Growth of Mathematical Understanding Through Pattern Finding Viewed through Coactions and Conversions

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Abstract
This report investigates two students working cooperatively on patterning activities and the learning that occurs. The collective mathematical understanding is viewed through the framework of Martin, Towers and Pirie (2006) with the use of coactional processes. Elements of Duval's (2000) framework are used to discuss mathematical understanding abstracted from the coactional processes.

Introduction

Classroom culture is defined by the combination of teachers and pupils, their knowledge, beliefs and the social interactions that result from those combinations. "If we are to gain anything from the study of the culture of the mathematics classrooms, it will come from an understanding of the factors that contribute to their productivity…" (Nickson, 1992, p 111). Comments such as "I just got that" and "right, right" are associated with instances of student productivity and we attempt to understand this productivity in terms of growth in mathematical understanding and social interaction. Our question is what mathematical understanding is associated with this productivity and what factors contribute to the mathematical understanding?

Frameworks

Duval (2000) presents a framework that can be used to explain student's mathematical thinking. Duval states that mathematical understanding requires the coordination between at least two registers of which one is multifunctional and the other monofunctional (p 66). In addition, mathematical understanding is achieved as students learn to discriminate and to coordinate between semiotic systems of representation. The semiotic systems of representation, or registers, refer to four classifications of systems of representation:

- Multifunctional – open to multiple interpretations
  - Verbal register - utilizes natural language, verbal associations, reasoning and non-algorithmic processes.
  - Geometric register - is defined by its use of geometrical figures as shape or configurations, actions on those shapes and non-algorithmic processing.
- Monofunctional – one interpretation
  - Symbolic Register - includes numeral systems, symbolic or algebraic notations, formal languages and algorithmic processing as characterizing properties.
  - Graphical Register - characterized by the use of Cartesian graphs, changes in coordinate systems, extrapolation and interpolation.

Changing representation within a register is a treatment and changing representation between registers is a conversion. A coordinated conversion is a bidirectional change between two registers. Treatments and conversions provide a picture of students as they strive to achieve mathematical understanding.

The discrimination associated with coordinating between registers is highlighted through the improvisational framework of Martin, Towers and Pirie (2006). This framework provides a lens for viewing social interactions particularly those social interactions which lead to collective understanding. "…collective mathematical understanding emerges through the ways in which the identifiably diverse and different understanding of individuals combine and coact to enable growth that is not simply located in the actions of any one individual but in the collective engagement with the task posed" (p 157). This improvisational view of group or collective understanding can be employed to identify productive social interactions, coactions. Coactions are meaningful and contribute to productivity in light of, and with reference to, the actions of others within a group, i.e. the actions of one student influence the actions of another.

Methods

For this study, a middle school classroom was selected to participate in a four-cycle teaching experiment that took place over the course of an academic year. The participants in this study attended a small, urban middle school located in the southeastern region of the United States. The participants were eighth grade students between the ages of 13 and 14 and were enrolled in an eighth grade math class. The class was diverse in terms of race and gender. The research team was comprised of seven doctoral students in mathematics education and a senior faculty member, all from a large university in the southeastern United States. Two members of the research team were responsible for classroom instruction and others were responsible for operating video equipment and taking field notes. Data collected in the teaching experiment included student artifacts, videotapes of classroom teaching episodes, and field notes.

This investigation focuses on two students, Barbara (B) and Carl (C), as they participated in a patterning activity. Using videotapes and student artifacts, transcriptions containing discourse and actions were produced for this activity. Each item of discourse was numbered and coded for representation: manipulative, drawing, number, table or rule. To analyze the data, each item number was placed by its representation within the appropriate register (see figure 2). The manipulative and drawing representations are within the geometric register whereas the number, table, and rule representations are within the symbolic register.

Results

This activity occurred on day two of cycle two of the teaching experiment and consisted of Barbara and Carl working on a patterning task in which they are asked to develop the relationship between the number of triangles in a triangle train and the perimeter of the train (see Figure 1).

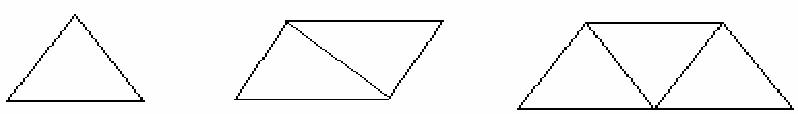

Figure 1 – Triangle Pattern Train

The transcription of the triangle train patterning activity produced 172 items. The tables below (Table 1 and Table 2) represent 25 of these items surrounding Carl's comment "…right, right…" Each item was coded for representation and evidenced by student discourse and actions.

Students discourse and actions leading to an instance of productive mathematical understanding

Displayed in Table 1 is Barbara and Carl's discussion on the development of the relationship between the number of triangles in a train and the perimeter of the train in the form of a rule. Prior to this excerpt, Barbara and Carl investigated the relationship between the number of triangles and the triangle train perimeters with less than 12 triangles.

In items 1-3 (Table 1), Barbara and Carl engaged in the same conversion: from drawings to number (i.e., "twelve is fourteen"). Once a significant quantity of numbers had been generated, Barbara (item 4) recognized that one (column) was "two bigger" than the other. This was the first discourse associated with the rule representation. Carl responded "so for every triangle, the perimeter is" and Barbara completed the sentence with the rule "plus 2". In addition, Barbara verified this rule by reading entries from the table (see item 6). Barbara continued to vacillate between the table and rule while Carl sought verification of the rule by testing larger numbers. Barbara indicated "that's how it worked in the table". Barbara's statement did not refer to table values; the numbers 100 and 102 were not in the table. Carl asked for repeated verification of numbers, and Barbara verified that each one was correct by referring back to the rule "cause its two bigger" until Carl grins and says "…right, right…".

Item	Student	Representations	Discourse and Actions
1	C	Drawing Numbers	(*counting from drawing*) one…twelve, thirteen fourteen, twelve [triangles] is fourteen [perimeter]
2	B	Drawing Numbers	Is nine [triangles] eleven [perimeter], I'm gonna check…(*drawing and counting*)
3	C	Drawing Numbers	(*counting*) …eight…yea that is… so if we had a rule to where it would be like, it would be
4	B	Table Rule	This one [column] is two bigger than that one…
5	C	Rule	So for every triangle the perimeter is…
6	B	Rule Table	Plus 2. its like, there are six triangles and the perimeter is eight. If there's one triangle then the perimeter is three, 1 plus 2 is 3.
7	C		So…
8	B	Table Rule	This row, everything is 2 more than this row
9	C	Rule Numbers	So are you saying 100 would be 102
10	B	Table Rule	I guess cause that's how it worked in the table…
11	C	Table Numbers	Going by our table 100 would have to be 102 and 102 would have to equal 104
12	B	Rule	Yeah, cause its two bigger
13	C		Yeah so if we – right, right – so if we figure it out – I really do not want to draw 100 triangles

Table 1. Discourse and actions of Barbara and Carl and their representations

Barbara and Carl began this activity in the geometric register and used drawings to represent the triangle pattern trains. Their counting led them to ordered pairs of values that were represented by numbers in the symbolic register. Change in representation between two registers produced a conversion. In item 8 Barbara changed from a table representation to a rule representation and produced a treatment, a change in representation within a register.

Students discourse and actions leading to a coordinated conversion

In the course of the conversation between Barbara and Carl, one of the teachers (T) enters and begins to ask questions regarding their findings. The discussion between the students and teacher is displayed in Table 2.

In item 14 (Table 2), the teacher asked the students to explain the change between the rows of the table, in particular the perimeter column. Barbara responded "it always goes up by one." The teacher asked Barbara to clarify her statement. Barbara (item 17) related the plus one rule to adding one side of the triangular manipulative. When asked by the teacher to identify where the plus one side, Barbara pointed to two sides. Carl recognized this discrepancy and held up two fingers to denote the problem. Carl moved the manipulative that Barbara had pointed to away from the others and as he moved it back, he explained how placing a manipulative (triangle) beside another produced a net effect of plus one side. Carl experienced difficulty finding a word to describe the phenomenon; he suggested the word "negate" but was unsatisfied with this term. Barbara attempted a summation which involved the words "covers up".

Item	Student	Representations	Discourse and Actions
14	T	Rule Table	I see your plus 2 [referring to their table], you said when I have plus one I add a triangle. [teacher uses manipulatives] when I see a plus one here [perimeter column]
15	B	Table Rule	It always goes up by one
16	T		By one what?
17	B	Rule Manipulative	Plus one triangle I think – like a side
18	T	Manipulative Number	That's one triangle side so show me that – put those [manipulatives] down, show me the one side. When I add a triangle where does this one side come from?
19	B	Manipulative	Here and here (B *points to manipulatives while student C holds up 2 fingers*)
20	T	Number	**but that's 2**
21	C	Rule Manipulative	but then if it did that it would only be plus one because its still one because you take out one of the sides (*moves manipulatives*)
22	T		Say that one more time
23	C	Rule Manipulative	Its like, I don't know the word for it, but its only plus one because that one takes the other side, negate – not that word…
24	T		What's he talking about?
25	B	Manipulative	Like this was 2 but when he stuck that there it covers up the side

Table 2. Discourse and actions of Barbara, Carl, and the teacher and their representations for each item

In Table 1 Barbara and Carl made a conversion from the geometric register to the symbolic register. In Table 2, the teacher asked them to look at specific rule representations and relate them to manipulatives in the geometric register. Items 21, 23 and 25 identify the change of representation which indicates a conversion from the symbolic register to the geometric register.

Systems of Registers

Upon completion of coding the transcript, a system of registers was created for each student and the item numbers associated with each representational coding were placed in the appropriate registers beside their corresponding representations (see Figure 2). When an item appeared in two registers, the extensions *a* and *b* were used to identify order.

Analysis indicated that both Barbara and Carl coordinated conversions between the geometric and the symbolic registers. Drawings were used in the geometric register to produce numbers in the symbolic register. Treatments involving numbers, tables and rules led to both students using the rule representation within the symbolic register. Barbara and Carl are then able to conversion from the rule representation within the symbolic register to the manipulative representation within the geometric register. This is a coordinated conversion between at least two registers, one monofunctional (symbolic) and the other multifunctional (geometric). Barbara and Carl exhibited a requirement which is necessary for mathematical understanding.

Student B

	Geometric Register Manipulatives:17b,19,25 Drawings: 2a
Symbolic Register Number:2b Table:4,6,8,10,15 Rule: 4,6,8,10,12,15,17a	

Student C

	Geometric Register Manipulatives: 21b,23b Drawings: 1a,3a
Symbolic Register Number: 1b,3b,9,11 Table: 11 Rule: 5,9,13,21a,23a	

Figure 2 – System of Registers for Barbara and Carl

Discussion

Mathematical understanding associated with productivity in developing a rule for a triangle train appear as 1) coordination of conversions between the geometric and symbolic registers and 2) treatments within the symbolic register as representations change to or from: numbers, tables and rules. Barbara and Carl's social interactions were factors that contributed to their mathematical understanding. Their discourse and actions produced the trajectory of representations which lead to growth of mathematical understanding. The coactional processes were important to the collective growth.

The ability to recognize different conceptions contributes to growth in flexibility, an essential element in being able to see useful patterns (Lee, 1996). This flexibility is demonstrated by Barbara and Carl as they change representation multiple times within the symbolic register.

The intervention of the teacher was a factor which contributed to mathematical understanding. Without the intervention it is possible that the students would not have attempted to conversion from the symbolic register back to the geometric register. Carl even stated that he hoped they were right because he didn't want to draw 100 triangles.

Results from this research imply that as students are encouraged to make meaning for variable, change and covariation through patterning activities, flexibility is enhanced by the diverse and different understandings of individuals. The diverse and different understandings provide the representations and change in representation contributes to growth of mathematical understanding. Attention to coactional activity can help to identify learning trajectories that are productive in promoting mathematical understanding within and among students. Analysis of other activities may provide additional insight into productive factors influencing mathematical understanding of patterning activities.

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