"Workshop of Mathematical Talent", a Wonderful Educational Experience.

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Abstract

From the year 2004, a group of more than 30 teachers, enthusiasts of Mathematics, have been carrying out the activity "Workshop of Mathematical Talent" ("Taller de Talento Matemático", TTM¹, in Spanish). We celebrate meetings every two Fridays from 18:00 to 19:30. Students aged 13 to 18 meet other fans of Mathematics of the same age. Our aim is that they enjoy aspects and approaches that habitually cannot be seen (for lack of time) in the classrooms of their centres: fascinating perspectives that make them meditate and bring to the surface the best of their good minds. Unlike other projects of this type, we do not do any type of previous selection in the election of the attendees. We send informative letters summoning for ALL the students interested in Mathematics, as well as for the voluntary teachers that want to take part as ALTRUISTIC mentors in any of the meetings. All the meetings can be found in http://www.unizar.es/ttm.

We are deeply satisfied with the positive outcome of this activity: that of encouraging the enjoyment in the resolution of Mathematical problems. We think that this is definitively a way of preparing our teenagers to solve with reason and logic other problems that they will have to face all along. We are sure to be EDUCATING by means of the Mathematics.

Statement of the problem that spurred the innovation

Students aged 13 to 18 generally do not enjoy mathematics. They see this subject as a hard mountain they have to climb if they want to get their degrees in order to be accepted in a university or starting a ruled professional training. Maths are normally in our education system just a mere collection of formulae, more or less understood, that must be used adequately in every case.

The increase of the mixed-abilities students in the classrooms, and the necessity to take care of it, has been misinterpreted almost always as a "reduction of levels" so that all the students can acquire the basic skills. For that reason, students with interests and aptitudes, are relegated

¹ www.unizar.es/ttm/

and put under routine tasks that discourage them little by little. Many times they pass unnoticed or are taken as "bad students".

If mathematical talent is not specifically educated and stimulated, by means of special programs, only those students who have a stimulating atmosphere within the family are the ones that end up developing it.

Evaluations of the Mathematical level of European students show Spain is below the middle level.

Students fond of Maths in Aragon^2 end with the 2^{nd} ESO³ Mathematics Olympiad and they do not have any other place where they can develop their mathematical skills in order to participate in other Maths competitions. There is an enormous gap.

Young students prefer learning things by making discoveries on their own rather than studying the given units in their books. They are impressed by teachers who know their subject, are authentically interested in it and transmit this enthusiasm to the students. They enjoy special experiences in which they can participate and give them the opportunity to investigate on subjects that attract to them and to develop personal techniques to solve problems.

Mathematically skilled teenagers are generally considered as weirdo among their schoolmates. They need to find other people with the same age and the same hobby. They need their "live forum" where they can express without complexes their ability.

There are plenty of activities for after classroom time: sports, music, painting, dancing... But what can a boy or a girl who likes Mathematics do?

All these facts justify the beginning of experiences that allow that these talents do not fail and render the fruits that our society needs and contribute to the scientific and technological cultural development. We must not think that caring these people with special talent is elitist and segregator. Usually students who have a privileged cultural situation in their family are those who will end up developing a mathematical talent on their own. For this reason, our workshop allows other students without this privileged environment to equally develop their capacities. Thanks to the TTM we believe to contribute to a democratization of Mathematical knowledge.

Description of the innovation

Investigating in Mathematics was during our adolescence an unimaginable possibility for most people who now are professors. Nowadays it is not like that. A teenager who feels the taste to discover and to create in the Science of Pythagoras has several channels to develop his/her liking. The Aragonese Society of Teachers of Mathematics carries on every year the Aragonese Phase in the Mathematical Olympiad for 2nd ESO (13/14 years old students). The Association TTM, Workshop of Mathematical Talent of Aragon takes over from the Olympiad, welcoming in the Department of Mathematics of the University of Zaragoza students aged 13 to 18. His present director, Alberto Elduque⁴, is the creator of the fecund bridge tended between both education levels, making possible that altruist university professors and Secondary School teachers open windows to the imagination and creativity of youngest, revealing unsuspected and exciting

² Aragon is a north-eastern region in Spain (Europe) with less than 1.500.000 inhabitants. Zaragoza is its most populated city.

³ ESO stands for "Compulsory Secondary Education". In Spain, Secondary School comprises four courses for students aged 12 to 16. Those students who want to have access to the University must take two more courses of "Bachillerato" (A-Levels) (17-18 years old).

⁴ elduque@unizar.es

aspects of the Mathematics. This activity is accompanied by other competitions like the Mathematical Kangaroo and the International Mathematical Rally Without Borders. Also, in the web pages of our schools and High Schools the Contests of Resolution of Problems proliferate.

The TTM is an after-school activity for Maths fans that want to spend a good time while training their brains. Organized by professors of Secondary Education and the University of Zaragoza, it goes directed to 3rd, 4th ESO and "Bachillerato" (13 to 18 years old). It is a different way of seeing Maths.

Our project

Any Friday, the parents of Jorge and Luis do not mind taking them 50 miles to Zaragoza. The younger brother is intrigued because that day the unit is for 3^{rd} ESO (14/15 years) "New

Geometries". Already in the Workshop of Mathematical Talent, soon after beginning, he is shocked. It has in his hands an orange crossed by three sticks and he is verifying surprised, that the three angles of a classic triangle do not add 180°, but 270°! The older brother attends meanwhile the practical session of Theory of Groups, for those of upper grades. Accompanied by Ángel, an expert in the subject, he wanders through the "Mudéjar"⁵ Zaragoza in search of 17 flat groups of symmetry. Enrique comes to Zaragoza from Utrillas (60 miles away). In 4th ESO (15/16 years) Enrique is devoted in that occasion, to solve chess questions by means of Mathematics. But may it be any relation?



TTM's Non Euclidean Geometry and oranges

Manolo, the teacher, talks about a certain Euler and a certain Gauss... Two weeks later, Cristina, a veteran of 2^{nd} year, comments with Yasmine that the session of the day was really extremely interesting. It is "Magic and Mathematics". "Great Alexander" surprises again the audience with his fantastic numerical predictions and his topologically nonexistent knots. And besides, he



explains how he does it! Also they will be able to tell their sons that in their youth they had the luck to watch how Don José Garay wrote by heart on the blackboard, thanks to his famous "Pi-Symphony", the 1000 first decimal of this ubiquitous number... And to hear them! The apotheosis arrives with "Mathematics and Cinema", prodigious selection of cinematographic and television cuts that presents/displays José María, an expert teacher in discovering hidden Mathematics in classical films, series of television and even in cartoons: Homer Simpson allows him to contradict to Pierre de Fermat with the calculation

 $1782^{12} + 1841^{12} = 1922^{12}$

(Try with your pocket calculator if you do not believe it!).

⁵ Local Christian-Arabian art style

From October 2004, more than 300 students and about 30 professors enthusiasts of Mathematics have attended the TTM. Every two weeks, Fridays of 18:00 to 19:30 in the Building of Mathematics of the University of Zaragoza, girls and boys from 100 miles around meet to share their liking to the Science of Pythagoras with other "colleagues" of their same age. We try that they enjoy aspects and approaches that habitually cannot be seen (because there is no time) in the classrooms of the centres. Fascinating perspectives that make run and get to the surface their better ideas. Unlike other similar projects, we do not make any type of selection nor demanded attendance. From Secondary-school teachers, Education inspectors, advisers of the Centres of Teachers and Resources, members of the Department of Mathematics of the University of Zaragoza, from scholarship holders of investigation to university professors come from other towns to develop the sessions. We enjoy sharing with the students the pleasure to create and discover in Mathematics.

The project was impelled by Alberto Elduque, Director of the Department of Mathematics of the University of Zaragoza and by Fernando de la Cueva, myself, Coordinator in Aragon of the Mathematical Olympiad. We were worried about the lack of general preparation of the Aragonese students who participated in the previously mentioned contests. We luckily had the true friendship of a great number of colleagues. Their generosity has allowed the formation of this magnificent group of voluntary professors who have given practical form to this activity. Every professor prepares his session for 1,5 h practical exposition and makes a summary to be hung in our web page. I coordinate the days in which every professor must teach. We have about 15 sessions of every one of the three levels: 3rd ESO (14-15 years), 4th ESO (15-16 years) and "Bachillerato" (16-18 years). We follow the attendance of the students. The last session is a special one. We have a great conference of about 1h; we give diplomas to the regular attendant students and some prizes to the winners of the contest sessions (T-shirts, bags, books, backpacks ...). After that, we have a dinner party in the cafeteria of Mathematics building.

We do not make any previous selection in the election of the assistants. We send informative letters and e-mails to all the Education Centres of Aragon, summoning to all the students with interest in Mathematics that want to attend, as well as the voluntary professors whom they loved to participate like altruistic collaborators. We also profit the Final Phase of the Aragon Mathematical Olympiad of 2nd ESO (13-14 years), to call personally more than 100 attendants.

To channel legally and adequately any contribution, we have formed the association "WORKSHOP OF MATHEMATICAL TALENT OF ARAGÓN ", legally registered. The purpose of the association is the spread of the Mathematics. Its activities are centred on the TTM and in the Aragonese Phase of the Mathematical Olympiad. The association has the headquarters at the Department of Mathematics of the University of Zaragoza. We have the support of the "Diputación General de Aragón" (our local government) and the Academy of Exact, Physical, Chemical and Natural Sciences of Zaragoza. The Department of Mathematics of the University of Zaragoza provides the classrooms, the materials ... Its Director is our webmaster.

We, as teachers and educators, are deeply satisfied with the positive outcome of this activity: that of encouraging the enjoyment in the resolution of Mathematical problems. We think that this is definitively a way of preparing our teenagers to solve with reason and logic other problems that they will have to face all along.

As we say in Spain "Best of life is free". We think ours is one of those free things. We are sure to be EDUCATING by means of the Mathematics. **Topics**

- Strategies in the resolution of problems: two sessions at different levels. Polya, Miguel de Guzmán and other great Problem Solvers brought to the practice.

- Congruencies, modular arithmetic, cryptography: two sessions at different levels including number bracelets and other numerical curiosities.



Deltahedra at the TTM



'Mudéjar' Art (Zaragoza.)

- Algorithms, logarithms and computers: two sessions by our computation expert. An approach to Mathematics through the computer.

- Progressions and other successions: two sessions at different levels. Geometric and arithmetic progressions, Fibonacci sequence, young Gauss 1 to 100 addition, chess inventor's prize, compound interest, Malthus population growing, number e ...

- Polyhedral, deltahedra: you can fold them...

- Mathematical Competitions: Mathematical Rally⁶, Mathematic Olympiad⁷, Mathematics Kangaroo⁸, Mathematical Gymkhana through Zaragoza⁹

- Metric Geometry: angles, circumferences, triangles (with wonderful Cabri presentations)

- Conjectures and tests in Mathematics. The Pick's Theorem and others.

- Mathematical Gymkhana through the town of Zaragoza: our city is full of maths.

- Algebra: groups and "Mudéjar" art in Zaragoza. Three sessions with a mathematical walk through the Arabian heritance of Zaragoza.

- Movements in the plane: homothety, inversion, translation, rotation
- Friezes and mosaics: tesselation of the plane.

- Conic curves: a Power Point Presentation with plenty of surprises.

With sound: Conics Surround Us!

- "New" non Euclidean Geometries: Saint Exupéry's Little Prince strange geometry and others.

- Topology: two sessions.

- Classic Problems with rule and compass. Greek classical.
- What you gamble? (Probability). Combinatory.
- Mathematics and:

Magic. Our students favourite session. Cinema. Amazing film cuts. Dominoes. Plenty of Maths. Graphs theory. Chess trough and for Maths. Origami: folding Maths.



Mathematics and Magic at the TTM

⁶ www.cprlanuza.org/rally07/index.html, by teams

⁷ http://platea.pntic.mec.es/~csanchez/olimmain.htm

⁸ http://es.geocities.com/canguromat/

⁹ http://es.geocities.com/humor_matematicas/RUTAS/rutas0.htm

Description of the extent to which the innovation was successful with respect to the targeted problem

Students tell us personally that this experience is quite grateful for them. The last day of the course, we pass a survey so that they value the activity and we ask them reasons by which they would animate friends to spend Fridays afternoons (so much valued by teenagers) in this project. Most of them indicate the privilege to be in a comfortable atmosphere, because they are with people before whom they do not have to hide their interest in Mathematics, where they meet new friends and they enjoy other of seeing Mathematics.

For the participant professors these are enough reasons to go on with this project. It is possible that mathematical level in aragonese teenagers may be rising thanks to this activity. What is sure is that more than 300 young people have enjoyed maths in a way they never suspected thought of.

Possibilities for transfer to different environments.

All we have explained previously is a guideline for other teachers who are interested in this model of activity. It is a FREE activity. We don't make students selection. The teachers who participate are ALTRUISTIC. We invest 15 to 20 hours preparing our session. Our web page is a guide of the topics. It is a good free starting point for the whole mathematical community of this kind of things. There must be a group of generous people and one or two coordinators. If possible, a webmaster is welcome.