Investigating Social and Individual Aspects in Teachers' Approaches to Mathematical Problem Solving

Erik De Corte (1), Fien Depaepe (1, 2), and Lieven Verschaffel (1)
(1) Center for Instructional Psychology and Technology< University of Leuven, Belgium
(2) Research Assistant of the National Fund for Scientific Research – Flanders, Belgium

Abstract

The study aims at investigating how teachers approach reform-based problem-solving lessons. The study is framed in a socioconstructivist perspective that focuses on both the social and the individual dimension of classroom learning. To grasp the social perspective we developed a coding scheme relating to (1) the focus of the learning environment, (2) the instructional techniques and classroom organization forms, and (3) the set of tasks. For understanding the individual level, we administered three instruments to measure teachers' and students' beliefs about the teaching and learning of mathematical word-problem solving and to assess students' problem-solving processes and skills. The paper presents the results with respect to both the social and the individual perspective.

Theoretical Background

Starting in the 1970s, a worldwide consensus grew that mathematics education should mainly aim at students' mathematical reasoning, problem-solving skills, attitudes, and the ability to use these skills in meaningful, real-life (application) situations, instead of focusing on the acquisition of definitions, formulae and procedures. This reform movement was also influential in Flanders and resulted in the formulation of new standards for primary mathematics education (Ministerie van de Vlaamse Gemeenschap, 1997).

As a contribution to the implementation of these new standards, Verschaffel, De Corte, Lasure, Van Vaerenbergh, Bogaerts, and Ratinckx (1999) designed, implemented and evaluated a learning environment that is powerful in eliciting in upper primary school children the appropriate learning processes for acquiring competence in mathematical problem-solving as well as positive mathematics-related beliefs. This learning environment emphasizes students' acquisition of an overall metacognitive strategy for solving mathematical problems involving five stages: (1) build a mental representation of the problem; (2) decide how to solve the problem; (3) execute the necessary calculations; (4) interpret the outcome and formulate an answer; (5) evaluate the solution. Moreover, it also aims at eliciting positive mathematics-related beliefs. To attain these goals, the learning environment is based on three pillars: (1) the use of a varied set of realistic, complex, and open problems; (2) the use of a varied set of powerful instructional techniques; and (3) the creation of a classroom culture aimed at establishing new norms about teaching and learning mathematical problem-solving. The results revealed that the learning environment was indeed powerful in producing significant and stable positive effects on students' beliefs and problem-solving capabilities.

In an attempt to generalize the benefits of this experimental learning environment, some Flemish textbook makers reconceptualized their materials by explicitly appealing to Verschaffel et al.'s (1999) work. However, implementation of this environment on a large scale is problematic for at least two reasons. First, translating research into practice is context sensitive (Burkhardt & Schoenfeld, 2003). Second, successful implementation of the innovative ideas requires not only good materials but also highly motivated and capable teachers who are prepared and able to use the materials appropriately (Remillard, 2005).

In this paper, we describe a study aimed at investigating how textbook problem-solving lessons that explicitly appeal to the above-mentioned experimental learning environment are implemented in regular classrooms. Based on the findings of previous research on curriculum implementation, showing that teachers, as active interpreters of the curriculum, create their own meanings concerning what should be implemented and how this should be done (Remillard, 2005), we first expect difficulties in the implementation of these lessons by the teachers as well as considerable differences in implementation quality between different teachers. Second, we anticipated that these distinct approaches to implement the same problem-solving lessons are related to teachers' beliefs, on the one hand, and students' beliefs and problem-solving capabilities, on the other hand.

Socioconstructivism

The socioconstructivist perspective, especially the approach of Cobb and associates (Cobb, Stephan, McClain, & Gravemeijer, 2001), enables us to meet the intended objectives, i.e., to investigate the way in which teachers approach problem-solving lessons and to unravel the (complex) relation between these approaches and individuals' beliefs and performances. Cobb et al. (2001) describe mathematics cultures in terms of classroom norms and practices, on the one hand, and teacher's and students' beliefs, conceptions and activities, on the other hand. These clusters of concepts reflect the social and psychological perspective underlying socioconstructivism: the social perspective refers to ways of acting, reasoning, and arguing that are normative in a classroom community, while the psychological perspective is concerned with the nature of individual students' reasoning and beliefs, and their particular ways of participating in communal activities.

Method

Ten sixth-grade teachers who all used the textbook *Eurobasis* participated in our study. Major criteria for the selection of that textbook series were its representativeness for the Flemish situation in elementary school teaching, and its explicit reference to the principles and materials of the aforementioned experimental learning environment of Verschaffel et al. (1999). For instance, the textbook applies explicitly the proposed overall metacognitive strategy for solving mathematical problems. In each classroom we videotaped the same two problem-solving lessons from the textbook that contained, besides some routine tasks, several non-routine problems. In addition, all students' materials (e.g., their workbooks) were collected. Moreover, we administered three instruments to measure teachers' and students' beliefs and attitudes about the teaching and learning of mathematical word-problem solving, and to assess students' problem-solving processes and skills.

In line with Cobb's framework we focused on both, the social and individual processes to understand how teachers approach problem-solving lessons. To grasp the **social perspective** we developed a coding scheme relating to three important characteristics of the classroom culture that are assumed to enhance students' mathematical beliefs and problem-solving competencies, namely (1) Is the focus of the lessons in line with reform-based ideas? (2) Do the teachers apply powerful instructional techniques and classroom organization forms that foster such a reform-based focus? and (3) Does the set of tasks and the way in which the tasks are addressed reflect these innovative ideas? The focus of our analysis was the teacher, i.e., each speech act[1] of the teacher was coded for the first two characteristics. For the third one, we analysed all tasks that were given to and discussed with the whole class.

Concerning the *focus of the learning environment* we focused on the extent to which the teachers explicitly addressed heuristic and metacognitive skills, on the one hand, and on their establishment of particular classroom norms, on the other hand. The teacher's manual that accompanies the textbook proposes some important heuristics to which teachers should pay

[1] In this context, a speech act is defined as the whole set of sentences, which is uninterruptedly spoken by one actor.

attention, such as “distinguish relevant from irrelevant data”; “reword the problem”; “contextualize the problem”; “draw a picture”; “make a scheme/table”; “guess and check”; “evaluate the solution”; “interpret the outcome”. Teacher’s speech acts were analysed with regard to the classroom norms that were explicitly negotiated between teacher and students during the lessons. We distinguished between ten norms, such as: “solving problems is enjoyable”; “solving problems is time-consuming, also for smarter students”; “a problem can be solved in different ways”, “a problem may have different solutions”, etc.

The second part of the coding scheme focused on *teachers’ instructional techniques and on classroom organization forms*. The instructional techniques distinguished in the cognitive apprenticeship model (Collins, Brown, & Newman, 1989) were used as coding categories. However, we added to the list the technique of praising, which relates to the motivational aspect of learning. This resulted in the following instructional techniques: modeling, non-directive coaching, directive coaching, scaffolding, articulation, reflection, exploration, and praising. With respect to classroom organization we distinguished between whole-class instruction, group work, individual work, and combined organizational form.

The last category pertained to the *set of tasks* the students were confronted with. Each task was analysed with regard to its degree of realism and its degree of complexity.

To better understand **individuals’** conceptions and activities, we assessed teachers’ and students’ beliefs about the teaching and learning of mathematical word-problem solving and students’ problem-solving processes and skills.

To measure *teachers’ beliefs*, we used a self-made questionnaire containing three parts. The first part asks for some personal information. The second part consists of a series of statements relating to teachers’ beliefs about mathematics and problem solving (e.g., “There is only one correct way to solve a problem”). Teachers had to express their level of agreement on a five-point Likert-scale. The third part addresses teachers’ evaluations of possible realistic and non-realistic student answers to two realistic mathematical problems: “Jan’s best time to run 100m is 17sec. How long will it take him to run 1km?”, and “Steven has bought 4 planks of each 2,5m. How many planks of 1m can he saw form these planks?”. As in the study of Verschaffel, De Corte, and Borghart (1997), teachers were asked to evaluate four possible student responses to these realistic problems; one of the answers did not take into account any realistic consideration (e.g., 4 x 2,5 = 10 planks for the above planks problem) whereas another answer could be considered as a (more) realistic one (e.g., 4 x 2 = 8 planks)

Students’ beliefs were measured by means of a questionnaire developed by Verschaffel et al. (1999), consisting of 21 statements that students had to value on a five-point Likert-scale. These statements represent two reliable factors: students’ pleasure and persistence in solving word problems (7 items, e.g., “I like to solve word problems”), and a problem- and process-oriented view on word problem solving (14 items, e.g., “There is always only one solution to a word problem”). The beliefs addressed in this questionnaire corresponded with the norms distinguished in our coding scheme (see above), which enabled us to investigate whether the approaches to implement these problem-solving lessons (and more specifically, the norms that are negotiated) are related to individual’s beliefs.

Students’ mathematical problem solving was measured by a paper-and-pencil test, containing 10 non-routine problems for which the solution process appeals especially to the application of heuristic/metacognitive skills and to a disposition towards realistic mathematical modeling and problem solving. This test was also developed and used by Verschaffel et al. (1999), but was slightly modified with respect to the current situation of our research.

Results

Social perspective

The analysis of teachers' speech acts from the perspective of the focus of the learning environment showed that some of the *heuristics/metacognitive skills* were frequently emphasized during the problem-solving lessons. In general, most attention was paid to "distinguish relevant from irrelevant data", "reflect critically on the problem", "make a scheme/table", "compare different solution processes", and "evaluate the solution". Other skills were almost never addressed, such as "guess and check", "simplify the numbers", "contextualize a calculation", "make a flowchart", and the "overall metacognitive strategy for solving mathematical application problems". Substantial differences among the teachers were observed in the frequencies with which they stressed the heuristics/metacognitive skills. Remarkably, in only a very few cases the teachers explained *why it is important* to use a particular heuristic or metacognitive skill.

Interestingly, little or no attention was paid to the deliberate and explicit establishment of mathematics-related *norms*. Only to a very small extent did some teachers emphasize that a problem can be solved in different ways and/or may have different solutions.

Teachers' speech acts were also analysed with regard to their *instructional techniques*. Apart from modeling, scaffolding, and exploring, most of the instructional techniques described in the model of cognitive apprenticeship were used frequently. Notable is the importance given by the teachers to articulation and reflection, which certainly fosters students' problem-solving competency. Teachers praising students was observed rarely. Again, substantial differences between teachers in using these instructional techniques were observed.

Although most of the total lesson time was devoted to whole-class instruction, the combined *classroom organization form* was also frequently applied. In this format teachers mostly guided a specific group of weaker children, while the others worked individually on tasks. Although the mathematics education literature emphasizes the importance of small-group instruction for the development of students' problem-solving skills, only two of the ten participating teachers used group work during the video-taped lessons

All *tasks* were analysed with regard to their realism and their complexity. Almost all tasks referred to contexts taken from students' experiential worlds. Conversely, the percentage of complex problems was much lower. Interestingly, some teachers systematically dropped the complex tasks provided in the textbook.

Individual perspective

The *teacher's questionnaires* revealed that, in general, the teachers did have positive beliefs towards mathematical problem solving. However, three out of the ten participating teachers believed that a good math problem always has an exact solution. With respect to the runners-item, teachers valued non-realistic answers more than realistic ones, whereas the inverse was found for the planks-item. These results are in line with the findings of Verschaffel et al. (1997).

Students' questionnaires revealed that our students' beliefs and attitudes about the teaching and learning of mathematical word-problem solving were not very well in line with the reform ideas reflected in the new standards. This suggests that attempts to innovate mathematical textbooks do not automatically result in an enhancement of students' mathematics-related beliefs. This is in line with the related research literature (Remillard, 2005).

Students' performance on the *mathematical application test* showed that their problem-solving competency had increased substantially. This good performance may be explained by a more problem-oriented instruction towards mathematical problem solving in our ten classes, suggesting that the innovated textbook had indeed a positive effect in this respect. Also here remarkable differences between classes were observed, with scores varying from 1.94 to 5.04 out of 10.

Attempt at Integrating Both Perspectives

Although our restricted data set does not allow us to draw any causal relations between the individual and social aspects investigated in our study, our results revealed already some provisional trends concerning the connection between the social and individual dimension of mathematical learning and teaching. First, only one teacher frequently stressed the norm that problems can be solved in different ways (e.g., "All roads lead to Rome, as long as you take the road that is the easiest one for you"). This was the only class wherein all students expressed the belief that problems can be solved in different ways (59% totally agreed and 41% agreed with this statement); in all other classes the overall agreement with this statement was considerably lower. Second, the class that performed worst on the problem-solving test had the least availing beliefs. The teacher of that class coached more directive than non-directive, whereas all other teachers (except one) did the inverse. This directive teacher also never explicitly negotiated an appropriate norm for mathematical problem solving, she never explicitly emphasized the importance of a certain heuristic, and she never used group work or individual work.

Conclusion and Discussion

This study confirms the gap between theory and practice, in this case the tension between the intended and the implemented curriculum (Remillard, 2005). In addition, the study yields a useful instrument for understanding the complex processes going on in the mathematics classroom.

However, our study has also several limitations. We restricted our analyses to low-inference coding, i.e. directly observable classroom activities. As an example we refer to our coding of classroom norms. We were especially interested in the normative aspects that are negotiated in the classrooms. However, as we only coded for norms that were *explicitly* negotiated between teacher and students (i.e., low-inference coding), we could only partly map out the ways of acting, reasoning, and arguing that are normative in a classroom community. Indeed, norms are also *implicitly* negotiated between teacher and students. Moreover, we videotaped and analysed only two lessons in each classroom, and, arguably, classroom norms develop and get only installed gradually over time. Although our coding of classroom norms has led to a reliable identification of these norms, it should be possible to achieve a better grasp of the normative aspects of classroom activities by looking for patterns of regularities (i.e., practices that frequently occur in a classroom) that implicitly reflect certain norms, and by conducting more in-depth case studies (involving video-based stimulated recall with teachers and/or students) spread over a longer period of time.

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