Mathematical Relations in Architecture and Spatial Design

Jane Burry, BA DipArch registered architect
Research Fellow, Spatial Information Architecture Laboratory (SIAL),
School of Architecture and Design, RMIT University, Melbourne, Australia
jane.burry@rmit.edu.au   http://www.sial.rmit.edu.au/~jburry

Abstract

This paper discusses **the introduction or application of mathematics in architectural and spatial design education**. The examples discussed are from undergraduate courses running in the Spatial Information Architecture Laboratory in RMIT University but the work draws on research in a wider international community. It specifically addresses the dual themes of ***effectively utilizing new paradigms in teaching and learning*** and ***applications of mathematics and modelling in the real world***. The work discussed is project- rather than problem– based. Problems that emerge and have to be solved, are a subset of the students' own world; the framing and definition of the problems are as much a topic for discussion and criticism as the student's approach to their solution. In the context of architecture and design, a disciplinary area where mathematical content is sparse, and, if anything, diminishing and there is no prerequisite for measured mathematical competencies, the paper reports on courses where the prevalent reticence to call on explicitly mathematical ways of defining relations in design modelling has been overcome to produce interesting design outcomes. It will conclude with plans to extend this research to combined course offerings in mathematics and design.

Introduction

The sixteenth century Academy of Arts in Florence……. was a kind of polytechnic college, where the teaching of mathematics was obligatory. Here mathematics was taught not in its abstract and pure form, but in its purposeful application as the leading science of the art of design (arti del disegno) which embraced all branches of the technique of arts and engineering" (Straub, 1952 p xvii-xviii).

Geometry has been defined as the *study* of space and architecture as the *creation* of space. (Blackwell, 1984) The definition of the word geometry comes from a Greek root meaning land measurement (Pedoe, 1976) or earth measure (Smart, 1994). Daniele Barbaro, the sixteenth century translator and commentator on the treatise of the Roman architectural theorist, Vitruvius was *"an eminent mathematician, poet, philosopher, theologian and diplomatist"* (Wittkower, 1952 p59). It is interesting that he is listed first as a mathematician. It seems that there has been something of paradigm shift in design education since the sixteenth century, and possibly quite a recent one. It is now arguable whether mathematics is generally thought to be at the core of the art of design.

Mathematics in Architecture and Design Education

As recently as two or three decades ago, a typical modernist architectural education included introduction to the proportions of the classical orders, not merely as historical context, but through undertaking applied exercises in Greek typography, for instance. At that time, knowledge of how to construct one, two, and three point perspective drawings, in other words *projective geometry*, was an essential skill for the craft. A rudimentary introduction to *statics* was also important, if only to know at what point the architect should call in the structural engineer. Over recent decades, the computer has increasingly subsumed many of the former 'mathematically based' activities of designers through their use of various types of drafting and subsequently, modelling software. The design classes that are reported here are underpinned by the research question: can the computer extend the potential for relevant application of mathematics in design?

Existing paradigms: project-based learning

One of the principle distinctions between architecture and design pedagogy and engineering, pedagogy in our university, apart from the differing status of mathematics in these two areas, has been the fundamental allegiance of the first to project-based learning and the second to problem-based learning. This is a subtle distinction that nevertheless arrives at very different approaches to design by the two groups of graduates. The project-based learners typically work from a given site, programmatic brief, and some kind of ideological or conceptual framework. They will start from an extremely large, experimental and conceptual *design space* and may approach conceptual design from many different starting points from which they will progressively try to synthesise: form generation, diagrammatic planning, environmental or social performance criteria are examples. The problem-based learners, by contrast, start work with a more defined starting point – to span a given space using concrete, for example - and will typically try to apply a range of given techniques and solutions to the problem to check for fit.

Introducing flexible parametric modelling in undergraduate design

While much complex mathematics is, for the most part, concealed behind the computer software interface, the computer does provide the opportunity to the adventurous design student to experiment with mathematical relations as a generative tool, for form finding, or for relating programmatic and performance criteria of the design explicitly to the generation of spaces and forms. They can do this through scripting for a 3D modelling interfaces or visualisation software, or using the interface of a parametric modeller. Those who become proficient, find these very empowering opportunities to iterate and make 'downstream' productivity gains. Scripting and simple computer programming have generally been introduced through elective 'technical' courses, part of a generic menu of skills- based offerings that support the design work in the studio. In the last five years, affordable educational licenses of powerful parametric aeronautical software have also allowed students to construct models with declared parameters and a tree of geometrical relations using a graphical program interface without the prior need for specialised scripting or programming skills. Students become familiar with the concept of flexible parametric modelling, using associative geometry, nominally constructing models with *stable topology* but *variable geometry*, for their own design projects.

Many students are content to build a fairly simple tree of geometrical relations. An abstract example of this is a circle of radius 'r', whose centre is a point, 'pt_cen' defined parametrically at a distance 'd' from the vertex of a rectangle and angular direction 'a' to one of the edges of the same rectangle. The rectangle has been constructed as a line, 'l_01', from a point, 'pt_01' in direction parallel to the y axis,, with a length, Len_01, a further line,'l_02', perpendicular to line 'l_01' and length, Len_02, and two further lines l_03 and l_04, translations of l_01 and l_02 by distances Len_02 and Len_01 respectively. The radius R could be the length of the rectangle edge Len_01 divided by 2 (Len_01/2).(see Figure 01) All these named parameters have variable values. The logic is simple but the process of construction is made labour intensive by creating a tree of related parameters, when compared with using the rectangle and circle commands in explicit modelling software. The delight expressed when a new initiate changes a parameter of the rectangle and observes the circle update, indicates that they have begun to grasp the potential of this upfront investment of time in the modelling process. The real examples in the exercises are less simplistic and quickly extend their knowledge to how replicate instances of their geometry, by re-parenting it on new parent geometry and parameters and how to link their parameters to spreadsheets in which they can more simply manipulate and relate the values than within the modelling interface. From here it is a straight forward matter to start using simple algebra to create more sophisticated relationships. But for many, mathematical notation proves a stumbling block. Typically they will look for a constructed geometrical work around that side steps algebra by, for instance, morphing a form by defining points by a ratio along a line. Figure 01 shows a way or creating a morph that can vary between a sphere and cube through varying this parameter between 0 and 1. They have neatly side stepped writing any algebraic expressions and kept a very concrete geometrical conception of the transformation. It is possible that this builderly approach will only take one so far, however. At the end of semester when the students have modelled their own projects, high achievement in this course is measured by notational economy through structuring a logical tree of relations that meets their own design performance criteria, has appropriate ‘high level’ parameters to provide the user controls they are seeking. A sophisticated hierarchy of relations, and ways of introducing subtle constraints on parameter values through conditional statements are all smiled upon. The model that moves but only through a whole series of value changes in a very shallow graph of relations is probably a sign that the power of the system has not yet been fully grasped. Clearly this way of modelling provides the opportunity to assess many more design iterations at the stage of design development than would be supported by traditional approaches to graphical and three dimensional representations. On the other hand, there is also no doubt that this slow systematic way of thinking has the potential to interrupt the creative process at the very early conceptual stages of design and that the mathematics of the models themselves can quickly become very complex and potentially over constrained or behaviourally unpredictable.

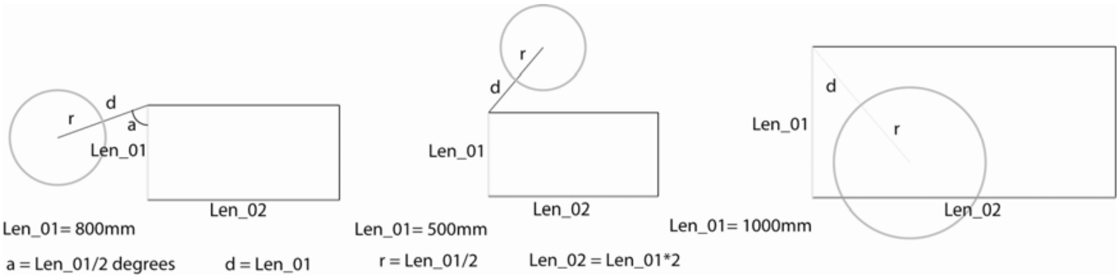

Figure 01 a) Relating a circle to a rectangle: parametric variations of the length of the rectangle side.

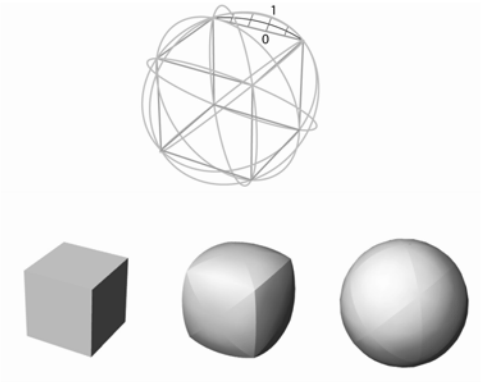

Figure 01 b) Morphing a sphere to a cube using simple synthetic geometry.

An experimental example in disciplinary détente

I will now briefly summarise an example of a mathematics application in an undergraduate design research class, which, I believe, takes the idea of the parametric schema further. This was a particular exploratory approach to using **mathematical surface definition** which provided a language and vehicle for design co-authorship by the unusual partnership of an architecture student and an engineering student.
In the example, the problem addressed is the strong epistemological divide between architects and structural engineers, a gulf reinforced through the pedagogy of the two disciplines, which, nevertheless, work closely together professionally.

This was a final year course combining architecture students in their penultimate semester, engaged in research leading to their major design project in final semester and final semester engineering students, who were to write a research report on a topic of their choice. The semester was divided into three sub projects: first, the partnerships were to explore their own declared combined interests through selecting techniques and applying these to their own simple design proposals (eg a nightclub space suspended over a laneway, a tower form developed through applying evolutionary structural optimisation software.(see Figure 02) In the second phase, they were to develop one technique to a higher degree, and, in the third, they were to introduce a challenging site. At the second stage one partnership cemented their common interest in mathematics when they discovered a software that gave them free access to a catalogue of mathematically defined surfaces, including three dimensional meshes of portions of these surfaces.(see Figure 02) They adopted some that were judged most promising and applied them to a series of projects: a tower complex, a railway station, an urban cathedral enclosed in small scale apartments where a surface based on a Gamma function mediated between one type of occupation of space and the other. In the third and final stage, this partnership chose an old causeway site across a sheltered maritime bay, now with a dredged shipping canal through it that had once been proposed as the route for a freeway bypass for the adjacent city. Taking on the idea of the mathematical surface as a mediator between different programs, they designed an inhabited bridge carrying a freeway bypass using a mathematical surface. The surface undulated to provide the structural piers, a site for housing, with service roads and ferry access. The surface itself provided the structure and mediated between the freeway overpass and the domestic program.(see Figure 03)

What is the mathematical pedagogical interest here? In the stage two projects they had been able to manipulate their adopted mathematical surfaces only through scaling, trimming and changing parameter values by introducing factors in the functions or varying the values of constants. The engineering student, applying his problem-solving training of nearly four years, was still valiantly attempting to apply finite element analysis to uncover the structural performance of the surfaces and to resolve the complex surface into linear steel members. In the final sited project in stage three, the two participants had, through playing with the functions, become more familiar with relationship between the function and the form of the surface. They were also seeking help, initially from our own mathematics department and subsequently from Paul Bourke, an astrophysicist at another university who had been involved in developing the program they were using to generate the surface. They were also faced with a very concrete set of shape parameters, related to structure, program and site (the height of the shipping canal, the maximum structural span over the shipping canal, the maximum gradient of the freeway, the width of the freeway, spacing of piers, angle of springing from the approach roads at each end of the bridge, the size and curvature of the undulations between piers to accommodate the housing, for example.) By now, they were aware of the effect of superposing other functions to achieve finer grain surface variation and to introduce new degrees of control. Paul Bourke showed them how to parametricise the function to give them the variables corresponding to their own performance parameters. (see Figure 03)

They could now play the function like a musical instrument, they were even able to add the lanes of the freeway within the function rather than attempting to add these graphically in the output surface geometry models

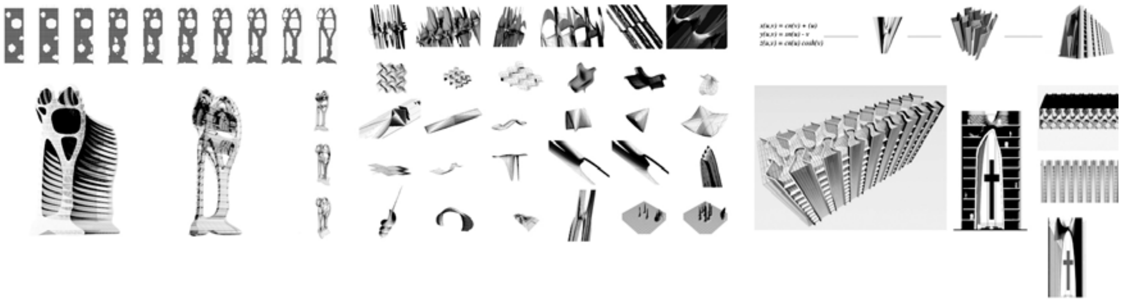

Figure 02 Images from stages one and two of the project: 1. an experiment with evolutionary structural optimisation software; 2. a catalogue of mathematical surfaces – a number were chosen including a combined Jacobi elliptic and hyperbolic cosine function; 3.The hybrid cathedral/housing project

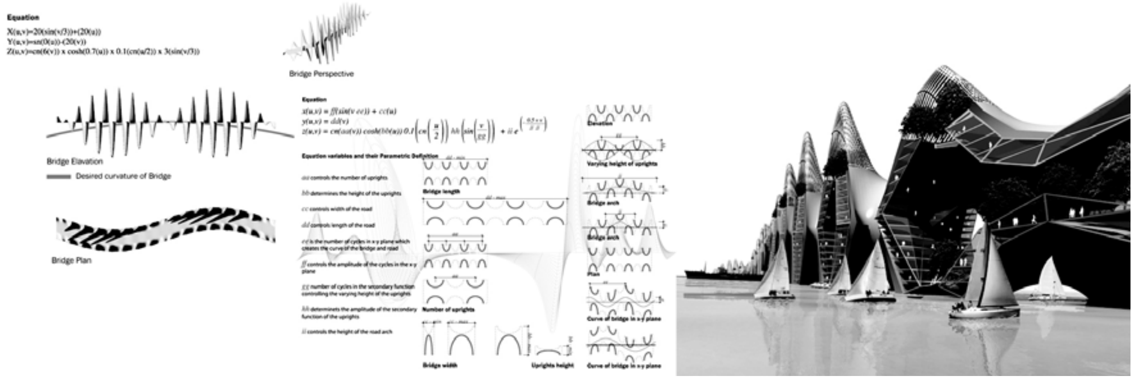

Figure 03 Images from stage three of the project: 1. the equation and the architectural problem; 2.the parameters; 3.rendered image of the bridge

Discussion + Conclusions

The paper has briefly described and contrasted two pedagogical innovations that have involved architecture, design and engineering students taking more consciously mathematical approaches to their design modelling.

The first was to introduce parametric software to a mixed group of design students to allow them to construct flexible associative geometry models, allowing them to define and explore a range of design outcomes within a given design space. Many are restricted in their ability to succinctly define relationships through their apprehension of the use of mathematical notation, but they work around mathematical descriptions by exploiting the construction of synthetic geometry.

The second was to combine a high level undergraduate research course in engineering with a high level undergraduate research course in architecture to try and break down the ontological and methodological boundaries between students of the two disciplines. Through their own initiative, one partnership in this course chose to work with mathematically defined surfaces.

They were attracted to this approach to parametric design by the notational economy of the mathematics, the very idea of three lines of function that contain all the characteristics of the main structural surface that could be passed simply between design and construction players. Currently there are many barriers to good and economic collaboration in the architecture engineering construction (AEC) industry, incompatibilities both in software and ways of approaching the same problems lead to frequent remodelling in different environments, poor reuse of data and information and expensive errors ensuing in construction. The single shared *building information model* is the holy grail but as yet, even after thirty years research, barely realised. Significantly, the architecture and engineering student were, in the closing stages of this project, able to design together, pushing and pulling the same (mathematical) model. For this, the engineering student had to move to a much more conceptual, *in principle,* approach, content to defer the definitive right or wrong answers to structural stability with which he was familiar and which engineering analysis software is designed to furnish. The architecture student, accustomed to a much more forgiving modelling software environment where it is easy to loft a surface over B spine curves and drag its control points to sculpt, discovered that he could manipulate surfaces almost as fluently and much more meaningfully in terms of control, repeatability and transmissibility through adjusting the mathematical functions.

Following on this work, we have two further proposed innovations starting in 2007. As a result of the successful outcomes of this joint architecture engineering course, we have decided to set up another joint course, a dual elective with a much larger cohort of more junior students. They will work in combined groups to design long span and high rise buildings. We are also at the early planning stages of combining architecture and design students in a design studio with a cohort of mathematics students undertaking an applied ‘professional practice’ project in their final general year. At this stage, initial evidence points to transdisciplinarity (contributing from both disciplines to project and learning outcomes that transcend either) as a fruitful and widely beneficial line of experimentation that has the potential to support the creative use of mathematics.

References

Blackwell, W. (1984). Geometry in Architecture, John wiley and Sons.

Glasersfeld, E. v., Ed. (1991). Radical constructivism in Mathematics Education. Mathematics Education Library. Dordrecht, London, Boston, Kluwer Academic Publishers.

Pedoe, D. (1976). Geometry and the Liberal Arts. Harmondsworth, Penguin.

Smart, J. R. (1994 (1973)). Modern Geometries. Pacific Grove, California, Brooks/Cole.

Straub, H. (1952 (trans) 1949 (German)). (Achievements in Steel and Stone) A History of civil engineering: An outline from ancient to Modern times.

Wittkower, R. (1952). Architectural Principles in the Age of humanism. London, Alec Trianti ltd.

Acknowledgements

I would like to acknowledge the contributions of Steven Swain, whose original award-winning student PreMajor work is featured in Figures 02 and 03, Sean Ryan the engineering student who worked with him on this project, and those who co-conceived and taught the architecture-engineering class: Andrew Maher, School of Architecture and Design, ,Saman De Silva, Engineering RMIT, Professor Y. Xie who leads the RMIT Civil Engineering program and finally, Paul Burke, Swinburne University for his direct assistance to the students in parametrising their functions.