

The Use of Writing to Teach and Learn Mathematics

Results of Research Testify to Benefits

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Abstract

The use of writing in content areas as a means to enhance the understanding of the content originated in the 1960s as an off-spring of the writing-across-the-curriculum movement in which writing in the content areas was used to improve students' writing skills. Research on writing-to-learn reached its apex in the decade from the mid-80s to mid-90s and has since diminished. Research reports have been mixed with anecdotal evidence supporting its benefits to students, teachers, and the classroom learning environment. The purpose of this review of literature is to summarize the benefits of writing-to-learn as a strategy for teaching and learning mathematics.

Introduction

Nearly forty years ago, Bruner (1968) advised that both writing and mathematics were "devices for ordering thoughts about things and thoughts about thoughts" (112). It is reasonable to believe that pedagogical techniques exist in which these two domains can work together to augment learning; that is, through writing one can learn mathematics and through writing about one's understanding of mathematics a person can become a better writer.

For many years, mathematics educators, mathematicians, and school teachers in several countries have explored pedagogical connections between writing and mathematics, specifically writing as a support to mathematics learning. It is easy to argue that writing benefits the construction of students' mathematical knowledge because it is a reasonable assumption; that is, writing requires people to think and organize their thoughts. Therefore, writing should help people to think about the learning of mathematics.

The purpose of this report is to provide an update on what research says about the use of writing to teach and learn mathematics. It does not include pieces that outline the theoretical underpinnings of how writing assists in the learning process. It focuses on those whose specific intent was to examine writing-to-learn activities in mathematics.

Findings have been categorized by benefits: to students in both cognition and the affective domain (attitudes, anxieties, views or beliefs about mathematics); to teachers through changes in pedagogical content knowledge and their attitudes towards students and views about teaching mathematics; and, to the classroom environment by virtue of improved student-teacher interactions. Several documents reviewed report findings in more than one category, resulting in some overlap in the categories. Limitations of space prohibit an acknowledgement of all 170+ documents read. The studies reported are intended to be a representative sample of the literature reviewed.

Benefits to Students

Believing that one learns from reflecting on experiences, Powell & Lopez (1989) conducted a case study in a developmental mathematics class in which a process-product paradigm was used to investigate the benefits of writing on concept development. Through more than one phase or drafting stage students were able to construct meaning and generate knowledge. Another data set was constructed from the collaborations in the analysis of the student's writings by Powell, the teacher, and Lopez, one of his students. The conclusions reported include: (1) writing is a heuristic tool with which one can negotiate meaning that leads to generating knowledge and learning; (2) reflecting critically, in writing, about the mathematics being learned gives students greater potential to control their learning and to develop criteria for

monitoring their progress towards understanding; and, (3) reflecting critically on one's mathematical experiences, in writing, creates an active learning scenario.

Upper division university mathematics students taking a course in the history of mathematics were divided into two groups by Johnson (1990) who wanted to investigate whether journal writing would assist students in their construction of mathematical knowledge. The experimental group kept journals throughout the course while the control group did not. Students in the experimental group scored marginally higher on an examination covering the first half of the course, but significantly higher than students in the control group on an examination covering the second half of the course. The researcher/teacher concluded that the benefits of writing in college mathematics are more probable if practiced over a long period of time.

Allen's (1991) dissertation research investigated the use of writing in a college-level remedial mathematics classroom. Students' written thoughts expressed that they believed they had understood a concept until they began writing about it. They also expressed an inability to evaluate their understanding of a concept before they encountered it on a quiz or a test. When asked if their efforts were matched by results, the responses disclosed that students can unknowingly work an entire assignment incorrectly. These students indicated they viewed writing as an effective means of self-evaluation. Allen concluded that the use of expressive writing has the potential to promote the self-awareness displayed by an expert learner. The benefit is in assisting students with the metacognitive process of learning.

A study conducted in statistics classes used writing as a means to help students learn interpretation. Beins (1993) asked students in an experimental group to write press releases free of statistical jargon. At the end of the study, he found that these students acquired better computational and interpretive skills than did students in a control group. However, emphasis on interpretation was not associated with greater conceptual knowledge. The merit of the writing assignments was students' improved ability to articulate the meaning of their data analyses.

Clarke, et al. (1993) report a four year study examining the use of journal writing in school mathematics, grades 7 through 11. The results of the Australian study provide a powerful demonstration of the link between language and mathematics and suggest a relationship between students' mathematical writings and their perceptions of mathematics and mathematical activity. The findings of the study are gleaned from students' reports of their behavior, their perceptions, and their beliefs, and their teachers' behaviors. In summary, students were guided through three phases of journal writing: a recount mode in which mathematics was described; a summary mode where connections between mathematics concepts are expressed; and, a dialogue mode where students constructed mathematical knowledge. The results indicate that journal writing leads to a progressive refinement of purpose from an initial state of simply listing events in the mathematics classroom to summarizing work done and topics covered. The process of journal writing implemented in this study resulted in students interpreting mathematics in personal terms, constructing meanings and making connections.

Shepard (1993) reports similar benefits to students' cognition by saying, "the creation of specific writing assignments may be best thought of as following a student's current developmental phase of understanding about a particular content area" (p. 292). Of particular importance is the inclusion of assignments that gradually move students from rote, mechanical learning to more abstract and integrative learning.

The impact of writing on students' attitudes, anxieties and beliefs about mathematics has also been investigated. Children's perception of mathematics is influenced by interaction with others in the home, the playground, and the classroom. Just as children are engaged in constructing mathematical meaning from the experiences and interactions that confront them, so they are constructing and establishing perceptions about mathematics. Ellerton (1988) asked

children to write a letter to a friend and explain what mathematics had been covered while the friend was away from school. Through their responses, the children revealed their perception of the mathematics and the teaching style they encountered at school. The lack of detail in many of the explanations was interpreted as either a lack of understanding on the children's part or as closely linked with the children's low self-esteem and interest in mathematics. Ellerton concluded that children's writings provide a good indication of their feelings and perceptions about school mathematics.

The results of pre- and post-survey data and the observed changes in students' attitudes convinced Ropp (1989) that writing in mathematics classrooms will result in benefits to students and teachers. Two types of writing were implemented in her college level mathematics course, prompts that solicited short, direct answers to determine students' understanding of a particular problem or question and journal writing. The results showed that students' attitudes towards learning mathematics improved.

Implementing techniques of ethnographic research, Borasi & Rose (1989) report an overview and categorization of the complementary ways in which journal writing can contribute to improvements in the affective domain. Potential benefits for students include a therapeutic effect on the emotional components of learning mathematics and a more appropriate view of mathematics by writing about and reevaluating their beliefs on the nature of the discipline.

The purpose of a study conducted by Shepard, Perry, Shepard and Frye (1993) was to determine the effects of writing on gender differences in attitudes and learning mathematics. Two groups of eighth grade general mathematics students were given identical instruction over a unit on number theory except for their homework assignments. A control group was given traditional drill and practice type problems to solve while the treatment group received a series of writing assignments. The effect of writing on attitude was not statistically significant for male or female students in either group. However, statistical analysis indicated that females' construction of mathematical knowledge benefited more from writing while males performed best under traditional learning conditions.

In summary, writing about learning mathematics and the experiences one encounters can have therapeutic value towards improving attitudes, alleviating anxieties, and clarifying one's beliefs about the discipline.

Benefits to Teachers and Classroom Environment

One field of knowledge important to effective teaching is pedagogical content knowledge. With experience, teachers become aware of how students comprehend or typically misconstrue mathematical concepts, skills, and generalizations. They become aware of common misconceptions and buggy algorithms constructed by students and of the stages of understanding that students are likely to pass through in moving from a state of having little understanding of a topic to the mastery of it. It is important for mathematics educators to find ways to expedite the development of pedagogical content knowledge.

Ellerton (1986) studied children's made-up mathematics problems to see if this type of creative writing would provide teachers a glimpse into the level of mathematical understanding a student possessed. The children, ages 11 to 13, were asked to make up a problem that would be difficult for a friend to solve. Ellerton reports that this type of writing is a rich source of information about children's use of mathematical language. If language is the conduit through which mathematical knowledge is constructed, then teachers need to use writing tasks to learn more about students' use of language.

Linn's (1987) unpublished master's thesis reports benefits to teachers because they were able to identify misconceptions and assess students' progress towards understanding by reading students' journal entries. The purpose of the project was to determine the effects of journal

writing on the thinking skills of high school geometry students. The results showed that the students' journals served effectively in various capacities. The benefits included: (1) each student became actively involved in his or her own learning process; (2) writing forced the students to synthesize information which led them to become more aware of what they did and did not know; (3) the journals served as a diagnostic tool for the teacher; (4) the writings opened lines of communication between teacher and students that had not previously existed; and, (5) the improved communications contributed to a more personalized learning environment.

In a study of secondary algebra classes, Miller (1991 & 1992) found that students' writings about their understanding of mathematics can accelerate the development of novice teachers' pedagogical content knowledge. The study used in-class, impromptu writing prompts to investigate what teachers learn about their students' understanding of school mathematics and how instructional practices are influenced as a result of reading students' writings. Miller concluded that the teachers' assessment of students' understanding of school mathematics was enhanced and their instructional practices were changed as a result of reading their students' responses to impromptu writing prompts.

Gordon and Macinnis (1993) worked with mathematics teachers in grades 4 through 6 to see if dialogue journal writing would gain teachers' insights into students' thinking about their mathematical learning. Using writing prompts and free writing, students made daily entries in their journals. The teachers read and responded to all entries. Through the journals, the teachers began to understand better what was happening in the classroom by "listening" and observing in a more focused way (here "listening" was accomplished by reading journal entries). As a result of this watching and listening, the teachers and students became a community of learners. As the students chronicled their frustrations and their confusions in their journals, the teachers altered their lesson plans to address the students' needs. The teachers saw the journal entries as a realistic way to "listen" to all the students individually. Information gleaned from the journal entries allowed guidance and instruction on a more individualized level.

Using an action research paradigm, Stewart and Chance (1995) investigated how journal writing influenced the learning environment in a first-year algebra classroom. The experimental group, where students wrote three days a week, was compared to two classes that did not engage in journal writing. The study lasted one academic year. Statistical and interpretative data analyses resulted in the researchers concluding that writing had a positive influence on the learning environment because students' mathematical understanding improved, their interest in mathematics increased, their mathematics anxiety decreased, student-teacher discourse increased, and the classroom became more student-centered and collaborative.

On-going work that will provide additional information about the benefit to students' cognition and the development of pedagogical content knowledge is being done by Kelly-Kembitsky (2006). Her research asks students do their own error analyses on quizzes and examinations. Writing about their mistakes or errors may help students and teachers gain a better understanding about the teaching and learning process.

According to the literature reviewed, the benefits of writing-to-learn for teachers were positively correlated to the development of pedagogical content knowledge and benefits to the overall classroom environment.

Conclusions

The results of research testify that writing is a beneficial tool for teaching and learning mathematics. It can be used as a vehicle to inform teachers about their instructional practices and how students are learning from these practices. Writing in mathematics takes more time than lecturing and some other methodologies, but if teachers value the notion that it is important to teach students more about less, then writing in mathematics is worth the time invested.

Writing-to-learn activities can benefit students' construction of mathematical knowledge and assist in building higher levels of understanding. Writing-to-learn activities can improve students' attitudes, reduce anxieties, and positively influence the views and beliefs that students have about their study of mathematics. Various reports indicate that the construction of pedagogical content knowledge can be enhanced and accelerated by reading students' responses to writing activities. All of these benefits, collectively, can have a positive influence on the classroom as a learning environment.

Writing-to-learn activities are not the cure all for mathematics students and teachers, but like any other instructional and learning strategy, writing will be highly effective for some students and teachers and should not be easily dismissed. Time constraints are often noted as the reason teachers do not ask students to write about their understanding of mathematics. Also, students complain about being asked to write in mathematics class. The results of research testify that the gains outweigh the pains for some teachers and some students.

References

- Allen, B. R. (1991, July). A study of metacognitive skills as influenced by expressive writing in college introductory algebra classes. Unpublished Doctoral Dissertation, Louisiana State University, Baton Rouge.
- Beins, B.C. (1993). Writing assignments in statistics classes encourage students to learn interpretation. *Teaching of Psychology*, 20(3), 161-164.
- Borasi, R., & Rose, B. J. (1989). Journal writing and mathematics instruction. *Educational Studies in Mathematics*, 20, 347-365.
- Bruner, J.S. (1968). *Toward a theory of instruction*. New York: W.W. Norton.
- Clarke, D.J., Waywood, A., & Stephens, M. (1993). Probing the structure of mathematical writing. *Educational Studies in Mathematics*, 25, 235-250.
- Ellerton, N. F. (1986). Children's made-up mathematics problems – A new perspective on talented mathematicians. *Educational Studies in Mathematics*, 17, 261-271.
- Ellerton, N. F. (1988). Exploring children's perceptions of mathematics through letters and problems written by children. In A. Borbos (Ed.), *Proceedings of the twelfth international conference for the psychology of mathematics education* (Vol. 1, pp. 280-287). Veszprem, Hungary: International Group for the Psychology of Mathematics Education.
- Gordon, C., & Macinnis, D. (1993, January). Using journals as a window on students' thinking in mathematics. *Language Arts*, 70, 37-43.
- Johnson, P. E. (1990). Enhancing learning by using writing in history of mathematics. *International Journal of Mathematics, Education, Science, and Technology*, 21(2), 259-263.
- Kelly-Kembitzky, K. (2006, April). Different ways of integrating writing into mathematics class. Paper presented at the annual meeting of the National Council of Teachers of Mathematics, St. Louis, MO.
- Linn, M. M. (1987). *Effects of journal writing on thinking skills of high school geometry students*. Unpublished master's thesis, University of North Florida, Jacksonville.
- Miller, L. D. (1991). Constructing pedagogical content knowledge from students' writing in secondary mathematics. *Mathematics Education Research Journal*, 3(1), 30-44.
- Miller, L. D. (1992). Teacher benefits from using impromptu writing prompts in algebra classes. *Journal for Research in Mathematics Education*, 23(4), 329-340.
- Powell, A.B., & Lopez, J.A. (1989). Writing as a vehicle to learn mathematics: A case study. In P. Connolly & T. Vilardi (Eds.), *Writing to learn mathematics and science* (pp. 157-177). New York: Teachers College Press.
- Ropp, C. (1989, April). Writing as a tool for learning in the mathematics classroom. *The Illinois Mathematics Teacher*, 40(2), 10-14.
- Shepard, R. G. (1993). Writing for conceptual development in mathematics. *Journal of Mathematical Behavior*, 12, 287-293.
- Shepard, R. G., Perry, C. D., Shepard, L. L., & Frye, M. (1993, April). *Gender differences in using writing to learn in mathematics*. Paper presented at the annual meeting of the American Educational Research Association, Atlanta, GA.
- Stewart, C., & Chance, L. (1995). Making connections: Journal writing and the Professional Teaching Standards. *The Mathematics Teacher*, 88(2), 92-95.