approximating function and make a summation or you compute the area by integration.



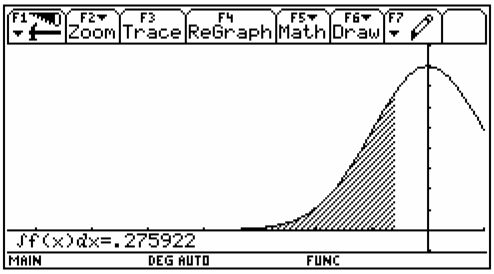
Integration of nv(x,μ,σ), corrected bounds



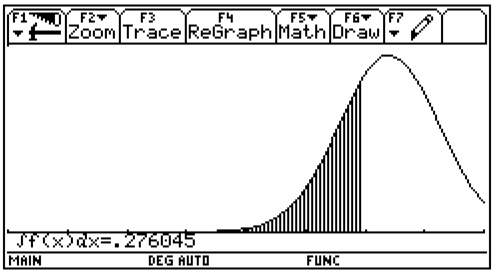
Integration of nv(x,0,1), corrected bounds

8. **The TI-92 can calculate in the graphic window.**

The TI-92 can plot the Gaussian bell-shaped curve of the normal distribution in the graphic window for every μ and σ. As soon as the curve is plotted, the TI-92 is able, with the command ***∫f(x).dx***, to calculate the area under the curve after the input of the lower and upper bounds.



Input of the standardized corrected bounds



Input of the corrected bounds 950.5;979.5

Many students like this way of solution because they can immediately see at once the area which is the measure of the probability on the screen.

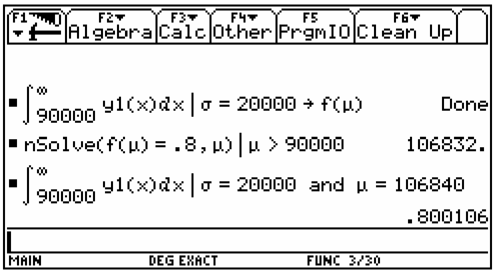
9. Solving a problem of the **normal distribution** with the TI-92 if **μ** is to be calculated.

Engines have an average life μ of 100000 km and a standard deviation σ of 20000 km. How is the average life to be improved so that by a constant σ more than 80% of the engines have a life longer than 90000 km ? (FIALA/MOSER, *1982*)

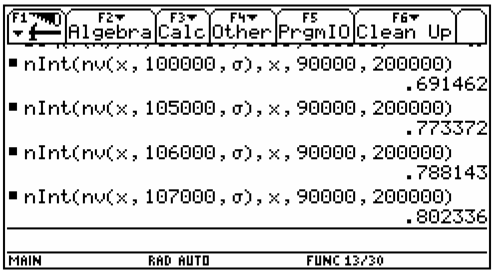
Such problems, if μ or σ is asked, you can only solve with CAS-calculators if you have an idea of the result. Then you can go two ways:

- first integration and then solution of the equation with the command **nsolve**,
- first calculated testing, then interpolation and at the end control with integration.

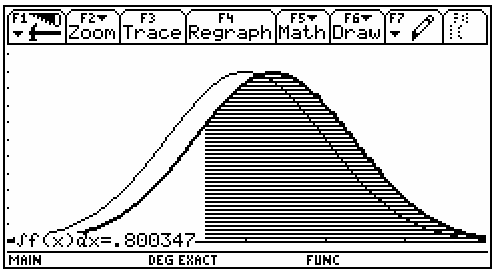
The better students like these two ways. An average student prefers the Φ(z)-tables, because it is a receptive way, but very often with a wrong result if there is no control at the end.



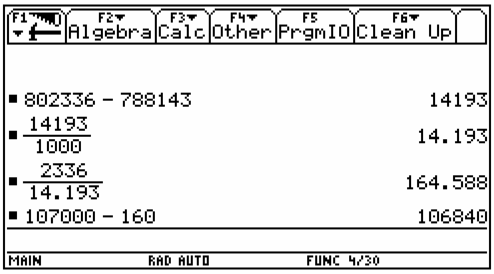
first integration, then nsolve with μ>90000



calculated testing from μ=100000 to 107000



bell-control with μ=100000 and μ=106840 km



linear interpolation between 106000, 107000

The average life μ of the engines is to be improved to 106840 km.

On the basis of the accompanying research of the CAS II project with the center of school development (ZSE), it results that **students who are mainly good at maths make positive observations and see advantages in the use of the TI-92** more frequently than those students who have good marks in their certificate in other subjects as well. Furthermore the report points out that contrary to the girls, **boys experience a stronger increase of fun and a better stimulation of their interests and suffer less additional stress with the use of the TI-92**. Besides boys hardly notice a loss of mathematical basis knowledge. *(GROGGER, 1999)*

All these reasons require a careful moving on when using utilities or functions and programs defined by the teacher. **There is a great danger that such functions and programs are quickly used as a blind tool by the students**.

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