A didactical proposal: perpendicular and vertical straight lines

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Abstract*:* The last studies in mathematical education reveal (Maier "Conflict between mathematical language and natural language in the pupils ") that the difficulties of understanding mathematical concepts are often caused by "closeness" between the natural and the mathematical language. For example in a mathematical context the word "vertical " is referred to a specific relation between two straight lines, that are perpendicular straight lines. The conditions that two perpendicular straight lines must satisfy are:

- System of reference;
- The formation of four right angles.

In the natural context the word "vertical " is referred to the direction of centre of earth.
The pupils often use this mean in a mathematical context and so when there is a change of the straight line position as regard which they must determinate the perpendicular straight line, it's possible observe wrong answers. It's very interesting to analyse these phenomena and search some answers to these questions:

- Have pupils the implicit model of plumb line?
- Are the difficulties of understanding the concept of perpendicular straight lines related to difficulties of linguistic kind?

We'll try to answers these questions using "the theory of situations".
This was an expedience lead within the "Progetto Spora" . The schools involved are "risk schools ", and the pupils live in strong hardships condition. The age of the pupils involved in the project was 3-11 years hold. But in this paper we analyse the results obtained in the Primary school
The theoretical frameworks of this research is the systemic approach "knowledge, pupil, teacher". The studies of the possible relation regards the theory of education research that has contents, methods and a paradigm.
Knowing the paradigm of the research (the theory of situation of Brousseau, revisited by F. Spagnolo in the part that regards the epistemological and storic-epistemological reflection) is necessary have:

- An adequate language;
- Adequate methodological tools;
- Adequate statistic tools.

From the point of view of teacher controlling the tools of the theory of education research means:

- Having the conception of the positive role of the error;
- building a theoretical framework to solve teaching problems;
- having autonomous ideas to communicate the process and the results of the research.

Theory of education research is an important elements for the teachers, it is important:

- Knowing the phases of the research;
- building the "analysis a priori" about the situation/problems;
- formulating hypothesis of research congruent with "analysis a priori".
- producting documentation for the person who haven't participated to the experience.

The aim of this research is to investigate about the conceptions of pupils to the point of perpendicularity.
The first answer of this research was: *"have pupils the implicit model of plum line in accordance with the concept of perpendicular straight lines is identifiable with the concept of verticality?"*
In according to this point of view the model of reference is model of plumb line which is referred to the concept of verticality. It is very interesting to analyse the definitions of vertical and perpendicular that we can try in Italian dictionary of mathematical terms (Rizzoli Italian dictionary).
Vertical (late latin verticalis, which go from the vertex to the basis, from vertex, top, summit): means the straight line which follows the direction of plumb line.
Perpendicular: from latin perpendiculum, plumb line, derived of hanging.
We can observe that the term "perpendiculum" means plumb line , but the direction of the plumb line doesn't define the condition of perpendicular straight lines but it defines the condition of verticality. In fact in the natural language perpendicular is an adjective that points an object which has the direction of plumb line, it is a synonym of verticality.
This research is based on this considerations. An obstacles to learning of mathematical concept is the closeness between natural and mathematical concept. In this case the closeness between the word perpendicular and vertical. Pupils used often terms of the natural language and they puts them in a natural context but they don't works the change of meaning of the technical-scientific language. They uses technical terms in the sense that they have in the natural context.
Starting point of this research was the follow hypothesis:

- Words perpendicular and vertical are synonymous for pupils;

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- When the teacher presents the concept of perpendicular straight lines (in mathematical sense) pupil learns it if he adapts opportunely the plumb line model to the new general situation that the teacher has proposed; if the breaking of this epistemological obstacles don't happen the pupils will be no able to do exactly the task.

It's natural to suppose that probably very strong obstacles are opposed to the right formation of the concepts, which are established preciously to mental-physical of the pupil and that didactical action is often inadequate superficial and it produces more instruction than a conceptualisation.
In The first phases of the research it was proposed to pupils an open text (you can see the text at the end) composed by four question.
It was used a strategy for avoiding that the pupil's answers were due to the experimental or teaching contract. In this way the teacher told to the pupils that they helped the teachers. It was a winning strategy because the pupils free themselves from every possible emotive tension caused by possible valuation. Participation was total and this was appeared to the teacher a victory independently of results of the research . In fact the schools involved in the project are a risk schools.
In the second phases of the research it was created an a-didactical situation called "gioco-percorso" (play-way) where pupils acted Tom and Jerry. The teachers realized the some ways indicated by the text and they asked pupils to determinate the position where Tom and Jerry had to stay for following Tom to take Jerry.
The pupils had therefore the possibility to experiment physically that the concept of perpendicular straight lines it doesn't connect to the plum line model but to the concept of mininum distance.
The "gioco-percorso" was organized in the following phases:

- Situation of action (the teacher explains the play and he simulates the play and the rules, the task was: *two of us are Tom and Jerry, Tom is waiting for Jerry slipping a little, Jerry is strolling quite. What is the position where Tom And Jerry are nearer. The other team components have to represent all the play in a sheet that is an help to understand what is the winning strategy. The team who finds the winning strategy wins the game. How can Tom understand when it is the right moment to take Jerry. Find the way to hallow Tom, in "any position", is Jerry to take him*); The different teams write the possible situation of the play in a sheet . The pupils try to found the solutions.
- Situation of formulation : teams socialize different strategies
- Situation of "validazione": teacher asks pupils to state their discoveries, that leads to the solution of the play. Some conjecture are necessary, and when all pupils agree to consider this conjecture the winner strategies, this conjecture became theorems. In this phases can be useful use of a thread.
- Situation of "istituzionalizzazione": this is the phases in which pupils enunciate a theorem, as solution or winner strategy share by all.

The question
In general, the children have been able to answer the question and to explain the cause of their choice. The test's analysis has allowed to suppose that children give the same meaning to the following concept:

- Minimum distance between two points
- Vertical line

The analysis of the cause of children's answer has allowed to suppose that children think that vertical line means "direction of gravity power"; this concept is suggested by everyday experience. Children know that if they leave an object from a height, it'll fall down toward the floor, therefore if Tom jumps when Jerry is under tom, he'll catch Jerry: children think that the right answer is the vertical position. This is proved by the following answers' cause written by pupils:

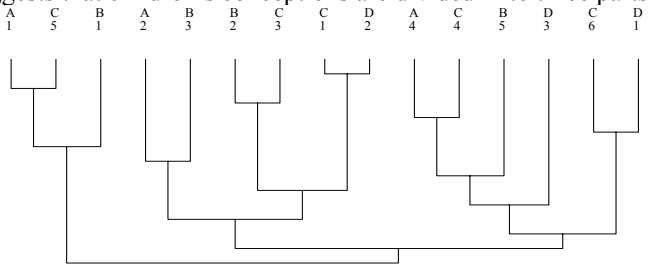
- *"Because the mouse walks and suddenly Tom jumps";*
- *"Because it is easier";*
- *"Because Tom was ready and in the right moment he has jumped"*
- *"Because he jumps and catches the mouse"*
- *"Because the cat is allinied with the mouse, therefore it is easier to catch him";*

Some children have understood that the vertical line wasn't always the right answer, and they have tought that Tom had to jump when jerry was in the nearest point: *"the cat waits that mouse is near to him"* . This is proved by the answers' causes written by pupils: they understand (these pupils haven't studied the perpendicolarity) the concept of minimum distance. You can read the following causes:

"Because it is near";
"because the cat waits the rigth moment";
"Because Tom waited that Jerry was next to him and cought the mouse";
"Because Tom wants to catch Jerry and when he is next to him he catchs the mouse";

Other pupils have tought that when the mouse is on the roof he is next to Tom, or they have found a hiding place for Jerry. A lot of pupils have partecipated to the test with their emotions, in fact they have supported the cat or the mouse. Achild has written that Tom catchs Jerry *"because he is hungry"*. Some children have tought that

Tom will catch Jerry if the mouse doesn't see the cat: to catch a prey it isn't necessary to minimize the distance, but it is necessary to be invisible.
Quantitative analysis of the test proposed in the first and the second classroom of primary school.
Data analysis was made by Chic software. The observation has been applied to 33 pupils. Even if few data we can try to find new ipotesys for future reserches.
After the a priori analysis we get toghether write and wrong conceptions.
The similarity graph suggests that children's conceptions are divided in to three parts:



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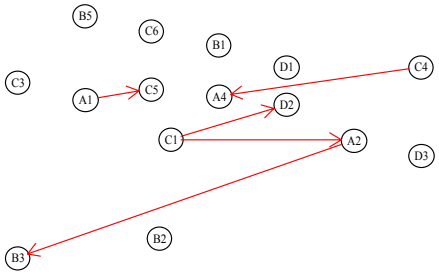
- ✓ A1, C5,B1
- ✓ A2, B3, B2, C3, C1, D2
- ✓ A4,C4,B5,D3,C6,D1.

The stronger similaritys are:

- ✓ A1, C5: child can sign the vertical line in exsercise 1 and understands the verticality in exsercise 3.
- ✓ A2, B3 are the wrong conceptions in exsercises 1 and 2; C1 and D2 are the write conceptions in exsercises 3 and 4.
- ✓ In the last similarity group we can observe that if child hasright intuitions but they aren't strong conception he will not be able to generalize to other contests.

If we analyze implicative graph we'll be able to say that :



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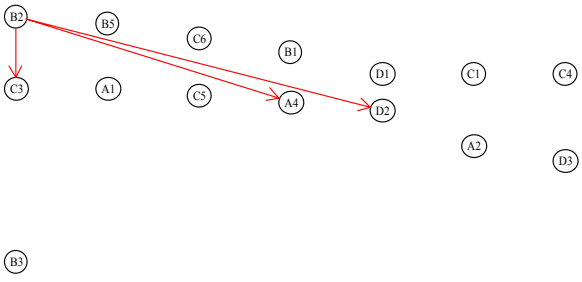
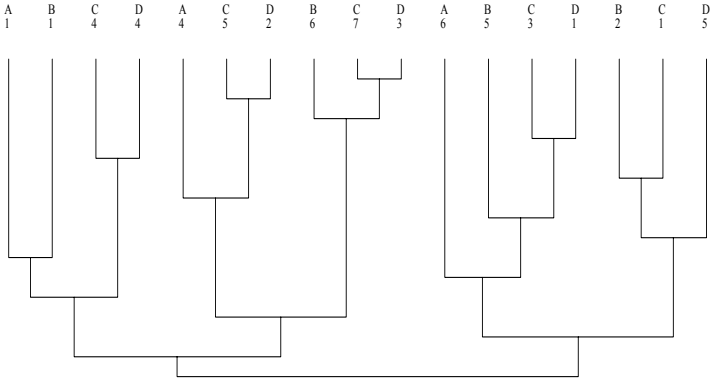


- ✓ If children sign the perpendicular line in exsercise 1, in other contests he'll be able to have right insight but he'll not always choice the right strategy (A1,C5)
- ✓ (C4,A4,D3) if child choices the wrong strategy in a contest he'll choice the wrong strategy in the other contests too.
- ✓ (C1,D2) if child sign the perpendicular line in exsercise 3, he'll be able to understand the right position that minimizes the distance between Tom and Jerry. If we consider the answers given by childre in exsercise 3 and 2, even if the questions are similar pupils give different answers (C3,B3). To explain this implication, we can suppose that the difference between the exsercises is the solution number: 1 in the first case, 2 in the second. According to didactic contract, child that doesn't understand this

difference, thinks that the exercise must be different: why did the teacher have to make two identical questions? Nobody has given answer C2 in exsercise 3.

Quantitative Analysis of the test proposed in the III, IV classroom of primary school and in the I classroom of "scuola media".



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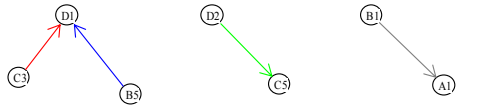
Data analisys was made by Chic software. The observation has been applied to 73 pupils. Because of the different social context, we were able to get together the data coming from the different classrooms.

The similarity analisys suggests that the pupils' conceptions are divided into 5 groups. The child:

- ✓ A1, B1: signs the perpendicular line, signs the vertical line
- ✓ A4, C5, D2: understands the perpendicular line, signs an inclined line from the cat to the inclined line, understands the perpendicularity
- ✓ B6,C7,D3: the child signs a line from the cat to the highest and nearest to the cat point
- ✓ C3, D1: signs the vertical line
- ✓ B2,C1: understands the verticality.

The implicative graph suggests that:



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- ✓ The children that sign the vertical line in exercise 3 (C3) and the children that sign a inclined line in exercise 3 (B5), sign the vertical line in exercise 4. They present the same verticality model in different exercises.
- ✓ The children that understand the perpendicularity in exercise 4 (D2), haven't the implicit model of verticality in exercise 3 (C5).
- ✓ If the child signs the vertical line in exercise 2 (B1), signs perpendicularity in exercise 1 (A1). This suggests that in the first exercise, the pupils sign the perpendicular line correctly because it coincides with the vertical line.

Qualitative analysis of the way-game

During the first step of the way-game, pupils have used a vertical line to sign the way covered by Tom to catch Jerry. The vertical implicit model was present.

During the second step the implicit model of minimum distance began to emerge.

In the third game's step, to help pupils to understand the concept of minimun distance we gave them a string, but we didn't explain how they had to use it. Pupils have used the string to measure the distance between Tom-child and Jerry-child: each children held a string's side, Jerry-child came along the way and both the pupils extended or abbreviated the string in according to the distance between them.

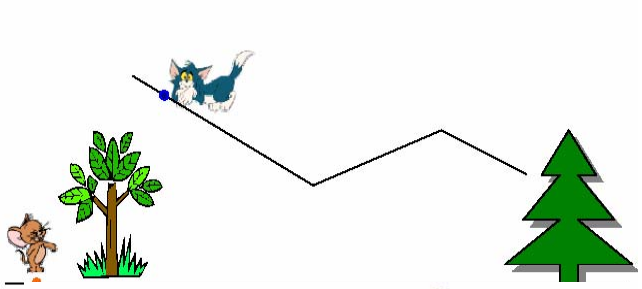
Pupils have noticed that there was a point of the way in which there was a minimun distance between Tom and Jerry.

During the "fase di validazione", pupils discussed each other and decided that the right solution was: "Tom catchs Jerry when the mouse is in the nearest point".

With some guide-question the teacher has introduced the concept of perpendicular line with the model of minimum distance.

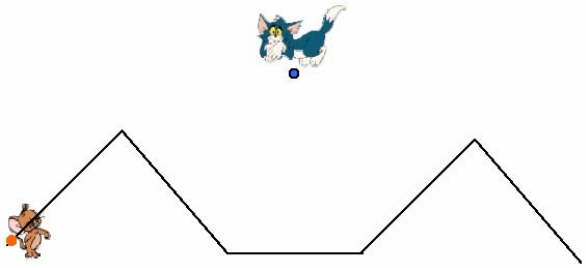
Text

- ✓ Tom (orange ball) is sleeping on the house roof when he see Jerry (blue ball) which is walking in the pavement. Where will Jerry have to stay when Tom jumps sure to take him, according to you.
  Cause your answers.

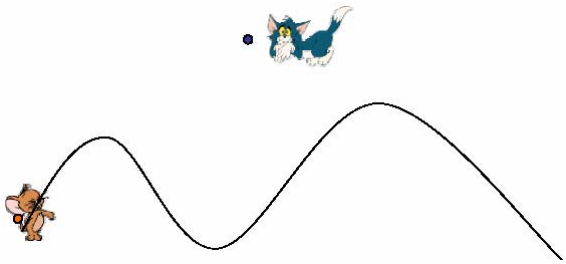


- ✓ Jerry (blue ball), is walking on the house roof , and Tom (orange ball) is waiting for him, whenTom jumps sure to take him?
  Cause your answers.

- ✓ The balls you see are Tom (blu) e Jerry (orange). Tom is waiting for Jerry, where Jerry will be when Tom jumps sure to take him.
  Cause your answers.



- ✓ Tom and Jerry are to the beach. Jerry runs in the beach and Tom relaxed himself in the lifeguard's chair. Where Jerry will be when Tom jumps sure to take him.
  Cause your answers.



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