Fuzzy Logic, Fuzzy Thinking and the teaching/learning of mathematics in multicultural situations

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The title of the present job is very ambitious in how much it tries to connect some distant matters among them and on which am working still.

I feel me honorable to be able to speak of this matter to the place native of Gödel. My relationship with the matters about logic goes up again to my university period when in the distant 1970 I faced during logic the reading of the "Gödel's proof".

The objective is that to try to understand as it is possible to be able to analyze and to study the phenomenons of teaching/learning of the mathematics in multicultural situations. The problem of the multiculturality is very diffused today and invests a big number of countries in the world.

We can consider this approach type ethnomathematic in the sense that U. Di Ambrosio from to this term. You will try to answer to the following questions:

1. that relationship exists among the Logical Fuzzy, the Thought Fuzzy and the common sense?
2. that role they have the natural languages in the construction of the mathematical languages?
3. that role they have the natural languages in the mediation in the processes of teaching/learning?
4. The a-didactic situations (G. Brousseau) they can be used for analyzing and to activate the processes of socialization of strategies and different trials in multicultural contexts ?

Naturally the project as he is introduced it is very ambitious. In this center they will be analyzed through some exemplifications theoretical and experimental, worked in the years, from the group in Palermo. A lot of job still stays to do and I will be thankful to you if you will signal me deficiencies and/or useful references for the pursuance of the job of search.

The intervention will follow the following run: Introduction to the logical fuzzy and the possible different approaches to it (paragraph 1), experimental considerations among logical fuzzy and common sense, theoretical-experimental considerations on the learning/teaching of the Chinese language and considerations historical-epistemological on the Arabic language, hypothesis of didactic proposals on the learning/teaching in situations of multiculturality.

1.0 Fuzzy Logic, Fuzzy thinking and linguistic approach

The Fuzzy logic has one history of his that is tightly tied to Lofti A. Zadeh[2] which initially introduces the whole fuzzy for then to pass about to matters the fuzzy logic[3].

The interesting problem is that the official science has welcomed with a lot of separation the ideas of Zadeh on the Fuzzy Logic. The problem is in first appeal of cultural nature. From the scientific revolution actually to halves 900 the reference on the processes of reasoning and demonstration within the scientific knowledges is almost always past through the ambivalent logic, Aristotle to intend us. This for different motives:

- Though the ambivalent logic they are had previously always some sure and controllable results. The tool of the ambivalent logic is some rough but sure.
- The structures some natural languages that have supported this cultural operation he is tightly tied to the Aristotelian logic (English, French, Italian, German, Spanish, the neo-Latin languages for instance).

The Fuzzy Logic has been lived from the westerners as a probabilistic logic, endless values of truth among the value of void probability 0 and the value of probability 1 (certain event).

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[2] Lotfi A. Zadeh was born on February 4, 1921, in Baku, Russia. He attended the American College in Teheran, Iran, and was awarded the B.S. degree in Electrical Engineering by the University of Teheran in 1942. He came to the United States in 1944 and entered Massachusetts Institute of Technology where he received the M.S. degree in 1946. Until 1965, Dr. Zadeh's work had been centered on system theory and decision analysis. Since then, his research interests have shifted to the theory of fuzzy sets and its applications to artificial intelligence, linguistics, logic, decision analysis, control theory, expert systems and neural networks. Currently, his research is focused on fuzzy logic, soft computing, computing with words, and the newly developed computational theory of perceptions and precisiated natural language. Il suo libro "Fuzzy Sets" é del 1965.

[3] Bibliographical references of L.A.Zadeh and some jobs can be unloaded by the following site web: http://www.cs.berkeley.edu/People/Faculty/Homepages/zadeh.html .

This vision is once more tied up to the ambivalent logic and that is when an event for instance has a certain value of probability 0.35, this he is read as the event that can have this value of probability (true) or not to have him/it (false).

1.2 Fuzzy sets and their representations..

The being Fuzzy represents the due vagueness to the human intuition, not the probability (Zadeh). You probability has to whether to do with to verify him some events, and when we have all the data a certain event or he is occurred or no.

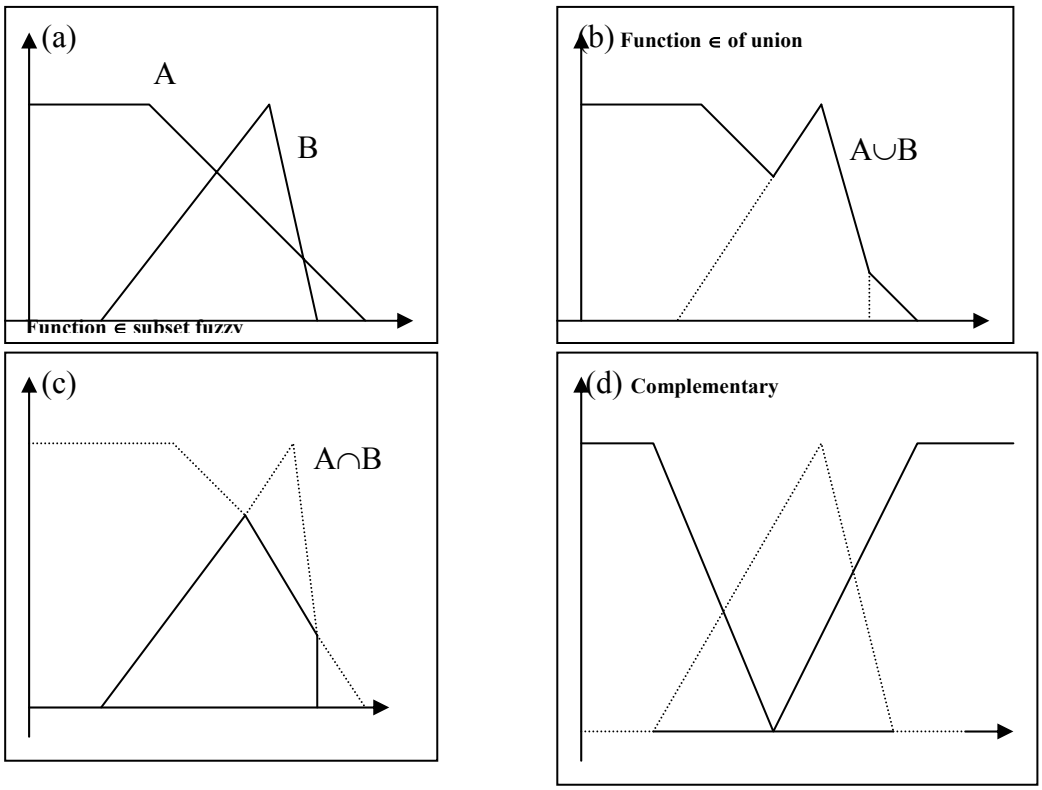
Certain whole fuzzy, those constituted by numbers, can especially be visualized drawing graphic. In mathematical terms, these are simply the graphs of the functions of affiliation y=A(x) in a system of Cartesian coordinates. These graphs are frequently of triangular form, trapezoidal and, occasionally, to bell or of some other form.

Among the definition of a whole fuzzy and his representation A(x) there is difference. As a line of well defined poverty doesn't exist that you separate the poor men from the others, so she doesn't exist "true" function of affiliation of the whole the poor people, or of anybody else together fuzzy. This doesn't mean but the choice of the degree of affiliation is completely arbitrary: in the choice considerations both theorists and empiriche as the context and the particular situation intervene of which he is looking for to give a model.

Operazioni con gli insiemi fuzzy:

- Inclusion $A \subseteq B$ "If all the elements of A are also elements of B"
- Degree of belonging:
- $(A \cup B)(x)=\max\{A(x),B(x)\}$
- $(A \cap B)(x)=\min\{A(x),B(x)\}$
- $\bar{A}(x)=1-A(x)$ (If x is an A, $\bar{A}(x)=0$; if x it is not in A, then $\bar{A}(x)=1$).

Some graphic exemplifications:



The fuzzy logic is a based mathematical method on the theory of the whole fuzzy that helps the cars to reason in more way similar to the that human.

The fuzzy logic usually realizes him through an algorithm or a program, on a conventional digital calculator, and, as such it is exact. But the method also has a subjective component (therefore essentially empirical and inaccurate) because it implies the translation in numerical form of the vagueness of the language and the human knowledge.

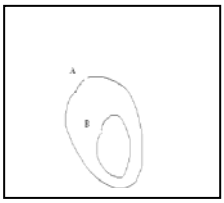
The fuzzy rules.

The fuzzy rules can express him with the natural language in the following way: if x is small and y is middle, then z is great.

The variables x, y and z are type linguistic. With the whole fuzzy these words can be translated in numerical relationships, and to perform calculations. "A method to calculate with the words."

Rules of inference:

If x is A, and y it is then B, z it is C *(Where: A, B and C are words that point out subsets fuzzy).*
*This rule of inference would seem to be analogous to the rule****: If x is A and y it is then B z it is C***
Premises: x is A', y is B', Conclusion: Z is C'.
The relationship fuzzy-probability according to the point of view.
1. from the point of view of Aristotle the logical fuzzy is seen with the tools of the probability. The measure than a subset B is contained in A (B/A).



The "subsethood", as Kosko calls it, it concerns the degree of affiliation of an element x to a determined together fuzzy. We can interpret this " subsethood " as:
1. measure than an element x belongs to a determined together fuzzy.
2. the degree of truth of the affirmation "x belongs to the whole A.", for instance. In this case the value of truth can be a whatever value understood among 0 and 1[4].

What we see is an event with the maximum probability.

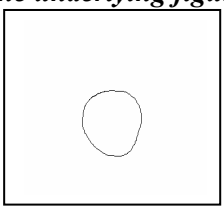
The probability as the free will, shape him as a collateral psychological effect regarding creatures "provident", or rather predisposed to expect a certain sequence of events. It helps us to organize our perceptions, the memories and the most greater part of ours attended. The probability from an ordinary structure to the causal forecasts, between them conflicting, on as it will evolve the future in the next instant, day, season or millennium. It assigns a place and a weight to our future.

The ability of forecast is an important element for the biological and cultural survival.

More knowledge and information I lead probability.

The probability as cultural constant that it makes to think about a biological substratum and this implies an evolutionary history.

The underlying figure is a circle fuzzy, in a certain measure a circle, in a certain measure an I don't hoop:



More knowledge and more information it emerges the nature fuzzy of the things.

Elementhood:

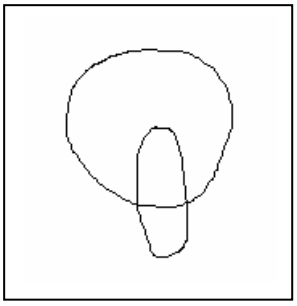
A whole that contains his/her elements in a certain measure. That is when it partially contains the element. (A person is happy about his job in a certain measure, hardly totally happy or totally dissatisfied)

Subsethood:

A set that contains another set in a certain measure.

(The index of implication of Régis Gras works on this aspect through the normal distribution and the probability)

[4] We suppose to want to train a car to recognize a signature. They are initially recorded the signature, the speed (or better the components horizontal and vertical of the vector speed in partnership to the point of the pen). In the moment in which is wanted to reproduce the same signature it is had to do it to the same speed. Insofar it will be impossible, also for the person that the first time has signed, to reproduce its same signature with a value of equal truth to 1. we can consider valid for instance values 0.95 or greater.



- The whole one contains the part. From the point of view of Aristotle and the whole one. Probability m/n. In the case of the figure we have the conditional probability. $\frac{|A \cap B|}{X}$
- The part contains the whole one in a certain measure. From the point of view fuzzy and of the part.

We can consider three approaches, according to Zadeh, to the fuzzy logic:

1. CTP Compuational Theory of Perception
2. CW Computing with Words
3. GCL Generalized Constraint Language

We consider the generic relationship now (implicative): X isr R
(X is the forced variable, R is the relationship that he forces, isr is the copula variable, where r is the variable whose value defines the road for which R forces X).

We can have different types of constraints that allow us therefore to analyze a vast range of approaches to the fuzzy logic:

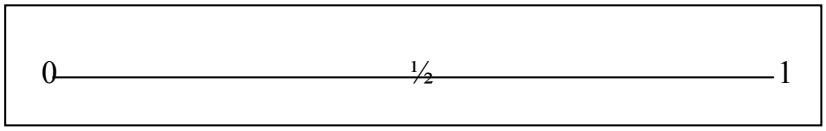
- Possibilistic;
- Veristic
- Probabilistic
- Random set
- Pawlak set
- Fuzzy graphic
- usuality

GCL is very more expressive than the language of the logic of the predicates. In the Computing Word, the initial and terminal whole, IDS (Initial Data Set) and TDS (Terminal Data Set) they are express in the natural language. The model for CW and TDS is the human mind. The approach CW represents therefore the approach more fuzzy next to that representable from the man.

1.3 The representative point of view of Kosko

The language, the mathematical language of the science, especially creates artificial demarcations between white and black, while the reason or the good sense fall through them: the reason works in light and shade.

Bivalence	***Polivalence***
Aristotle	Budda, Yin/Yang
A or not A	A and not A
Exact	Partial
All or nothing	In a certain extent
0 or 1	Continuity beetwen 0 and 1
Digital elaboration	Neural network (brain)
Fortran	Naturale Language Italian, English, …
bits	Fuzzy units: Fits



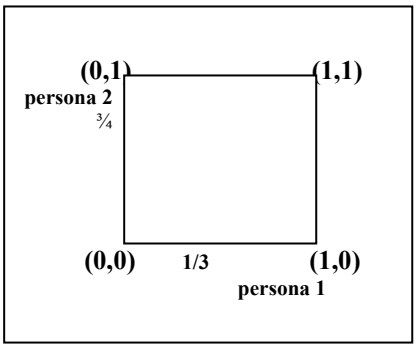
The middle point of the line is a "paradox" for the Aristotelian logic. For the logical fuzzy it is the point where of the half truths are and where A. is equal to not A. (equation yin-yang).
In the ambivalent logic the paradox is neither truth nor forgery, it doesn't have sense inside the language taken in examination and of the context.

We consider the following concrete situation:
An audience: two people.
Question: Are you happy about your job?

You lift the hand to answer to this question.

The hand won't always have lifted equally, entirely raising, partially, etc…

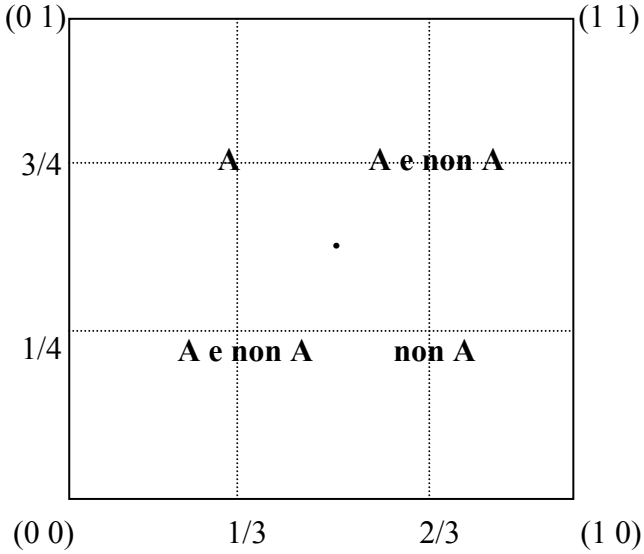
The following scheme analyzes the possible situation with the presence of two only people, the model he can certainly widen for n people. To every vertex of the square it corresponds a situation limit: (0 1) only the person 1 lift the hand, (1 1) they lift the both hands, etc…



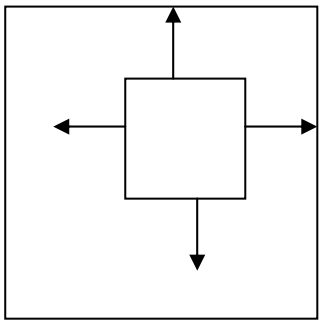
Person 1: raises the hand to 33,3 % (1/3)
Person 2: raises the hand to 75 % (3/4) (1/3, ¾) A
The answer of the opposite audience is (2/3, ¼) not A.

A and not A is equal to (1/3,¼) and not is (0, 0) that is the empty set. The first position is that of the thought fuzzy (yin and yang), the other one is the Aristotelian position.

We represent this in another scheme:



If the answers are less fuzzy, the smallest square it widens toward the angles not fuzzy.
And in this case we have Aristotle.



If instead the answers become more fuzzy then the inside square tightens him toward the middle point. In the case limit the square collapses in the middle point (all the people lift the hand to 50%). Then A. and not A. (yin yang) to 100%. A. = A. or not A. = A and not A. = not A. (it doesn't distinguish him the full half glass and an empty half)

Aristotle dominates in the contour, Budda in the center.

The middle point is the black hole of the theory of the whole.

The idea of the type of representation on exposed it is of B. Kosko that it consists of representing every under together fuzzy of X through a point in a system of Cartesian coordinates.

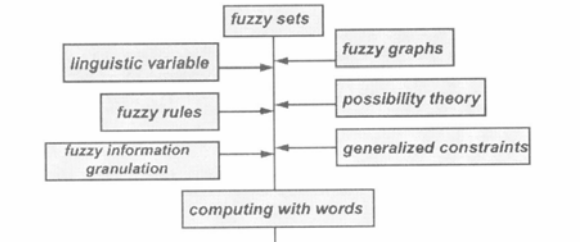
1.4 Some epistemological reflections on the approaches to the fuzzy logic.

L.A. Zadeh recently in a conference on the Artificial intelligence to Palermo[5] he has redefined some all the possible approaches to the Logical Fuzzy. We can synthesize this way the possible approaches to the Logical Fuzzy:

Approach with Words: Linguistic approach	Possibilistic Approach	Veristic Approach	Probabilistic Approach	Random set Approach	Pawlak set Approach	Approach with fuzzy graph
Computing with words (CW). Computational theory of perceptions (CTP). Perceptions are expressed as propositions in a natural language. CW-based techniques are employed to translate propositions expressed in a natural language into what is called the Generalized Constraint Language (GCL).	CW-Based possibilistic	CW-Based veristic	CW-Based probabilistic	CW-Based Random set	CW-Based Pawlak set	CW-Based with fuzzy graph
	X is A Partial knowledge	X isv V Partial truth	X isp P Partial certainty			

The role model for CW and CTP is the human mind.

Computational theory of perceptions (CTP).



The linguistic approach to the fuzzy logic seems, for the time being, one of the best approaches both for the resolutions of problems of the artificial intelligence (simulation of the human behaviors) that for the resolution of problems in the field of the technology.

2.0 Fuzzy thinking and Common Sense: analysis of a experience.

Pupils, during the proposed activity, did not use their mathematical and computer science knowledge and freely used natural language to express their models, in this way they ended up using actual fuzzy rules to describe the control instructions for some expected actions. The lack of sequential schemes has also encouraged fuzzy thinking and creativity. This, in a nutshell, was the main result of the experience. But it is necessary to comment further this experience in order to reveal its own specific characteristics:

- The way the activity was carried out can be easily lead back to the so-called "a-didactic" Brousoau's situations rather than to any conventional didactical approach (drills and class tests and so on). The pupil can talk with the class mates and can express his own opinion as well as face the proposed problem using the approach he thinks it is the best. He knows that will not be evaluated the

[5] Lotfi A. Zadeh, From computing with numbers to computing with words – from manipulation of measurements to manipulation of perceptions, Human and Machine Peception, Vol. III, Edited by V. Cantoni, V. Di Gesù, A. Setti & D. Tegolo, Palermo, Italy, 2000.

process and the method he uses to get the solution, but what really matters is a convenient solution to the problem is found.

- There are no objectives referring to "logic capabilities", and everyone knows the programmatic choices that were made. It was created an atmosphere in which everyone had the feeling of being free to use whatever kind of thinking he liked, linear or parallel.
- In this work, speaking with the class about the fundamental choices, it was preferred to use terms such as "different mathematics" rather than simply "mathematics" also it was abandoned the idea of knowing the "reality" in favour of knowing the "realities".
- And finally, uncertainty, indetermination, incompleteness are terms that get to be familiar to pupils since their first year of school, facing, for example, games and problems which have no solutions or more than one, or situations that require a probabilistic approach, or open tests and so on, all situations that can easily happen during laboratory activities that are integrated in the ordinary scholastic work.

LINEAR THINKING	*PARALLEL THINKING*
• It is an intentional process • It is a mental attitude • It can be learned, put in practice, used	
• It is selective • It chooses a path excluding the others • It selects (or it searches for?) the best point of view	• It is productive • It does not select a path but tries to open new ones instead • It produces alternative approaches within the feasible solution space.
• It activates only if there is a clear direction indicating where to go • It is analytical • It is consequential • When it moves step by step, each step must be clearly justified	• It activates in order to produce a direction where to go • It is stimulating • It can jump from a place to another • It can move freely without taking into consideration any contradictions, provided that the final conclusions are right
• It uses negation in order to prevent from following some paths • It explore the most probable paths • It is expected to find an answer	• It does not use negation, thus it is possible to follow a wrong path • It explores the least probable paths • There could be an answer, but there could be many others too, besides there could be partial solutions.

3.0 The example of the Chinese language and the dubiousness of the relationship Natural Language and Mathematics.

In the cultural integration the relationship between mathematics languages and natural languages has a very important role.

In mathematics education there is an important problem about the relationship between Epistemology, History and communication of mathematics.

If the interpretation of phenomena learning/understanding utilises the semiotic approach of mathematics we have the instruments to analyse verbal messages and non-verbal messages.

A classification of semiotic interpretations of history of mathematics as 1) history of syntax of mathematics languages, 2) history of semantic of mathematics languages, 3) history of pragmatic languages, give the instruments of interpretations a priori of understanding/learning phenomena.

The representations historic epistemological are the possible routes to knowledge of pupils (a priori and a posterior)

In this prospective the history of mathematics is of service in research in Mathematics education: the researcher has special requests for the historian of mathematics.

In paper (Spagnolo, 2002) we present an experience of preparation of a Chinese man in the examination in the last class of college. We compare structure of Chinese language with algebra. The history of mathematics is a basic key to interpret the didactic problem.

History of Mathematics in China: Algebraical Thought	**Chinese Language**	**History of Mathematics in Western: Geometrical Thought**	**Natural Languages in Western**
• Chu Chang Suan Shu:	≈1000 after C.	Euclid: Axiomatic	The Aristotle's

Roots, false position, solution of particular systems. 300 before C.- 200 B.C. • Chao Chun Chin, Commentary to Chou Pei: 200-300 After C. • Wang Hsiao Tung, Equations of 3° degree. 625 a.C. • Chhin Hiu-Shao, Numerical solutions of equations of degree>2. 1247 b.C. • Ricci and Hsu Kuang-Chhi, Tradution of Euclide Geometry. 1607. Lorfi Zadeh: Fuzzy sets (1965)	Chinese language like is in present. In the Chinese hieroglyphic language the 80% of hieroglyphics[6] are for association.  This hieroglyphic MAN is composed in two parts STRENGTH and FIELD (a mental equation: says Needham)	geometry 300 before C. • Geometry as first interpretation of physical world; • Geometry as science of to argue. Fibonacci 1200 introduce the algebra in western "Liber abaci". • Geometry as hypothetical - deductive system (Hilbert, Grunlagen der Geometrie, 1899)	Logic like a model of western languages. In particular the Euclidean Geometry like a model of Aristotle's Logic.

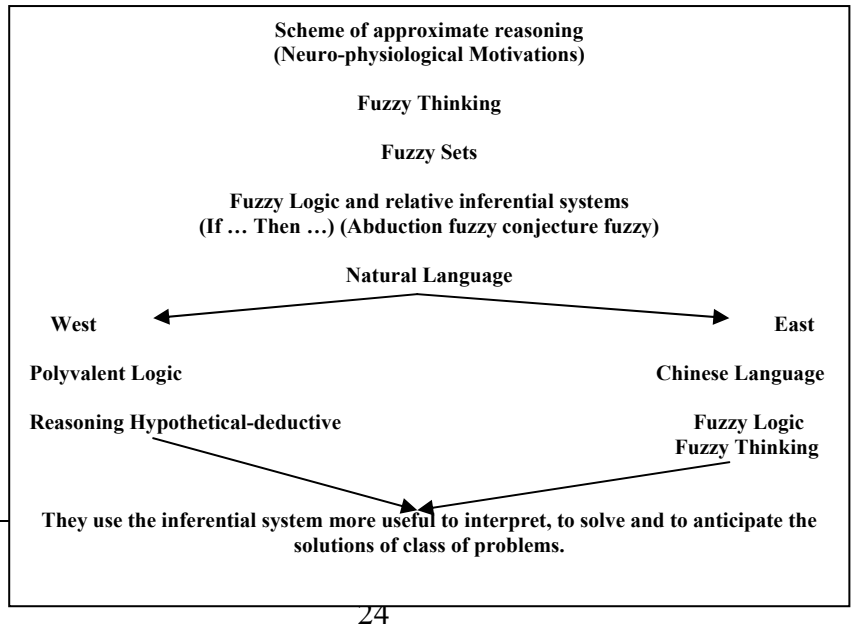
We can certainly affirm that in the case of Tong the algebraic structure of his natural language (Chinese) he has allowed him to render explicit entirely processes of resolution of problems in different way from those generally developed in the western culture. Its final examination has been, in mathematics, excellent.

The study of the case of "Tong" he has put above all in evidence the necessity to have to use all the possible cultural tools to our disposition: the inside and external history of the mathematics, the knowledge of the structure of the natural language, the logics references different and to different inferential systems.

All of this puts in evidence a series of open problems that they can be counterfeit is from the experimental point of view that theoretical:

1. actually to that point the knowledge of the natural language can engrave on the schemes of reasoning?
2. which the contributions of the natural language and which those of the social culture (beliefs, etc…) underlying?
3. in a multicultural situations of teaching/learning of the mathematics would be gotten the same results?
4. the study of the inferential systems produced in environment Fuzzy could give a theoretical contribution to this type of searches?

The following scheme tries to schematize the relationships among fuzzy logic – natural languages and argumentative-inferential schemes. He wants to be a problematic matter of opening to the discussion and not a result.



[6] See Needhman.

3.1 The example of the Arabic language.

The relationship between natural language and mathematical languages has two meaningful examples in the Arabic language:

1. the analysis of a magrebin manuscript of a manuscript of Jerba of Mahdi Abdeljaouad;
2. relationship between combinatorics and structure of the Arabic language.

They will be delineated therefore the traces of the job without going down in the particular ones and postponing to the literature for the close examinations.

3.1.1 The processes of symbolism in the Arabic language through the historical and epistemological analysis.

Through the analysis of an Arabic manuscript Mahdi Abdeljaouad (2003) it has well put in evidence in analytical way the relationship among process of symbolism in algebra and Arabic language.

Particularly it puts in evidence three periods of the symbolic magrebin writing:

1. *La première correspond à un usage intensif de la lawha pour tous les calculs initiaux et intermédiaires : Le recours aux initiales de l'inconnue (Shay) et de ses puissances pour les représenter sur la Lawha est une nécessité graphique, ayant pour but d'abréger l'expression et de la synthétiser. Les symboles de base (chiffres ghubar et signes associés aux inconnues) sont d'abord regroupés sur la planche à calcul en un assemblage correspondant à une expression arithmétique ou algébrique. Le calculateur transforme alors cette expression en utilisant l'une des opérations (addition, soustraction, multiplication, division, mise au même dénominateur, simplification, extraction de la racine carrée, ...) et travaille directement sur la planche à calcul. Une fois les calculs terminés, il recopie le résultat tel quel sur la parchemin en le précédant de l'expression wa suratuhu (c'est-à-dire "son image"), image exacte de ce qui est inscrit sur la planche à calculer. Nous avons signalé cette manière de procéder chez ibn al-Yāsamin et paradoxalement, dans la première oeuvre d'al-Qalasādi. Dans cette étape, peu d'assembleurs sont nécessaires; en fait, les seuls utilisés sont les assembleurs à une place, ش- , م- , ك- et ج- , initiales des inconnues (chose, carré et cube) et du jidhr (le radical) et subsidiairement le signe de la négation, ceux-là mêmes que l'on avait à écrire sur la lawha pour identifier l'inconnue. Nous avons vu que la possibilité de se tromper, d'effacer des résultats intermédiaires et de remplacer un coefficient par un autre est signalée de temps en temps.*
2. *La deuxième étape correspond à l'apparition, dans les textes, des assembleurs à deux places pour séparer les nombres ou les expressions polynomiales, l'identification de chaque opération par un signe univoque et l'utilisation du signe de l'égalité. Nous retrouvons ce type d'assemblages dans les textes écrits à partir du XIV^{ème} siècle. Il s'agit d'une symbolique syncopée à caractère d'illustration, toujours introduite par l'expression : wa suratuhu (c'est-à-dire "son image"); son rôle essentiel est d'éclairer le texte et de le rendre accessible du premier coup d'œil et elle peut éventuellement être ignorée car elle n'est pas indispensable à la compréhension du texte rhétorique. Ce sont donc des symboles d'illustration et non de substitution.*
3. *La troisième étape correspond à la production de textes symboliques totalement autonomes et dynamiques, qui, pour leur lecture ne nécessitent plus de recours au texte rhétorique. Ces textes symboliques utilisent un système clos de symboles, constitué des chiffres ghubar et de quelques lettres arabes, et soumis à une syntaxe bien établie :*
   - *un assemblage par ligne représentant la mise en équation du problème, une expression algébrique, une équation, un calcul intermédiaire, l'algorithme de résolution d'une équation canonique ou le résultat d'un calcul,*
   - *une hiérarchie des assembleurs,*
   - *des vincula délimitants pour les inconnues, les radicaux et la division,*
   - *des séparateurs entre deux assemblages successifs,*
   - *la lecture du haut vers le bas*

(pagg. 60-61)

3.1.2 Combinatorics and arabic language.

Roshdi Rashed (2002), historical of the mathematics of the Islam, it sustains that the combinatorics is used in the phase of setting of the Arabic language. In the history of the Arabic language the problem to be able to establish a bijective correspondence between the whole the words and the voices of the dictionary has been the stimulus to use the combinatorics. Among the X century and the XIII century ends an essential action for the history of the combinatorics: the explicit

recognition that the rules applied by the linguists are the same established by the algebristis and that they can be applied in different situations as the arithmetic, the philosophy etc…

Everything this brings us to reflect on the relationships between natural languages and constructions of mathematical languages with the possible didactical implications.

3.2 Do we have the possibility to recover the processes of learning/teaching in the difference?

The interpretation of the phenomena of learning/teaching they are very complicated to be analyzed. The situation is complicated when the trial to be analyzed is in a multicultural situation.

The interpretative tools have to keep in mind:

· of the history of the Mathematics;

· of the reflections on the structure of the natural language;

· of the Ethnomathematics as I break down theoretical interpretative;

· of the reflection on the interpretation semiotics of the mathematics is in the epistemological aspect, both in the interpretative aspect of the phenomena of learning/teaching.

For Ethnomathematics, according to the interpretation of D'Ambrosio, we intend etymologicaly: ethno (cultural context in the amplest sense, language, slang, codes of behavior, myths and symbols), mathema (to explain, to know, to understand), tics (reliquaries, art and technique). "Ethnomathematics is the art or the technique of explain, to know, to understand in the different cultural contexts."

The point of view that seems to offer quite a lot opportunities for the interpretation of the phenomena of learning/teaching could be that semiotic. The choice that is to interpret with semiotic the mathematical languages through a parsing, semantics, pragmatic (Spagnolo, 2001, 2002c). This choice is not the only possible but it is that that allows us to be able it describes better the models of the phenomena of learning/teaching. Such models they must to be able to analyze the different disciplines that compete to an interpretation of the phenomena.

The paradigm of the search in didactics is that of the "Theory of the Situations" of Guy Brousseau (1997), revisited by the group of search in Palermo (Spagnolo, 1998, 2003a) for an insertion in the tradition of the "didactics of the mathematics" from the end of the '800 to today. Such paradigm is absolutely an open model and in evolution that keeps in mind both of the matters concerning the epistemology and the history in an analysis a-priori of a didactic situation, both of the behaviors attended from the student.

3.3 A possible tool for the processes of socialization of the cultural differences and linguistics: the a-didactic situations.

In the processes of learning/teaching in multicultural situations a very important role is played by the fact that it has to exist a moment in which there is moments of socialization of the decisive strategies of a situation/problem in the class.

The a-didactic situations (Brousseau, 1997) (Spagnolo, 1998, 2003a) they can represent one of the possible tools. I shortly remember their structure. Three phases are generally pointed out:

- Phase of action, corresponds to the mathematics in the reality. And' the moment to put on in a concrete situation of action with the problem list of the situation/problem;
- Phase of formulation, corresponds during the debugging of decisive strategies of the situation/problem through the debugging of a code of effective communication;
- Phase of validation, corresponds during socialization of the decisive strategies. Such moment will put in evidence the optimal strategies with a collegial share of the class. And' the moment of the destination-knowledge in the mathematical activity in which different ways of think are compared, different cognitive styles.

And' in progress an experimental job that keeps in mind of this model and that it will try to underline the positive aspects, negative and the possible limits. Experiences are conducted in elementary schools and averages of the city of Palermo and in presence of different linguistic-cultural origins: Chinese, Arabs, Sicilians, etc…

I hope to be able to bring the results of these experiences in the next conferences of our international group.

3.4 ***Open problems and future Perspectives.***

What the problems open of search.

Surely many and all legacies to the difficulty to be able to interpret the phenomena of learning/teaching.

But we can already individualize some concerning of it:

1. The theoretical reflections can concern:
   (i) A study more deepened of the structure of the natural languages (also with the help of the linguists);
   (ii) A close examination on the use of the epistemology and the history of the mathematics for the interpretation of the phenomena of learning/teaching;
   (iii) A study on different tools of interpretation of these so complex phenomena is from the linguistic-communicative point of view (semiotics) how logical-linguistic (fuzzy logic);
   (iv) A study on the relationship of the neuro-physiology searches and the mathematics.
2. The application reflections can concern:
   (i) The study of the particularly functional situation/problem to the problem list of the multicultural. In the sense that can put in evidence the most greater number of decisive strategies, schemes of reasoning, trials…;
   (ii) The qualitative and quantitative study of the processes of socialization of the runs individual cognitive of the students (analysis of the phase of validation of the a-didactic situations);
   (iii) The study of the didactic innovations like source and stimulus for the creation of opportune didactic situations for the multicultural.

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