

**Promoting Teachers' Changes: Examples from an Educative Process in Early Algebra****Nicolina A. Malara**

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*After some notes about the socio-cultural aspects implying a reshaping of the mathematics teaching, we speak of our ArAl project, devoted to an early algebra approach, focusing our attention on the teachers. We sketch out what we have done in order to bring the teachers to acquire awareness about the theoretical frame and aims of the project and, in the same time, how to put into practice new and interactive ways of teaching. We discuss two excerpts of classroom discussion which are meaningful from the point of view of the decisions-actions of the teacher and of the classroom dynamics. We conclude with some considerations on the teachers' changes and some reflections on the complexity of their role.*

**1. INTRODUCTION**

Nowadays development is based on very fast transformations that require a constantly higher control of scientific knowledge. This implies the need for a more widespread (not only) scientific culture and involves, in a great challenge, the educational institutions of all countries. In particular, mathematics teaching is fundamental not only for the development of science and technology, but also for bio-medical and socio-economical disciplines. The first challenge is therefore the need to modify the aims and methods of its teaching, also because mathematics, even if surrounded by respect, has a much distorted image within society, i.e. that of a discipline based on knowledge which has been long crystallized, enigmatic and initiatic.

Often, in every day's teaching, the prevailing aspects are the passive acquisition of mathematical facts and the rigid application of rules, which don't reach the understanding of their sense. On the contrary (and the reasons for it are, according to teachers, manifold: lack of time, pressures of different kinds, lack of interest in the pupils', etc.), it is very rare to find a didactics of problematic situations that shifts the attention from the results to the processes that determine them. This shift of perspective would allow to build in the pupils – and therefore spread in society - a more realistic image of mathematics as a discipline born from and for the study of (sometimes very complex) problems which in the various ages men faced by constructing specific semeiotic systems of representation, which themselves became object of study and gave vent with time to unifying abstract theories.

In order to highlight the cultural and deeply human character of mathematics, it is necessary to revitalise at all teaching levels its *linguistic-epistemological dimension*. Moreover, in order to educate the pupils to flexibility in reasoning and in the elaboration of new data, it is necessary to give more room to *metacognitive* aspects – i.e. the reflection on knowledge – and to *metalinguistic* ones - the reflection on language.

Implementing this change of perspective is not easy and therefore the *key element* is the teacher. In this sense, it is meaningful that many industrialized countries have been investing on mathematics education for decades already through the creation and experimentation of innovation projects in the classrooms and the continuous and long term activation of teachers' training processes. This is documented by many researches focused on the teachers' development and changes (Krainer & al. 1998, Fennema & Scott Nelson 1997, Jaworski & al. 1999, Lin & Cooney 2001) and taking also into account teachers' beliefs, attitudes, emotions and values (MacLeod 1992; Thompson 1992, Vinner 1997, Zan 2000). Other more specific studies focused on teachers' disciplinary knowledge (Fennema & Fraenke 1992), teachers' ability to understand students' mathematical thinking and performances (Even & Tirosh 2002) or to orchestrate classroom discussions (Bartolini Bussi 1998, Yackel 2001). There are also studies which emphasize the importance of the teachers' awareness on all these aspects (Mason 1998, Jaworski 1998, Lerman 2001, Malara & Zan 2002, Malara 2003).

Our topic is framed in these studies and is based on our ArAl project (Malara & Navarra, 2001, 2002, 2003), which can be seen as an *integrated system* for teachers education, even if it has been conceived for the renewal of the teaching of arithmetic in the perspective of early algebra.

**2. SOME INDICATIONS ON THE ARAL PROJECT****2.1 Theoretical frame**

Over the past twenty years, research has focussed on a large number of possible approaches that increase the *meaning* of algebraic processes and objects (Arcavi, 1995). Some of the principal forms are: *problem solving* (where emphasis is given to the analysis of problems and equations); the *functional* approach (the use of letters to indicate measurements and the formal coding of relations among measurements); the *generalization* approach (the use of expressions to represent geometric patterns, numerical sequences, "rules"). A determining role is attributed to the *linguistic* approach and

to research that faces the didactical developments starting from the *concept of algebra as a language* (Arzarello & al., 1993; Malara and Iaderosa, 1999; Radford, 2000; Menzel, 2001). This role becomes even more significant if it is associated with the idea of an early approach to algebraic education beginning from the didactical revision of the relations between arithmetic and algebra. Many researchers indeed have underlined that the difficulties in the approach to algebra are rooted in the scarce attention paid to the relational or structural aspects of arithmetic, which constitute the basis of elementary algebra (Kieran 1992, Linchevski 1995). Recently, many studies have been developed, operating from various stances and with different approaches to the matter (Ainley 1999; Brito Lima & Da Rocha Falcão, 1997; Brown and Coles, 2000; Carpenter & Franke, 2001; Carraher & al., 2000, 2001; Da Rocha Falcão, 2000; Kaput and Blanton, 2001); these studies are now constituting the well known corpus of 'Early Algebra' in which ArAl Project is framed from the disciplinary point of view.

## 2.2 Hypotheses and aims

The ArAl project, has been developed on the results of our own previous studies (Malara & Iaderosa 1999, Malara 1999). It is based on the hypothesis that the main cognitive obstacles in learning algebra are to be found in the pre-algebraic field, and that many of these spring up from unsuspected arithmetical contexts and they then become conceptual obstacles to the development of algebraic thinking, because of the weak conceptual control which many students have over the *meanings* of algebraic objects and processes. In the perspective of a conception of algebra as a language, the hypothesis of the ArAl Project is that there exists a strong analogy between the methods for learning a spoken language and the algebraic language.

In order to explain this point of view, we shall employ the "babbling" metaphor. When a child learns a language, he or she masters the meanings of words and their supporting rules little by little, developing gradually by imitation and self-correction, right up to the study of the language at school age, when the child begins to learn to read and reflect on the grammatical and syntactic aspects of the language. Traditionally, in the teaching of the algebraic language, one starts with the study of the rules, as if the formal manipulation should have precedence over the understanding of meanings.

The syntax of algebra therefore tends to be taught while overlooking its semantics. The mental models for algebraic thinking should be built rather more in an arithmetical framework - beginning from the first years of primary school - through initial forms of *algebraic babbling*, teaching the pupil *to think of arithmetic in algebraic terms*, in other words, building her/his algebraic thinking *progressively into close interlacement with arithmetic*.

To this end, it is necessary to construct an environment which stimulates even informally the autonomous elaboration of the algebraic babbling and therefore the playful, experimental and continually redefined acquisition of a new language in which the *rules* may find their place just as gradually within a didactical context which is tolerant of initial syntactically "shaky" moments.

In order to make it possible to implement all this in the classrooms it is necessary that teachers undergo an ad hoc training.

## 2.3 The role of the teacher and the problem of its training

That of the teacher is a 'grounding' profession at social level, and of great responsibility. The mathematics teacher of primary school is considered to be the *first responsible* of an educational chain that at its top produces scientific and mathematical knowledge being inadequate to the models of technological development that the industrialized world is pursuing, to the performances that it consequently requires. In this context, the challenge consists in leading the teacher to a revision of his/her role towards a *critical review* of classrooms behaviours that are often worn out by habit or tiredness, devitalized by the absence of the fruitful doubts induced by a *meaningful* study, replaced by the certainties of a repetitive daily practice.

The possibility to take part in the project therefore aims at facing teachers with a moment of *reflection on beliefs and stereotypes*. The general questions they must deal with are: *Which arithmetic am I teaching? Which algebra am I teaching? Is my working method appropriate?*

The teacher's role in the management of the activities of the ArAl project is delicate and it is made more difficult by the fact that almost all compulsory school teachers do not have a mathematical university background, as the majority come from areas of humanistic and pedagogical education (primary teachers) and scientific education (junior secondary teachers). Their cultural background and beliefs naturally influence their choice of the ways through which they impart pupils basic knowledge. Therefore one of the main aims of the project is the rephrasing of the teachers' conceptions, so that

they can get a deeper competence<sup>1</sup> and acquire a new, more adequate professional identity. In particular the teachers have to become aware of the big variety of roles they have to face in the classroom, precisely:

- *provoker* in search for stimulating activities that promote reasoning and reflection;
- *model* who in front of problematic situations suggests and explains a wide range of strategies and reasoned trials, so as to induce attitudes that with time can become effective behavioural habits.
- *maieutic operator* who, through discussion, guides the pupils towards constructing their knowledge and reflecting on their thinking processes.
- *orchestrator* who leads the social sharing of achievements by coordinating individual interventions;
- *ratifier of knowledge* that makes sure that the solutions reached may gradually become institutionalised knowledge;

Of course our work aims at making them succeed in putting these roles effectively into practice. Furthermore, they can become for their students:

- *a cultural element of reference* in communicating them the pleasure of discovery, curiosity, intellectual challenge, and in building the awareness of how the growth of ‘local’ knowledge brings to their framing within *theories*;
- *an affective element of reference*, aimed at gaining the pupils’ trust by letting them feel the care and commitment for their harmonic growth and their human well-being.

#### 2.4 The “Units” of the ArAl Project

An important result of the ArAl Project is the creation of various “Teaching Sequences”, roughly called “Units” to facilitate communication among teachers. These Units can be seen as models of teaching processes<sup>2</sup> of arithmetic in an algebraic perspective. They are structured in such a way as to make the teaching process transparent in relation to the problem situation being examined (methodological choices, activated class dynamics, key elements of the process, extensions, potential behaviour of pupils and difficulties they may encounter). The final goal is therefore to offer teachers the opportunity to reflect on their own knowledge and *modus operandi* in the classroom, before actually providing them with didactical pathways that they should follow. Thus, the Units are not tools for immediate use in the classroom, but require a theoretical study, before being put into practice. To this end, the Project’s two key tools were created: the Theoretical Reference Framework and the Glossary, which contains more than 70 terms. Through the combined use of these tools, teachers can attain a double goal: the first, immediate and local, concerns the guiding of pupils in the collective exploration of proposed problems; the second one, more general and attainable in the longer term, concerns the objectivation of “hypothetical learning trajectories”<sup>3</sup> (Simon, 1997) as to the subject in question, according to the spirit of the Project.

But teachers who intend to embrace these innovative teaching approaches must be prepared to combine their existing knowledge, competences and beliefs with a mix of far-from-marginal methodological and organizational aspects – to stimulate activities with a high metacognitive content, to favour the reflection on language, to promote verbalization and argumentation, to reach a fine analysis of protocols. All these aspects operatively support an actual *culture of change*.

#### 2.5 The methodology

As referred to in the introduction, the ArAl Project is meant to offer an integrated training system within which teacher can:

- take part in meeting of study of theoretical reference frames together with the researcher;
- verbalize and comment the activities carried out (on the basis of recordings);
- assess together with the researchers the state of the project and cooperate in the organization of the materials;
- compare the works with teachers who are experimenting ArAl units in their classes;
- present the activities in seminars, congresses, workshops

<sup>1</sup> We use the term competence in the Abrantes sense (2001) that is as the “*process of activating resources (knowledge, skills, strategies) in variety of contests ... and related to the capacity to improvise ... and to make emancipatory action, based on reflection and implicating some degree of autonomy*”.

<sup>2</sup> Of course, these models are not theoretical tools for researchers (Schoenfeld, 2000), but tools for the renewal of classroom practice.

<sup>3</sup> According to Simon, “*The hypothetical learning trajectory is made up of three components: the learning goal, the learning activities, and the hypothetical learning process – a prediction of how the students’ thinking and understanding will evolve in the context of the learning activities*” (1997, p. 78).

- widen their knowledge of realities of school and didactic research through meetings and cooperation with teachers of other countries.

We are not lingering any further in the methodological structure of the project (for this see Malara (2003)). Here we shall concentrate on the articulation of a work carried out with a group of four teachers who were external to the project: two elementary school teachers (both women, V.I. and E.M.) and two middle school teachers (a man, R.N. and a woman, R.F.). These teachers were selected by MIUR (Ministry of Instruction, University and Research) for a one-year training stage as researchers within our university. Though being new in the field of academic research, these teachers already had achieved a high professional commitment within school and sometimes of autonomous research.

Experimentation in the classes is preceded by a theoretical-critical study of the ArAl project and of recent related papers (for example Radford 2001, Section 'Early Algebra' in Cick & al. 2001), papers on the orchestration of classroom discussion (for example Bartolini Bussi 1998, Yackel 2001), or peer discussion (Sfard & Kieran 2001) and also papers of historic-epistemological kind (for example Rogers 2000). An a-priori critical analysis of the Units of the project is carried out in order to compare the different setting choices and the more delicate points of their implementation.

The discussions carried out in the classroom are recorded and each teacher has the task to transcribe them. Then, the discussions are analysed and critically discussed together with the researchers. This joint analysis encourages a hot confrontation between teacher and researcher in the face of emerging habits, stereotypes, convictions, misconceptions, etc., and encourages the teacher to express points of view, doubts, perplexities, important indicators of his or her conceptions, and provides an opportunity to disclose conceptualisation gaps in the mathematics education of teachers.

This phase of the work ends up with a reflection essay of the teacher of this revision. This moment turns out to be of particular importance for the teachers' awareness of their way of being in class and for a first assessment of their role (didactical choices and interventions, word turns to the pupils, reintroductions, timings, etc).

There is also a phase of common comparative analysis of the four paths. This phase turns out to be the climax of the whole experience: comparing their experience path with that of the colleagues in the same steps of the didactic sequence, each of them detects important elements of difference and reflects on the effectiveness or the limits of their work, which makes them acquire deeper awareness of their way of being in class, of keeping one's behaviour under better control and possibly changing it.

At the end the teachers carry out a global reflection on the experience with the explicitation of the difficulties, the crisis and the awareness achieved<sup>4</sup>. This gives us the possibility to observe the different incidence of the experience on each of them and the influence of each personality on the educational process enacted.

### 3. ASPECTS OF THE IMPLEMENTATION OF THE ARAL PROJECT

We now examine two episodes of the implementation of the Project Unit "From the Scales to the Equations" (Grades 5<sup>th</sup>-6<sup>th</sup>). This Unit was meant to work from experience to theory, and uses the well-known scales scheme as an aid to a symbolic representation that can create a semantic basis for the introduction of algebraic formalism<sup>5</sup>. For reasons of space, we shall concentrate on two single class episodes, though many would deserve being mentioned. These are extracts of discussion, which are to be read from the viewpoint of the teacher's decisions-actions (see Table 1 and 2). At that point in time, the teachers had changed their conceptions of algebra and its teaching, had learned to appreciate the value of theoretical study, and had already started on Unit experimentation. The episodes evidenciate

<sup>4</sup> The material produced is very wide and meaningful, so that now we are organizing it in a booklet that we plan to use in future teachers' training.

<sup>5</sup> The Unit starts with the simulation of problematic situations on the scales, which are then solved by subtractions or splitting up of same quantities from both balance plates. Reflecting collectively on the actions taken to find a solution, students discover 'the principle of equilibrium' and the two principles of equivalence. The problem then arises of how to represent the situations already examined. This phase involves the progressive simplification of the representation of the scales, slowly arriving at the equal sign and the choice of representation of unknown quantities, which leads to the 'discovery' of letters in mathematics and equations. Even the procedures for the solution of equations are progressively elaborated and refined through collective and individual activities, during which students elaborate and compare various representations, refine their competence to translate sentences and, moreover, become accustomed to using letters as the unknown entity. A sequence of appropriately organized verbal problems of different levels of difficulty leads students to investigating how to solve problems using algebra.

the influence in the discussion of the personality of the teacher and of the net of socio-affective relationships among pupils. (The sentences in italic are notes or comments by the teacher.)

### 3.1 First episode (6<sup>th</sup> grade, teacher R.F.)

The episode takes its place at the end of the first phase of the Unit, in which a set of problematic situations on the scales are faced and where the weight of objects on the pans have to be found.

**Table 1**

#### **Excerpt on the collective solution of a problem invented by a pupil.**

The teacher poses the pupils the following question written by their classmate Mattia:

On one side of the scales there are a weight of 600 grams and 3 packets of salt, in the other there are a weight of 700 grams and a packet of salt. The scales are in balance. The packets are equal and we have to find the weight of one packet.

#### **Discussion**

Giulia B.: In my opinion a packet of salt weights 50 grams, because first I have taken away a packet of salt from the two pans, then I have taken away the weight of 600 grams from both the sides, considered that 100 grams and two packets of salt remain I have divided by two.

Federico: I would like to express my reasoning

Teacher: Do you need the initial scene?

Federico: ok (the teacher simulates the configuration of the pans of the scales)

Federico: If I have 700 grams and a packet of salt here and 600 grams and 3 packets of salt there, if the scales are in balance it means that to arrive from 600 at 700 we need 100 and there we have 3 packets of salt. If I use two packets I arrive at 700 and considering that it remains a packet and another here it means that I have 750 grams in each pan.

Teacher: I am not sure I have listened carefully enough to what you have said. Repeat more slowly

Federico: Here I have 700 and there I have 600, I miss 100 to arrive at 700. Then I have thought I use two packets of salt, which weight 50 grams each, in total 100, but considering there is another packet its weight is 50 grams too. Then I have 750 grams on the left and 750 grams on the right. (*Federico speaks in hurry and his words merge with the checks*)

Anastasia: Federico practically does not say it, but mentally excludes a packet from a pan and another from the other pan. When he says that from 600 to arrive at 700 we need 100 he thinks that here 100 grams are 2 packets, he has already taken away the other two (one from a side, another from the other side).

Federico: yes, we can also put it this way

Mattia: my reasoning is: if we take away a packet from both parts, if we calculate well, then in the first pan I have 600 grams and 2 packets. I suppose that the packets weight 50 grams each, in the first pan I have 600 and 100 more, so I have 700 in total and in the second pan I have 700 grams. Then 700 grams is in both pans, after there is the second principle. But now I've got lost... (*He goes to the desk and repeats the initial scene*) I take away a packet from both sides, so 600 more 100 gives 700, from the other side I have 700 and the scales are in balance. Then two packets of salt weight 100 grams and a packet of salt weights 50 grams. I have supposed that the weight 50 were grams (Mattia's intervention is "unclean", because he refers to his strategy in constructing the text rather than to the process of solution. Then he starts to express the process of solution clearly but in the end – referring to the choice of the weight of a packets – he returns to the previous model)

Jasmine: I explain my way of solution. I have taken away a packet from both pans, I have 600 and 2 packets and 700. Then I take away 600 and I have 100 and 2 packets. (*she uses gestures to represent where weights and packets remain*). Then I divide by 2 and I find that a packet weights 50 grams. I have done like Giulia. I couldn't understand well Federico's reasoning.

Federico: I had said right (*he repeats his reasoning grumbling and he concludes by saying that in the two pans there are 750 grams in total*)

Teacher: your conclusion is right but we have to find the weight of one packet.

Federico: here we have 700 grams and a packet of salt, there we have 600 grams and 3 packets of salt. If I think about 3 packets of salt and 600 grams to arrive at 700 and a packet of salt we need 100 grams. From this calculation I think that 2 packets of salt are as 100 grams and adding all the weights in each pan I have 750 grams. If 2 packets of salt are like 100 a packet of salt weights 50 grams.

*The teacher asks the pupils if they agree with the processes exposed by Giulia B., Jasmine and Federico.*

Mattia: I agree with Giulia B. because she has used the first principle when she takes away the same things from the two pans and the second principle when she divides by the same number. Federico, said Anastasia, does the same things but he does not say everything.

*All agree with Giulia B. From the class does not come any comment on the solutions of Jasmine and Federico.*

For assessing the individual learning realized in the discussion, the teacher poses the students the task to invent some problems similar to the ones explored in class. The reported discussion (table 1) regards the sharing of the solutions of a problem invented by a student (Mattia). This excerpt shows a class trained to compare their ideas and able to sustain different and coherent argumentations. It is also interesting as far as the teacher's behaviour is concerned (silence, reintroductions, turns). The episode develops around Federico's strategy. Facilitated by the simplicity of the numerical data, he sees immediately the solution operating mentally the subtractions of the same quantities (packets and weights) from the two pans and, in the same time, checking the correctness of the solution through the comparison of the total weight in each pan. At the moment the teacher does not grasp the student's reasoning and she drops it in spite of the clarifying intervention of Anastasia.

This episode evidences one of the most delicate points in the discussion: the teacher's flexibility to adapt herself to the flow of the thoughts which arise in the class, to grasp their potentiality to develop and insert them in the context of the work. In the following phase of reflection she writes: *On re-reading I notice I should be able to bring out the reasoning that Federico does, facilitated by the transparent numbers. My intervention was important because if I do not exalt the alternative strategies the students cannot appreciate them and keep relying on standard procedures.*

### 3.2 Second episode (5th grade, teacher V.I.)

This discussion was inserted at the beginning of the second phase of the unit, devoted to the representation of the problem situations examined, and concerns particularly the choice of the way in which unknown entities are to be represented. The classes had already tackled and solved the problem of representing the scales in balance. The discussion deals with ways of representing the weight of a packet of salt, rice, etc.. In it there is the autonomous choice of the teacher to address the students towards a unique code. At this point many students are still in the iconic phase and this discussion is to some extent premature. The students have neither the possibility to compare critically their representations and discover analogies and differences nor to appreciate the ductility of the letters as to other representations.

In the discussion we can see (Table 2) teacher's naiveties and above all the poorness of the motivation she gives for the problem posed, which is crucial for the disorientation of the students. She does not know how to manage this *impasse* and tries to overcome it by proposing the vote. She loses also the occasion to bring out some important intervention like the conclusive one of Laura.

The discussion highlights the classic conflict quantity-quality about the unknown data. Some students focus themselves on the quantity, several on their quality, certain students declare that both have to be considered. The discussion is very expressive from the point of view of the relational dynamics. Giacomo is the leader of the boys, Laura of the girls. From the beginning they set out on different choices. With her choice of the letter U, neutral as to the kind of the things in play, Laura appears having developed a higher, more abstract vision, almost an intuition of the unknown. The argumentations in favour of the letters of the other girls reveal a conception of this as a semantic anchorage. The boys become fixed in their leader's position, but they have no plausible arguments to support it; then ironies towards the girls and fears for the bigger number of these arise <sup>6</sup>.

A third episode, concerning the implementation of the unit in 6<sup>th</sup> grade (teacher R.N) is reported in Malara (2003). There, the discussion excerpt highlights how difficult it is for a teacher – however culturally and emotionally committed – to move to an innovative classroom practice.

As one can see when reading it, it is a problematic discussion, since the teacher, very probably suffering from latent anxiety, and affected by his usual way of being with the class, repeatedly intervenes, approves correct hypotheses at their first appearance, tends to interrupt those contributions he considers less than productive, anticipates the reasons why certain hypotheses must be discarded, does not ask pupils for justifications of their hypotheses, and decides conclusions *de facto*. The positive aspect is that, after a transcript analysis with the researcher, the teacher writes in his reflection commentary: *"I tend to impose too strongly the path we must follow. ... Perhaps I tackled the problem of the introduction of the letter too hurriedly; but it is important to be aware of this. It will come up again on other occasions, and then we can carry on the discussion."*

<sup>6</sup> Teachers state that complicity between individuals and rivalries between groups are elements that strongly condition the development of a discussion. Emblematic are, to this point, the reflections R.N. made on the experimentation of the same unit with two different classes (Malara 2003), where one can see that in one of the two classes some good female pupils refuse to take part in the discussion since they fear that discussion could jeopardise their leadership.

Table 2

**A discussion on the choice of a unique symbol to represent an unknown quantity (5<sup>th</sup> grade)**

Teacher: In the experience we have made during our last lesson we have seen many ways to represent a packet: with drawings, with circles, with squares, with some letters; now we unify these symbols so that we can have a symbol which is as much schematised as possible so that we can use it many times. Are there any proposals?

Giacomo: a circle

Dimitri: the letters, F for flour, S for sugar, in this way

Andrea: a triangle

Rifat: a square

Laura: the U of units, so it can be the same for all things (*the choice is made on the base of the experiences of measure*)

Teacher: are you in favour of the letter?

Laura: yes

Giacomo: ok, geometrical shapes, I have said a circle, so

Teacher: is for you a geometrical shape better?

Giacomo: yes, it is faster

Andrea: It not possible to understand with the letters

Teacher: we should explain even the geometrical shape, which in any case has to be explained

Laura: we make the legend

Michele: perhaps both of them is better

Teacher: Now, those who still haven't spoken try to express her/his opinion. What do you think: if we want represent the scale with the equal sign is the geometrical shape or the letter better?

Carol: for me it is better the letter, the initial letter

Gerardo: even for me the letter is ok

Veronica: for me it is any letter ok, for instance the letter U of unit, so for all the packets, because for us it is not important what it is inside but the weight (*with this intervention she shows she is on a more advanced level, towards abstraction*)

Gerardo: I am in favour of the initial letter, because if we use a geometrical shape we have to write the legend (*he is more connected with the experience. He agrees with the letter for having an easier semantic control*)

Pupils: But even with the initial letter is the same thing

Teacher: Those who still haven't spoken please express their opinion. It is a thing that concerns us all.

Claudia: I am in favour of the initial letter

Federica: me too

Linda: me too

Giulia: I am not in favour of the geometrical shape because we cannot understand if it represents salt or flour

Andrea: I am in favour of the geometrical shape, mainly the circle, because one does early and then it is not interesting what there is inside but its weight

Veronica: but, if you do a shape, afterwards you do not know what it is

Michele: then it happens also with the letter. There are 21, do you know what they are?

Andrea: we are not interested in what there is inside but in its weight

Damiano: then we put immediately the weight so we do it quickly

Laura: But if we put immediately the weight, then it is not interesting to find it

Giacomo: (*with ironic intonation*) we want it to be interesting!

Teacher: I do not understand Damiano, what did you want to say when you have said 'we put immediately the weight'?

Damiano: instead of putting the letter or the little circle or other geometrical shapes we put directly the weight

Pupils: but you do not know the weight!

Giada: that's what you have to find out!

Michele: teacher, why do we need a system that is as simple as possible?

Giacomo: So we can do quickly

Teacher: because we are going towards situations which are more and more complex to study and to represent

Giulia: Andrea had said that we are interested only in the weight, but actually what is inside the packet is also important

Laura: Andrea said that only the weight is important, but he said that one can understand better using a geometrical shape; however if we use the letter 's' of salt one understands, but if we use a letter by chance it is the same as the geometrical shape

Veronica: Using the letter is faster

Michele: For me both things are not good, because one cannot understand, you need the legend  
 Giacomo: we need the legend, because otherwise, also using the letters, we cannot understand  
 Andrea: we use a geometrical shape  
 Giada: I prefer to use the letter 'U', because you cannot count each corn of rice to know how much is in its weight, then it is not important what there is inside  
 Veronica: (*trying a mediation*) we can put also 'O' for the shape, we are interested in the weight not in what there is inside the packet  
 Giacomo: But (the girls) are flunking themselves, they are saying that it is not important what is inside. Why do we have to put the letters, it is better a geometrical shape (*it is emerging a kind of defence of male power, he is the leader among the boys*)  
 Giada: It takes longer to draw a shape  
 Boys: eh, yes really  
 (*In the discussions the conflict between boys and girls emerges. This conflict developed in the free time of play. In some moments their voices are very loud.*)  
 Teacher: For this experience we decide in a democratic way. I see that there are three proposals. If we want we can vote  
 Giacomo: yes, yes, of course the 'S' are more. (*He is worried to lose the voting*)  
 Teacher: now we'll see  
 Laura: I support the use of 'U', which expresses everything, not only the salt (*she goes up to the abstraction*)  
 Rifat: I support the use of geometrical shape  
 Teacher: Let's not support the ideas of your friends!  
 Girls: all the boys say ...  
 Michele: I support nobody  
 Teacher: the proposals are three: to use a geometrical shape, to use the initial letter of the contents, to use the letter 'U' of unit. Reflect well, we are 18, each of you express your choice, in the next experience we shall use the more voted code  
 Michele: ok, we lose  
 Giada: I support the use of 'P' the initial letter of 'packet'  
 Michele: but 'P' can mean 'pen', 'pile',  
 Giada: if there are boxes, we put 'B'  
 Teacher: in the voting we consider also the Giada's proposal, that is the initial letter of the container  
 Resulting votes  
 geometrical shapes: 5 votes; initial letter of the contents: 3 votes; letter 'U' of unit: 7 votes; initial letter of the container: 3 votes.  
 (*Laura, who proposes the use of 'U', is the leader of the girls, this can have influenced the choice of the pupils*)  
 Teacher: in the next experience we use the letter 'U', as Laura has proposed. Laura please explain why you have done this proposal  
 Laura: if we have packets or books it is the same thing, we use 'U', it is like a unit (*she senses that the quality of the object is not important*)  
 Andrea: Laura says unit, but what if there are 5 things?  
 Laura: we put 5 U (*5 letters or 5 followed by U? I had to clarify this point*)  
 Boys: yes, thousand, UUUUUUUUU.....

#### 4. THE TEACHERS AND THEIR CHANGING

According to their different personalities, the four teachers have a different behaviour in the management of the didactical paths. These are their profiles in short:

V. I. (primary teacher) is anxious and speedy, generally she is little flexible even if she gives voice to the students and takes into account the contributions of all. Not always she is able to identify the potentiality of an intervention and therefore to give it value. Unsure, she always feels the need to discuss with the researcher in order to have confirmation of the correctness of her own way of working. She is very receptive, she appreciates commentary that she always succeeds in elaborating.

E.M. (primary teacher) She is very creative and harmonic in her work; she leads her class with ability appreciating all the contributions, however she tends to stay isolated from the other colleagues, even if seeing her work appreciated. She clearly expresses her uneasiness, doubts and difficulties.

R.F. (middle school teacher) is very sweet and silent, very refined in leading discussion: she never imposes her point of view and simply suggests opinions and hypotheses for validation, promoting the justification of the reasons expressed. She is very careful to the affective aspects and to the relational dynamics among the pupils (though being sometimes a victim of such dynamics). She pays particular



Table 3

**Reflections by the two teachers documenting two different ways in which the experience is lived**

**R.N.** The methodological choice was centered on the process of sharing: sharing among colleagues (ideas, methodological and operative choices, materials, criteria of analysis, ...); : sharing among pupils (of ideas, interpretative choices, proposals, criteria of analysis, and criteria of assessment, ...); sharing among teachers and students (of collective growth processes, emotions, debates, ...). the sharing appears as a real key for working. But we also have to see the other side of the coin. The sharing in the class generates also: a) a strong process of inner debate, of comparison among singles and groups, which produces tensions that must be controlled and addressed; b) a trend to underestimate ideas coming from 'less strong' subjects and an overestimation of the proposals coming from 'stronger' subjects. This process is dangerous and it has to be controlled; c) a lengthening of the work time in class that worries (this factor has to be carefully assessed). The experience made allowed us to evidenciate the value of the processes of sharing that can be created within a community of teachers. I think that the question of how to harmonize the individualities in the sharing process must still be faced. The process of work has to guarantee a reinforcement of the self-esteem of the teachers. The personal education cannot to be neglected. It is important to think about paths for deepening the study via a specific bibliography, visits to exhibitions, participation in conventions and seminars. In our own small way, we have had significant experiences in this regard. The relationship with Nicolina [the researcher] has been a very special one: of dialogue, but with strong theoretical and methodological connotations, and based not only on experience, but also on a wide-angle intellectual opening and true personal commitment. This inevitably involves moments of crisis, disagreements, and lively discussions - a SEISMIC TREMOR!"

**R.F.** For me, globally, the experience has been very positive, it has taken me much energy, but it has given me satisfaction and it made me to teach in a new way. In my work usually I give room to the discussion but I had never used the tape recorder. At the beginning its presence has given me emotion and partly I got stuck, whereas the students expressed their opinions, their point of view without embarrassment, but with the relish to speak and listen to them in the discussions made. ... I have noticed the students' richness and how they feel motivated and strongly stimulated to take part in the discussion when their opinion is expressly requested, that is when one brings them to become protagonists. The monotone lesson at the only teacher's voice changed into an interesting lesson at many voices, awaited with pleasure. Then I think this experience has to be repeated also in other ambits. However I think that it is not possible to face each topic with this methodology, due to the high number of hours that it requires.

For my growth as a teacher it has been very important to listen again to the dialogues, I could realize how an intervention has been missing or little clarifying or how I have supported a solution rather than another, maybe only for having said "well" or having showed a particular face. One has to dose one's role very well: the teacher's intervention has to be carefully measured, because one should not forget that silence can facilitate the student in expressing his/her thought.

I have tried to give wide room to the discussion among students, reducing my interventions. In this I have been helped by my shy and introverted character and by my habit to speak little and to listen much. I am aware I have to learn a lot in orchestrating the discussions I noticed I have not always consciously made sure that all the students took part into the discussion. In the class there are some students who do not speak spontaneously and they do not give any answer when one poses them a direct question. In the moment I do not feel like forcing them to speak, but I should have been able to involve them by asking them to write what they would have wanted to say. However, in order to show the class that also these pupils had taken part, even if silently, into the discussion, on opening the discussions I have often begun with some excerpts of their notebooks.

The recording of lesson is a very useful tool to highlight merits and lacks of an activity (rhythms, dynamics, boredom, etc.).

Another positive aspect of the experience was the comparison with colleagues of other schools, a deep confrontation on what we are doing or we shall do. It has been possible to discuss about our difficulties and our different ways to pose ourselves in front of the students or to formulate them the questions for promoting the involvement of all. It is an experience which should be suggested to those colleagues who yet wish to put themselves in play and to change.

attention to individual learning and always makes sure that the teaching has been effective by testing the pupils through homework she creates purposely.

R. N. (middle school teacher) could be defined as an 'enlightened dictator'<sup>7</sup>. Self-confident in the classroom, he charms the pupils with his strong personality, still he doesn't really care to involve the class into discussion, which is usually carried out only by a small group (8-10 pupils of 21). His way

<sup>7</sup> Being very talented in organizing, he is the leader of the group (writes down and sends through e-mail the verbals of the group meetings and the summaries of the examined works, realizes also some video-recording of our meetings).

of leading the discussion shows the features of traditional teaching: he usually calls a pupil at the blackboard in order to summarize the different contributions, but he ends up to leading a dialogue with him minimizing the other pupils' contributions. A positive aspect in his approach is the fact that he gives the pupils a cultivated vision of mathematics by underlining its historical dimension.

As already mentioned, a very interesting and productive moment for teacher's changing is the cross-comparison of each intervention in the class. This offer to each teacher the opportunity to observe his/her way of being in the class, so that he/she can reflect on the actions-decisions taken, on the emerging bad habits, on the underlying emotions. Emblematic is, for this aspect, the comparison between R.N. and R.F., both middle school teachers who have chosen to go on in parallel. In particular R.N. has been able to notice his little flexibility in giving voice to the students because of his frequent and decisive interventions. R.F., instead, has acquired trust in her thanks to the general appreciation of her actions (the equilibrium of her silences, her attention in calling upon even the more reserved pupils to speak, her ability to put again under examination students' observations).

As to the impact of the whole activity on the teachers, we report in table 3 two excerpts by the two middle school teachers, which testify their different feelings in living the experience. The first excerpt (by R.N.) evidences what we can call 'a positive crisis of growth' through the expression of perplexities about the organization of the classroom work and the sharing process with the colleagues. On the other hand, the second excerpt testifies an effective teacher's growth and the ripening of her new knowledge.

### 5. SOME CONCLUSIVE CONSIDERATIONS

In conclusion, as far as the teachers' overall professional growth is concerned, we can say that thanks to the shared critical theoretical study all the teachers got to emphasize the multiplicity of representations and at conceiving the productivity of restructuring the arithmetic teaching in pre-algebraic perspective, stressing relational and generalization aspects. The question of the management of classroom work - particularly the mathematical discussion - is a more problematic question.

The reported case and others we have analysed – in which teachers show that they do not grasp a student's reasoning or fail to give due value and let drop significant contributions, or are conditioned by some students' invasiveness, or are even unable to use appropriate silent pauses – clearly show how rich and at the same time also how dangerously delicate the classroom discussion is, precisely because in the midst of the overwhelming energy of a participating class, "traps" for the teacher lie everywhere (unforeseeable diverging solutions, potentially fruitful but perhaps not too clearly expressed; time that flies; the need to keep alive the students' general attention; the need to consolidate achievements, rather than disperse them, etc.).

All this shows us very clearly the importance of a fine teachers education on listening to their students. This condition poses to us the hard challenge of how to best help them to "fine-tune their antennas" and acquire that "local flexibility" which enables them to adapt to the flux of thoughts which emerges from the class, to grasp the potentialities, to develop them and adequately insert them into the working context.

The task is far from being easy, since it is not a matter of dialogue on a mathematical knowledge, but on the more complex and delicate level of behaviour – mostly subconscious – that is rooted in the teacher's past life experiences. Furthermore, it is not a question of giving teachers an awareness of what is wrong with the way they operate (what they tend to anticipate or, on the contrary, even to omit in the midst of live classroom action), but rather more a question of heightening this awareness, in order to create a new, more adequate behaviour.

These experiences have made us aware of the fact that we have to implement even finer modalities, to encourage teachers to reflect upon their own actions, thus acquiring new abilities towards "knowing-to-act in the moment" (Mason & Spence, 1999). For example, we deem it indispensable to make use of tools such as video recordings of class interventions (up until now only marginally used in Italian research), to help teachers reflect on their micro-decisions and to analyse the use and incidence of non-verbal language. Needless to say, this "local flexibility" of teachers – once they have made a commitment to innovation – represents the result of a process which can in the final analysis be defined as "joint (self)education", involving study, comparison and experience.

A further, completely different, and important ground for reflection is for us the incidence of the network of socio-emotional relationships within the classroom (leaderships, power groups, median roles, and singles) in the development of discussions. In many cases, we observed rivalries between

groups of different sexes<sup>8</sup>, complicities between singles, or even a refusal on the part of pupils to have themselves involved. This led us to considering the class as a social system (Lerman 2001, Fele & Paoletti 2003).

To sum up, our experience allows us to assert that our interventions for and with teachers slowly modify their knowledge, beliefs and classroom behaviour but overall bring them to acquire a greater awareness of the complexity of their role and to get a new conception of their own profession, where the study and the peer comparison find a big room. Of course, the depth of change in each teacher depends on her/his receptivity and willingness to re-appraise her/himself, and, moreover, on her/his being on the same wavelength as the leaders of innovation.

What appears more problematic is the relapse of these experimentations at large in all schools, which are in many cases a world apart from any movement of renewal. Moreover, we have to consider that at the moment in Italy there is a highly confused situation. The new government has delivered new proposals of the curriculum for compulsory school (2003), which overlap to the ones by the previous government (2001) and which depict a cultural backing as to the present curricula and aboveall as to what the research in mathematics education has promoted in the last thirty years. In the culturally committed ambits it is circulating the idea of a convinced resistance to these changes. To have an idea this spread feeling we can take into account the last images of François Trouffaut's film "Fahrenheit 451", where a small community of people defends books about poetry, thought and values of culture from the effects of an alienating power, by constantly repeating them by heart.

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<sup>8</sup> The social equality between sexes, which prevails in our country, is reflected in the way in which teachers – mainly women – regard their pupils. However, we have been able to ascertain that sexual differences affect aggregations and subsequent performance in the development of discussions.

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