

Using the ClassPad as a Dynamic Learning Environment to Teach Mathematics (Workshop)

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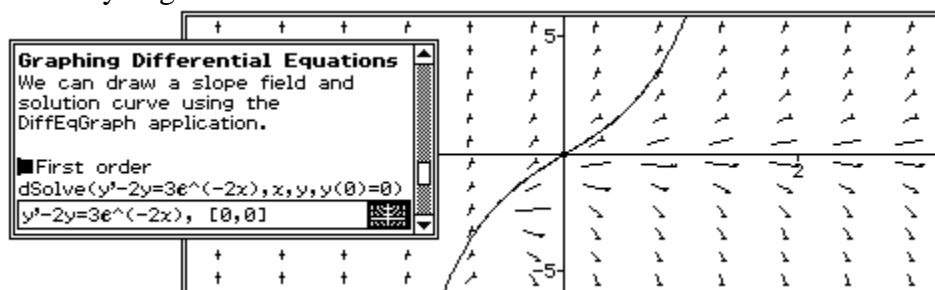
Abstract

How can technology be used in teaching to enhance understanding of mathematics and encourage learning through exploration? The ClassPad provides a way.

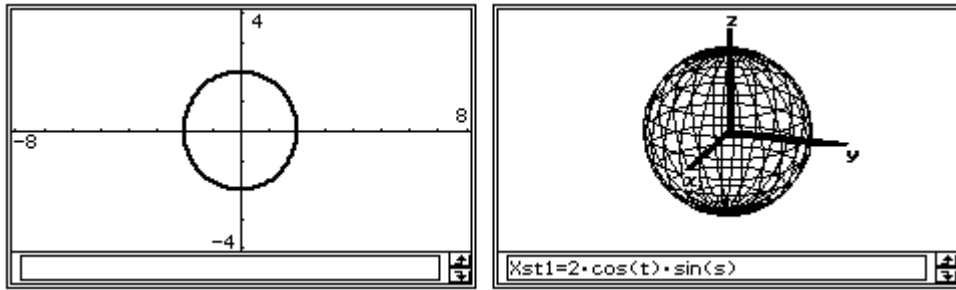
We have seen many beautiful technologies emerge for teaching mathematics over the past several years to assist students in visualizing mathematics. Seeing mathematics come to life was exciting, but the technology was often times difficult to use and time consuming to learn. Recently, a new version of technology has emerged that is simple to use. Within a short time, dynamic lessons that contain algebra, geometry, graphs, dynamic links, text, and more can be created within one file. The interactive activities, classed eActivities, can be created and used to share mathematics with students in the classroom or across the web.

In this hands-on workshop, you will learn how easy it is to create and use dynamic eActivity lessons to enhance mathematical understanding and share the beauty of mathematics multiple ways in one file. Activities will be based on many of the new features in the latest version of the ClassPad Manager Professional software. We will explore vector fields, 3D parametric curves, linking polar curves and more. No experience is needed.

The workshop will begin with an introduction to the ClassPad Manager Professional version and the resizable window. We will then explore various forms of differential equations using an eActivity to guide us.



To gain an understanding of the ease of creating an eActivity to use in lecture or as an activity, we will create a simple eActivity to experiment with curves and surfaces in parametric form.



We will also explore, as time permits, the derivative of a polar curve, Fourier series and answer any questions participants may have about the software.

Fourier Series - CAS

■ Definition

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(nx)) + \sum_{n=1}^{\infty} (b_n \sin(nx))$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx, \quad a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx$$

File Edit Insert Action

$\frac{d}{d\theta}(f(\theta)*\sin(\theta))$

define f_diff(θ) = $\frac{d}{d\theta}(f(\theta)*\cos(\theta))$

To graph each:

r=f(θ)

r=f_diff(θ)

□

Expand the Geometry window below and then drag & drop:

f(θ) and f_diff(θ)

r=θ

$$r = \frac{\theta \cdot \cos(\theta) + \sin(\theta)}{\cos(\theta) - \theta \cdot \sin(\theta)}$$

Alg Standard Real Rad

Part I: An Introduction to Using eActivities



We will begin this part by demonstrating an important use of eActivities; using an eActivity to assist in a lecture. Following the demonstration, we will open a second eActivity to explore different types of differential equations that the ClassPad can solve.

1. Demonstration using **01_SlopeFields-(eActivity).xcp**
2. (Hands-ON) Open the **ClassPad folder** located on your desktop.

3. **Double-click** the file named **02_DiffeQ_Examples-(eActivity).xcp**.
4. Explore the different forms of differential equations.

Part II: An Introduction to Creating eActivities

As an introduction, we will use the eActivity application to review parametric equations in 2- and 3-dimensional space and then save the file.

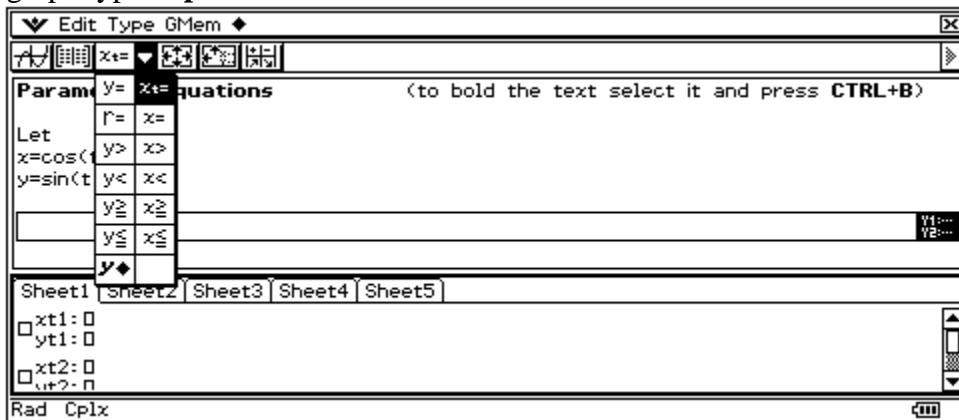
1. Select File/New to clear the eActivity window.
2. Consider the different types of lines in an eActivity:
 - A **text line** begins as an empty line and the toolbar shows the  button.
 - A **math line** begins with a small box (\square) and the toolbar shows the  button.
 - An **application line** is a long box (strip) with an icon to the far right to expand or minimize the window.
3. To begin, type the following into eActivity using the computer keyboard:

```

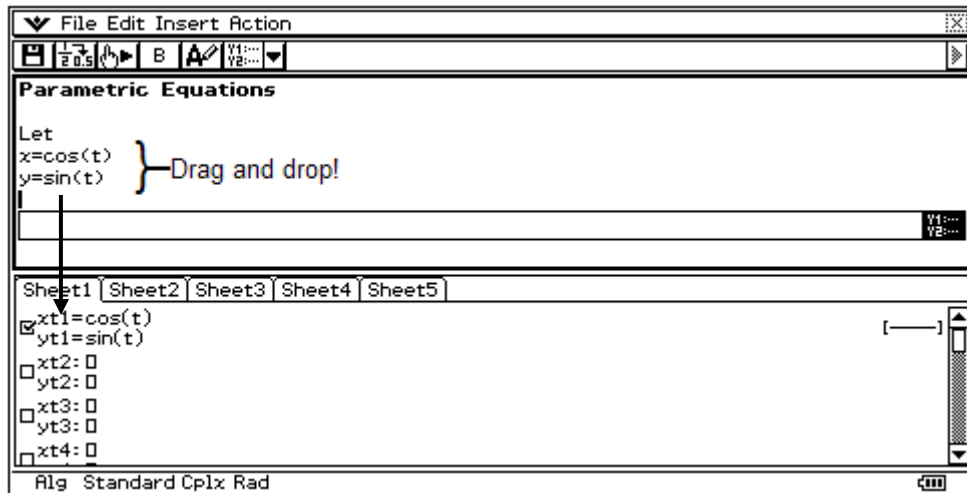
Parametric Equations (to bold the text select it and press CTRL+B)
Let
x=cos(t)
y=sin(t)


```

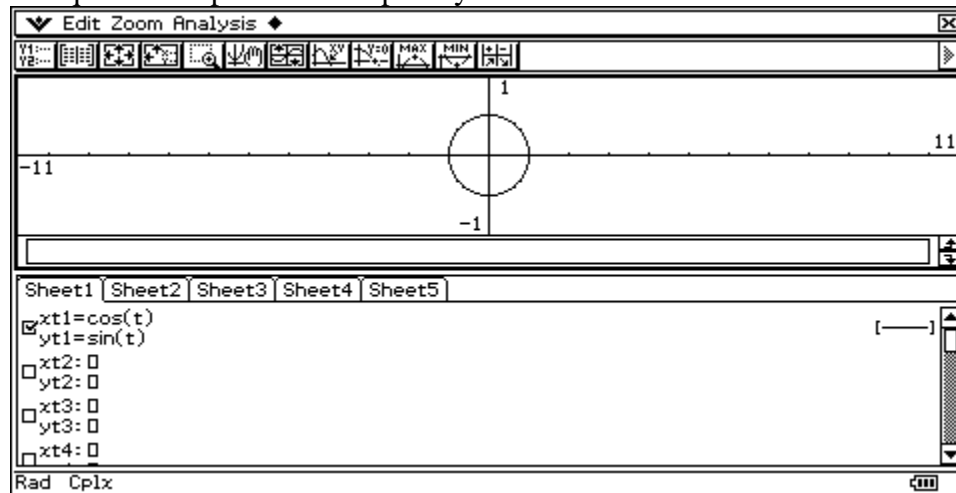
4. Open the **Insert** menu and select **Strip/Graph Editor**. When the window opens, change the graph type to **parametric**.




5. Select the $\cos(t)$ and then $\sin(t)$ and drag each to the Graph Editor window. Just select, release, press and drag.



6. After checking the box, click on the first toolbar button () to graph. Select Zoom/Square to improve the shape of your circle.



7. In what direction is the curve traced as t increases? [Hint: Click the  menu and select Graph Format. Next, check Leading Cursor.]
8. How can we reverse the direction the curve is traced in? There are multiple answers!


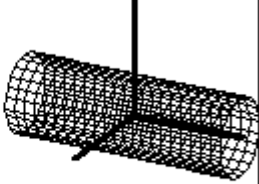
In 2-D, we have one parameter, t , to map two variables onto a curve. In 3-D, we have two parameters, s and t , to map three variables onto a surface. The ClassPad makes exploring parametric equations in 3-D fun and somewhat easy to understand!

Assume you do not know the equation of a cylinder. But, we do know $x=\cos(t)$, $y=\sin(t)$ is a circle in 2D. With this knowledge, can you guess the parametric equation for a cylinder?

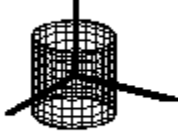
1. Continuing with the same eActivity, close the Graph window and click below the Graph Editor strip in the eActivity window.
2. Select **Insert/ Strip/ 3D Graph Editor**.

3. Click the **Z=** button to change to parametric form.


4. Complete: Can you discover the equation for a cylinder opening:

<p>a. Along the z-axis?</p> <p>Xst1=_____</p> <p>Yst1=_____</p> <p>Zst1=_____</p> 	<p>b. Along the y-axis?</p> <p>Xst1=_____</p> <p>Yst1=_____</p> <p>Zst1=_____</p> 
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5. Complete: How can we limit the range of the z-values (using a function) to [-1,1]?

<p>Without changing window settings, how can we limit z-values to [-1,1]?</p> <p>Xst1=_____</p> <p>Yst1=_____</p> <p>Zst1=_____</p> 

6. Complete: How can we pull in the top and bottom to form a sphere?

<p>At last, a sphere!</p> <p>Xst1=_____</p> <p>Yst1=_____</p> <p>Zst1=_____</p> 
--

7. Experiment with other forms!

8. You can save your eActivity by opening the File menu, selecting Save and inputting a name.

Part III: Graphing Polar Curves and their Derivatives

Next, we will use an eActivity to help us view and understand the derivative of a polar curve. To begin,

1. Open the **ClassPad folder** located on your desktop.
2. Select the file named **03_PolarCurve&Deriv-(eActivity).xls** and drag it onto the eActivity window or you can double-click the file.
3. You should now see:

Exploring the Derivative of a Polar Curve

Let:
 $x=r*\cos(\theta)=f(\theta)*\cos(\theta)$
 $y=r*\sin(\theta)=f(\theta)*\sin(\theta)$

Taking the derivative with respect to theta:
 $\frac{dx}{d\theta}=\frac{d}{d\theta}(f(\theta)*\cos(\theta))$
 $\frac{dx}{d\theta}=\cos(\theta)\cdot\frac{d}{d\theta}(f(\theta))-\sin(\theta)\cdot f(\theta)$
 $\frac{dy}{d\theta}=\frac{d}{d\theta}(f(\theta)*\sin(\theta))$
 $\frac{dy}{d\theta}=\sin(\theta)\cdot\frac{d}{d\theta}(f(\theta))+\cos(\theta)\cdot f(\theta)$

From this we can define dy/dx for $dx/d\theta \neq 0$.

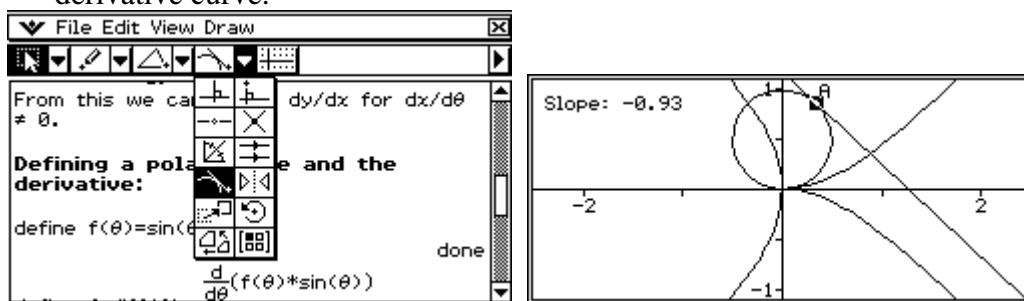
Defining a polar curve and the derivative:

```
define f(θ)=θ
done
define f_diff(θ)=
  d
  dθ(f(θ)*sin(θ))
  d
  dθ(f(θ)*cos(θ))
done
```

To graph each:
 $r=f(\theta)$ $r=\theta$
 $r=f_diff(\theta)$ $r=\frac{\theta\cdot\cos(\theta)+\sin(\theta)}{\cos(\theta)-\theta\cdot\sin(\theta)}$

Expand the Geometry window below and then drag & drop:
 $f(\theta)$ and $f_diff(\theta)$

4. Experiment by, for example, changing **define f() =** to **define f() = sin()** and then press **EXE** or **Enter**.
5. Clear the Geometry window and drag the new results in.
6. You can also add a tangent line to your curve to help in the visualization of the derivative curve.



Part IV: Exploring Fourier Series using CAS

For the final activity, we will use the CAS to help us better understand Fourier series. This is an excellent eActivity to use to introduce Fourier series to students.

To begin,

1. Open the **ClassPad** folder again.

2. Select the file named **04_Fourier-CAS-(eActivity).xcp** and drag it onto the eActivity window or you can double-click the file.
3. You should now see:

Fourier Series - CAS

■ Definition

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(nx)) + \sum_{n=1}^{\infty} (b_n \sin(nx))$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx, \quad a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx, \quad b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx$$

■ Example (in Complex)

Define the original $f(x)$ and try to have the Fourier for $n=1,2,3$.

Define $f(x)=x$ done

{1,2,3}⇒n {1,2,3}

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx + \text{sum} \left(\frac{\cos(nx)}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx + \frac{\sin(nx)}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx \right) \Rightarrow F$$

$$2 \cdot \sin(x) + \frac{2 \cdot \sin(3 \cdot x)}{3} - \sin(2 \cdot x)$$

□

Graph of Fourier Series [A]

4. Here are a few ideas to use this eActivity:
 - a. Expand the Graph strip.
 - b. Change {1,2,3} n to {1,2,3,4,5} n and press EXE or Enter.
 - c. Click in the Graph window to update.
 - d. Or, change **Define f(x)=x** to **Define f(x)=x^2** and press EXE or Enter.
 - e. Click in the Graph window again.

Thank you for attending our workshop!
 We hope that you will drag, drop and explore new areas in mathematics using the ClassPad.