# Helping Students Understand Technical Calculus via an Online Learning Supplement and Group Learning 

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#### Abstract

Many students have many difficulties with learning calculus. Some of the difficulties stem from not thoroughly learning algebra, lack of problem solving skills, or lack of study skills. During the spring 2005 semester, the researcher worked with $20+$ students in voluntary group discussion sessions, to help them to become more successful in technical calculus, overcome deficiencies in algebra, and improve their problem solving skills. The students worked with the researcher to become more familiar with the course material and to become more independent in their own learning by using a method called the three-step method, and an online learning supplement called the Technical Calculus Learning Supplement (TCLS). Many different results came out of the study through both qualitative and quantitative data analysis. For example, students that participated in the study performed significantly better in the course than non-participants, were motivated to learn, and had a positive attitude towards mathematics. The researcher will discuss the difficulties that students face when learning technical calculus, their perceptions on how the discussion sessions and TCLS helped them to perform better in the course, and the benefits students received from the discussion sessions and TCLS. In addition, some of the main results of the study will be discussed briefly.


## Introduction

The calculus reform movement was one of the first movements that began to question how calculus had been taught for years in college and whether calculus was meeting the needs of its audience. The calculus reform movement began in the early 1980's and did not attempt to challenge the content of calculus, but to examine the way calculus was being taught (Long, 2004). The beginning of the movement was concerned with changing calculus from being a filter to a pump for the further study of math, science, and engineering. Long (2004) stated that calculus was "the one class that seemed to be making or breaking students in mathematics and science" ( p .3 ). The calculus reform movement tried to enhance the way students learned mathematics by using pedagogical techniques that were radically different from the norm of how calculus had been taught in the past (Long, 2004).

Marcoff, cited by Douglas in The Importance of Calculus in Core Mathematics, stated "calculus cannot be learned passively." As the subject builds, the student must continually master ideas and techniques in order to profitably continue" (Marcoff, 1985). The National Council of Teachers of Mathematics (NCTM) state in Professional Standards for Teaching Mathematics, that "when students learn from the experience of doing, they are much more likely to retain and use what they have learned" (NCTM, 1991, p. 3). Furthermore, NCTM states "student’s learning of mathematics is enhanced in a learning environment that is built as a community of people collaborating to make sense of mathematical ideas. It is a key function of the teacher to develop and nurture students' abilities to learn with and from others" (p. 58). Moving Beyond Myths: Revitalizing Undergraduate Mathematics (1991) recommends that instructors "explore effective alternatives to 'lecture and listen' " and "involve students actively in the learning process" (p. 34). There is something about being actively involved that engraves things into the minds of students.

## A Background and Statement of the Problem

The statement by Ainsworth (1994), "students who come to college without an adequate background in math will likely withdraw from or quit performing when a math class becomes difficult," was a very
accurate description of some students who enrolled in a calculus course for technology majors called Technical Calculus. The majority of students in technical calculus were majoring in either: 1) Fire Protection and Safety, 2) Electrical or Mechanical Engineering Technology, or 3) Construction Management Technology. One of the problems that the researcher observed over the years of teaching technical calculus is that many of the students have a weak algebra background. In essence, many of the struggles that students face in the course with calculus can be traced back to their lack of understanding of algebra. Ainsworth's statement held true for technical calculus students who withdrew from or quit performing when technical calculus became difficult. The non-success rates during the timeframe from spring 1998 to spring 2005, defined as the percentage of D, F, and course withdraws, ranged from $24.1 \%$ to $60.7 \%$ and averaged $42.98 \%$ with a standard error of $1.38 \%$. Also data was collected for non-success rates for two comparison courses from spring 2002 to spring 2005, and revealed that the average nonsuccess rate for college algebra and calculus was $36.6 \%$ and $45.1 \%$, respectively. It was determined, using a two-tail t-test with $a=0.05$, that there was no significant difference between the average non-success rate for calculus ( $p=.1220$ ) and technical calculus, however there was a significantly difference between the non-success rate for college algebra and technical calculus ( $p=0.0008$ ).

To help students to be more successful in the course and understand the course material better, the researcher designed an online learning supplement called the Technical Calculus Learning Supplement (TCLS). In addition, the researcher worked with students voluntarily at various times in a group setting in order to help students understand the course material and so that students would be introduced to the TCLS. The group sessions were scheduled twice a week for no more than two hours at one time. Two major questions that will be addressed in this paper are: 1) what are the difficulties that students face when learning calculus, and 2) does the online supplement and discussion session significantly help students to be successful? Near the end of the semester the researcher conducted 30 to 45 minute interviews with each participant, non-participant, and a few students that stopped attending the discussion sessions. Themes that emerged from the coded transcribed interviews were: the difficulties that students face when learning technical calculus, their struggle throughout the semester to learn calculus, and the benefits students received from the group sessions and TCLS.

## A Brief Review of the Literature

Supplemental Instruction, denoted SI, is a system that concentrated on offering academic assistance to the whole class through voluntary peer-led discussion sessions instead of the individual assistance that a student could get at his or her college or university academic learning centers. Arendale (1994), Martin, Blanc and DeBuhr (1983), Widmar (1994), and SI staff (1997) discuss the SI model in more depth for interested readers. SI has been used with most courses in college and in particular there have been numerous studies involving SI and mathematics. The dissertation study by Miller (2006) references these studies in more depth and Kenny's study (1998) will be mentioned in this paper. Kenny attached SI to a business calculus class through discussion sessions. Out of six discussion sessions, two were labeled the treatment group and SI was implemented in that treatment group, another two were labeled the control group and SI was not implemented and Kenney did not implement SI in the discussion for the control group, and the last two were not included the study. The results of this study, which agree with the results of many SI studies, were: 1) students who participate in SI earn higher mean course grades (a half of a grade or a whole grade) in the course than students who do not participate in SI, 2) students who participate in SI reenroll in college at a higher rate, and 3) the group of students who participated in SI had a lower level of D or F grades and course withdrawals. In addition to the quantitative studies of SI and mathematics, there have been other studies that have focused on SI and mathematics from a qualitative methodology that are discussed in detail in Miller (2006).

There are many studies in mathematics that have examined collaborating group learning and are discussed in more depth in Miller (2006). One of the most known is the research study by Treisman
(1985). Treisman (1985) developed mathematics workshops for the Professional Development Program (PDP) to help black students succeed in calculus. The "mathematics workshop" emerged from his pilot study where workshop leaders worked with minority students and in particular, black students that were "interested in engineering, medicine, business administration, and other mathematics and scienceintensive professions" (Treisman, 1985, p. 62). For the workshops, Treisman developed problem sets that he called "worksheets" that consisted of difficult problems drawn from old exams and course textbook for the honors section of calculus. One of his main research findings were that "black Workshop students earn grades in Math 1A at a level comparable to class average, and about a full grade point higher than black non-Workshop students; and that black Workshop participants persist to graduation at rates comparable to the campus as a whole, while black non-Workshop students drop out at a substantially higher rate. Currently the workshop model is being used at numerous institutions with very positive outcomes.

## Participants in the Study

The participants for this study are students from Technical Calculus (Math 2123) sections 1 and 2, spring 2005. The students participated in the study on a volunteer basis by signing an individual consent form, after the researcher had clearly articulated the study's purpose and the research expectations to the participants. The participants attended some of the voluntary weekly discussion sessions for the course and were interviewed by the researcher near the end of the semester. In addition, near the end of the semester, the researcher asked non-participants if they would like help studying for their final exam. In return, the researcher asked the non-participant volunteers to agree to an interview with him prior to finals week. There were a total of 25 students who participated in some way to the study, which was comprised of 20 students who attended the discussion sessions throughout the spring 2005 semester, three students who attended some sessions, but stopped coming before the end of the semester, and two students who did not attend the discussion sessions, but agreed to an interview at the end of the spring 2005 semester in return for some help studying for the final exam. The researcher collected demographic information on each participant by using the student information system.

The demographics of the 20 participants that attended discussion throughout the semester with the corresponding demographics of the whole class stated in parentheses are: $90 \%$ ( $87.6 \%$ ) male and $10 \%$ (12.4\%) female; 20\% (21.8\%) freshman, $40 \%$ (46\%) sophomore, $25 \%$ ( $23 \%$ ) juniors, and $15 \%$ ( $9.2 \%$ ) seniors; 90\% (78.16\%) Caucasian, 0\% (3.45\%) African-American, 5\% (11.49\%) American Indian, 0\% (1.15\%) Asian, 5\% (4.60\%) Hispanic American, $0 \%$ (1.15\%) other ethnicity. The average age of participates with the corresponding average age for the whole class in parentheses is: 22.75 years (21.83 years). The demographics of mathematics courses taken in college for all participants with the corresponding demographics of the whole class stated in parentheses are: 75\% (73\%) - college algebra, $90 \%$ (67.4\%) - trigonometry, 5\% (7.87\%) - algebra and trigonometry, 30\% (16.9\%) - intermediate and/or beginning algebra, 25\% (22.5\%) - business calculus and/or calculus, 35\% (17\%) - other math courses, and $10 \%(19.1 \%)$ - technical calculus in a previous semester. All of the participants and $89.9 \%$ of the entire class met the prerequisites for the class.

## Results

Students face many difficulties when learning mathematics. Participants in this study discussed their difficulties in learning calculus with the researcher during interviews. These difficulties coming into the course and during the course had an impact on students' attitude towards mathematics. One difficulty that is not isolated to calculus is that participants believe that math is their weakest subject. One participant stated it this way, "I hadn't had a whole lot of math and math definitely has been my weakest subject. I was dreading taking it (calculus)." This belief or the difficulties in past math classes placed a barrier that students have to overcome when trying to learn in a future math course. Having a fear of calculus is another difficulty that students have to face. There is a mystic surrounding calculus of how hard it is and students communicate how they fear calculus to their classmates. This aurora that is produced results in a
barrier that students have to deal with and overcome. Another participant stated a prime example of this barrier when they stated that "I was really nervous and I have always heard horror stories about how hard calculus was ... So it made (me) really nervous." Two other difficulties that students revealed were that they dreaded taking calculus and that calculus is tougher than other math classes. This is a barrier because many students coming in with low expectations in the course and a built-in excuse why they will not do well in the course. A difficulty that students faced during the course was that they had a hard time working on the homework, studying for exams, or working through the exams during the course. In addition other difficulties were: students found that calculus problems were not as easy as step by step problems in algebra, students had a long elapsed time between a previous math course and technical calculus, students had a hard time comprehending calculus, and students had trouble understanding calculus by themselves. These participant difficulties might be similar to the difficulties that other students face when learning calculus.

Student perceptions on the difference between how well they were doing in the course compared to how well they would be doing without the help of the discussion sessions and TCLS, revealed the impact the discussion sessions and TCLS had on students’ performance in the course. Analyzing the interviews revealed that 15 participants articulated to the researcher clearly that they would have made at least a letter grade lower in the course than their grade perception at the time of the interview. The interview took place approximately two weeks before the final exam and most students had a good idea of their grade in the course. Of the 15 students, nine students believed that they would have earned one letter grade lower, four students believed that they would have earned two letter grades lower, and two believed that they would have earned three letter grades lower in the course. Furthermore, some of the students made comments like "I probably wouldn't be in the class right now because I would have failed it (the course)" or "I would have probably been withdrawn right now." Therefore the discussion session and TCLS went a long way in helping students not only to do better in the course, but in some instances, have enough confidence to earn a non-failing grade or keep from withdrawing from the course. In addition, students ended up staying in the course and performing a letter grade or two better than they thought they could earn. This is reinforced by the fact that no participants failed or withdrew from the course and only two participants (10\%) earned a grade below a C. The reasons why the discussion sessions helped so much is that students would gain an overall idea of how to work problems and would practice different problems individually or in small groups to make sure they understand the concepts in the course. The goals of the discussion sessions were to push students to become more actively involved in working problems and to become more independent learners which in turn helped students be more successful in the course and understand the material in more depth.

Participants and non-participants were given a pre and post-algebra assessment to establish a baseline on algebra proficiency, to determine the gain from pre to post in algebra competency, and determine if there are any differences between the participants’ and non-participants’ algebra skills. The average score on the pre algebra assessment exam for participants and non-participants out of a total of twenty 1-point questions were 9.79 and 9.69 , respectively. It was determined that there was no significant difference in mean pre-algebra assessment scores between participants and non-participants. The participants' and nonparticipants' post-algebra assessment score were 11.80 and 11.30 , respectively, which equates to a gain of 2.01 and 1.61 , respectively. It was determined that there was no significant difference in gain from pre to post-algebra assessment for participants and non-participants. Furthermore, the researcher compared the previous college grade point averages of participants (2.69) and non-participants (2.58) and determined that there was no significant difference in mean college grade point averages between the two groups. Therefore, participants and non-participants entered the study with a similar proficiency in algebra and academic performance and had similar gains in algebra proficiency during the semester.

The researcher quantified the grades $(\mathrm{A}=4.0, \mathrm{~B}=3.0, \mathrm{C}=2.0, \mathrm{D}=1.0$, and $\mathrm{F}=0$; withdraws were not quantified) for all participants and non-participants in the technical calculus class. The researcher
determined that the grades of the participants were normally distributed drawing a normal probability plot. The researcher tested if the mean course grade for the participants (2.47), with standard deviation of 0.77 , was equal to the mean course grade for the non-participants (2.07), with a standard deviation of 1.42 , by using a two sample right tailed t-test assuming unequal variances. The t-statistic came out to be 1.57 with 57 degrees of freedom and the p-value of 0.0610 , so the mean course grade for the participants is significantly different than the mean course grade of the non-participants ( $a \quad 0.1$ ) and would be more significant if withdraws were quantified! Therefore, participants earned a higher mean course grade than non-participants.

Overall students benefited in more ways than just being more successful in the course. Students learned technical calculus concepts by seeing detailed examples worked on the TCLS and during the discussion session and by becoming more actively involved in working problems individually or in small groups. Students could review various examples on the TCLS at anytime to help them when working on homework and preparing for exams. In addition, the researcher used the three-step method to show different problems on the TCLS at the beginning of each discussion session to students, work another problem with the help of students, and have students work from 1 to 3 problems by themselves to make sure they understood the concepts in the course. Students learned that in order to become successful in the course, they had to work hard and make sure they understood concepts well and were comfortable working a variety of problems.

Students had a change in mathematics attitude, and an increased confidence in their ability to "do" mathematics. The change of attitude towards mathematics became apparent during the interviews and during discussion sessions. One way some students had a change of attitude towards mathematics was with respect to the way they approached their homework. Many times students either had a hard time working through their homework or they worked the homework and believed they had mastered the topics. As a result of using the TCLS and the three-step method, some students stated that they had an easier time completing the homework and had better understanding of topics in the course. In addition, some students stated that they had a change in the way that they worked the homework. They realized that in order to do well on exams, they needed to work on all upcoming exam topics continually. Other benefits that participants received from the TCLS and discussion sessions were the ability to finish problems and an increase confidence in their mathematical ability. Students felt like they were better equipped to work through and finish problems on their homework and exams and had more confidence that they would be more successful in the course because of their understanding of the course concepts.

## Conclusion

Active student learning sessions coupled with online resources helped participants to perform better in technical calculus than non-participants who worked on the course material in a more individual manner. The sessions and online resources helped students in multiple ways. Many students, who otherwise might have earned at least a letter grade lower in the class, learned that they could understand mathematics and do well in the course as a result of their own effort, being actively involved in learning the course material in the discussion sessions, and using the online learning supplement. Furthermore, participants had a better attitude towards mathematics because of their experience and benefited other ways from their participation in the study. Many of the participants articulated that they would like more of their college classes to be taught in this way.

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