Keeping All Students on the Learning Path

Judy Mousley, PhD
Deakin University
Geelong
Australia
judith.mousley@deakin.edu.au

Peter Sullivan, PhD
Monash University
Melbourne
Australia
peter.sullivan@education.monash.edu.au

Robyn Zevenbergen, PhD
Griffith University
Gold Coast
Australia
r.zevenbergen@griffith.edu.au

Abstract
A powerful notion to guide thinking about whole-class mathematics teaching is Vygotsky's zone of proximal development (ZPD). Our research with primary and secondary teachers over the last six years has identified roles of teachers in relation to the ZPD, and ways of overcoming some typical barriers to students' movement through their zones. Methods have included focus groups of experts, video analysis of classroom interactions, classroom observation, and analysis of lesson plans and teachers' reflections teaching processes their outcomes. The research has involved the gradual development, trailing, evaluation, and adjustment of a six-component model for planning and teaching mathematics. The focus of this paper is on the use of one of its components, "differentiated learning trajectories".

INTRODUCTION

An underpinning principle of Vygotsky's theorising was one of unity between mental functioning and activity, with the development of the mind resulting from goal-oriented and socially determined interaction between human beings, their tools, and their environments. Vygotsky's key to learning is participation: "[The] only way … to acquire knowledge, is by doing so, in other words, by acquiring knowledge" (Vygotsky, 1997, p. 324). He described learning as movement through a series of zones of proximal development, and defined the ZPD as the "distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined by problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978, p. 86). Here, the "social dimension of consciousness is primary in time and in fact. The individual dimension is derivative and secondary (Wertsch, 1985, p. 58). Thus Vygotsky's work on processes of development contrasted at a foundational level with constructivism, in that internalisation was not seen as a course of plotting external reality on to a pre-existing cognitive structure, but as a process of learning to use the intellectual tools provided in social contexts.

Vygotsky's theory implies that the challenge for all teachers is to pose problems that most students are not able to do so, that students learn mathematics by solving problems through their own thinking, but to support those who are not ready for the level of independent problem solving required by the task. One way to support this development is through scaffolding, which involves structuring the ideas to be understood in an order that is likely to lead children to develop further and faster than they would on their own (Bruner, 1996). By introducing words, ideas and learning activities in a logical order, the teacher draws on what he or she knows the learner can do, gradually building bridges "from children's current understanding to reach new understanding through processes inherent in communication" (Rogoff, 1991, p. 351). Planning for such a process involves imagining a learning trajectory, and this is the basis of mathematics lesson planning. Even very open-ended inquiry-based lessons have starting points and target learning objectives where the aim is to teach aspects of mathematics.

To describe any proposed learning pathway that lies between children's current understandings and the new knowledge that it is believed that they need to learn, Simon (1995) used the term "hypothetical learning trajectory".

Learning Trajectories

Planning for lessons involves teachers making predictions about how students might learn specific mathematics curriculum content. Simon (1995) explained that it involves "a prediction of how the students' thinking and understanding will evolve in the context of the learning activities" (p. 136) and that this forecast also drives decision-making during a lesson.

A hypothetical learning trajectory provides the teacher with a rationale for choosing a particular instructional design; thus, I [as a teacher] make my design decisions based on my best guess of how learning might proceed. This can be seen in the thinking and planning that preceded my instructional interventions … as well as the spontaneous decisions that I make in response to students' thinking. (Simon, 1995, p. 135)

Since Simon coined the term, "learning trajectory", this has become a common way of describing the transformation of learning that results from participation in mathematical learning activities over time. It may refer to learning that takes place over longitudinal curriculum strands across a range of grades, such as the sequence of ideas from initial fractioning of wholes through fraction operations, leading to percentages and ratio. It is also used for a series of lessons, such as a week or two's work focused on percentages. A further interpretation is a trajectory of activity for one whole lesson, and another revolves around a particular learning activity that may use only part of one school mathematics class. We use the term flexibly, incorporating all of these ideas, using the term as a general concept.

Any proposed learning trajectory can only be hypothetical because experienced teachers make decisions and adapt aspects of planned activities in response to evidence of students' thinking and learning. Different aspects and levels of understanding become apparent as teachers conduct on-going evaluations of students' performance of classroom tasks. A concern that we have is that teachers are not always aware of potential barriers, and they rarely plan what to do when if students meet different types of barriers along the way. Potential barriers and how to overcome them have been the focus of our work over the past six years.

OVERCOMING STRUCTURAL BARRIERS TO MATHEMATICS LEARNING

With the support of the Australian Research Council and the Victorian Department of Education, we have aimed (a) to identify and describe aspects of mathematics pedagogy that create barriers to learning mathematics; (b) to design, implement and measure teaching approaches that attend to desired mathematical outcomes in ways that address the learning needs of a wide range of students; and (c) to determine the extent to which such approaches address the needs of particular student groups.

Open-ended questions

Throughout the research period, we have used "open-ended" questions because of their potential to be inclusive of a range of students' abilities. Sullivan, Clarke and Wallbridge (1991) found that pupils with various levels of competency can respond effectively to open-ended tasks, and that typically they provide different types of learning opportunities for a range of students. Let us give an example.

A conventional classroom task is: *Find the mean (average) of 3, 4, 8, 12 and 15.* One corresponding open-ended task is: *Write down a set of 5 numbers with a mean of 10.* Tasks such as the latter are useful for mathematics teaching in that they can be used with students of varying abilities. Further, use of such problems allows students to investigate, make decisions, generalise, seek patterns and connections, communicate, discuss, and identify alternatives that are within their ZDP. It has been shown that such social interaction around open-ended tasks can engage students in productive exploration (Christiansen & Walther, 1986), enhance motivation through increasing the students' sense of control (Middleton, 1995), assist in building robust knowledge structures (Sinclair, 1990), and contribute to teachers' appreciation of mathematical and social learning (Stephens & Sullivan, 1997).

Differentiated learning trajectories

In many of classrooms, there seems to be an assumption that all children will follow the same learning trajectory, from relatively common current knowledge to a learning goal that is the target of the lesson. Common alternatives, of course, are "ability grouping", "setting" or "streaming" with their self-fulfilling prophecy effects (Brophy, 1983). In our experience, most teachers who use groups in this way are aware of these potential effects. What we have been experimenting with is the use of "differentiated learning trajectories". That is, open-ended problems that allow all students to make a start are used as the initial stage of a lesson, and if students meet a barrier, they undertake a short activity (that we call an "enabling prompt") that enables them to learn what is needed and then proceed on the planned trajectory.

This means that teachers need to think of potential barriers when planning the lesson, and consequently to plan appropriate prompts. Teachers also need to plan, as part of the task differentiation that caters for a range of students, extending prompts: activities that allow some students to be gainfully employed exploring the same task at higher levels. For example, for the students who complete the original task quickly, the teacher might pose some challenging, additional questions. We hypothesized that this would allow teachers to maintain a sense of the class as a coherent learning community, even though the tasks posed would have been adapted to individual students' needs. With such an approach, the whole class would still be expected to works on the same basic task, and all pupils could participate in discussion and class reviews, and (most importantly) have enough knowledge in common to use as a basis for the ZPD in the subsequent lesson.

It is easiest to explain these ideas and terms with reference to an example, but we should first outline the methods used in this research.

Methodology

We have focused our research mainly on upper primary and junior secondary levels because of their potential for the results to be most easily transferred to other levels. More recently, we have used older secondary and younger primary classrooms as well. We have worked with teachers in schools where there is a mix of SES pupil backgrounds. In all, about sixty teachers were involved.

The data collection was guided by a framework, developed from Clark and Peterson (1986), that has teacher beliefs and understandings interacting with the opportunities, constraints, intentions and actions. The research approach is a combination of (a) interpretive analysis of teaching and teacher development, and (b) broader quantitative data collection. Focus groups and classroom-based research have been used, but this paper reports only on the latter. Here, teaching experiments using the processes established by Sullivan, Bourke, and Scott (1997) were carried out, involving modelling and coaching. That is, initially the researchers prepared lessons for the teacher, and taught some to illustrate principles such as "explicit pedagogy", then the teachers took over the planning of the lessons, some of which were observed. (See Guskey, 1986, and Clarke, 1994, for a rationale for this modelling and coaching approach.)

In each classroom, one researcher worked with the teacher, with collaborations following a reiterative approach similar to the action research cycle. Each researcher and teacher met periodically to discuss the implementation of new pedagogical features, and a record of these discussions was transcribed. In each classroom, naturalistic observations of the implementation were recoded, using a format developed as part of the investigation.

Research questions included ways that the strategies described above can be implemented easily in mathematics classrooms, whether teachers find them easy to use, what additional demands they place on teachers, and ways in which they change the teaching and learning experience. In particular, we wished to identify ways that explicit pedagogy and task differentiation may improve outcomes for students from lower socio-economic backgrounds.

AN EXAMPLE: THE “AREA AS SQUARES” LESSON

The goal of the model lesson for a Grade 4–5 that we called “Areas as squares” was for students to use squared paper to gain a sense of area as covering in square units, and to have them count units that can assist in calculating areas of shapes.

To introduce the idea of area being measured as squares, the lesson first had the children draw, on squared paper, letters of the alphabet using exactly ten whole squares and to think about the idea of working out the area of shapes by counting squares. As a first enabling prompt for students experiencing difficulty, some squared paper with the letter L already drawn was available. Also available as a more basic prompt were some square counters so that the teacher could ask children still experiencing problems to make a letter using ten counters. Students who finished the task quickly were given the challenge of making their name using letters of area ten units.

The students were then asked to draw, on squared paper, other letters of the alphabet using a total of ten squares, using some half squares to make the letters easier to read. As enabling prompts, squared paper with a letter O drawn on it was available, and also some whole and some half square counters for children’s use. Students who finished quickly were asked to choose the hardest letter they could think of and draw it in different ways, then to make some words with each letter having an area of ten units.

The students then completed a worksheet. The first worksheet task involved working out the area of some given colored rectangles. As a first prompt, students having trouble were given a worksheet with all the lines in the first rectangle shown, dividing it into rows whose units could be counted. The second task was to work out the area of some triangles. The final challenge for the students was to draw many different triangles with an area of twelve square units.

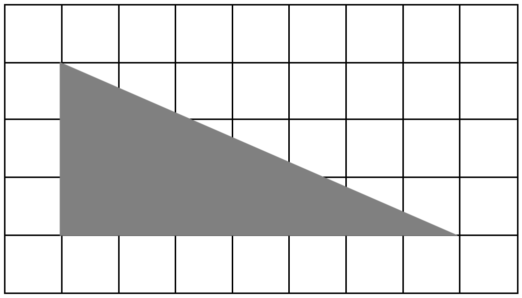
There was class discussion after each task, with children explaining what they had done and teacher’s questioning to draw out salient points.

Virginnia’s Lesson

Before the lesson, Virginnia had said that most of her Grade 5-6 students would need all of the prompt sheets. “They won’t be able to do that” (find the area of a rectangle), she said. However, when the children were asked to draw a letter using ten squares, none asked for help so the enabling prompts were not needed. Her pupils also all coped well with the activity where they could use both squares and half squares. Virginnia circulated, challenging but not helping: “Fabulous. Have a go at drawing a z again, but a different way”.

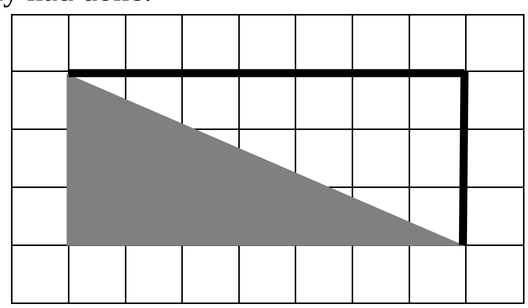
When introducing the “area of rectangles” task, Virginia pointed out that there were some “help sheets” available, and she left them in a pile on the carpet. Four children chose to use these enabling prompt sheets and helped themselves—one immediately, and three after thinking about the task. These prompts provided enough of a stepping stone for them to complete the original task independently. The children’s methods were varied but successful. For example, Frank really surprised Virginnia when he counted 18 rows of 7 squares then multiplied 18 x 7 quickly and correctly mentally; and when asked how he worked it out said, “I know seven nines. Then I doubled it”.

The most challenging part of the worksheet was to work out the area of a right-angled triangle.



Jack had counted the whole squares, and counted parts of squares as halves. Virginnia was circulating, and asked, “Are they exact halves? They seem to be different shapes, not halves. Some are more like quarters. Can you think of a way to work out exactly what the total is?” Jack pointed to coloured bits in adjacent squares along the hypotenuse: “That bit looks like it goes with that one. Them two—that one”.

Virginnia commented that she thought he was guessing, and he admitted it. She then said to the whole class “Hands up if you are up to the last triangle. No guessing. You have to try to work out exactly how many squares the area is. I’ll give you a clue. Try to use what you have been doing with the rectangles to help you.” A child, Dorothy, said “Oh!” Within a few minutes, many children were saying, “It’s ten and a half”. One child explained to another that, “You make it into a rectangle and then take half.” After a few minutes, most had drawn the full rectangle, just as Dorothy had done:



Many of the children finished the sheet quickly and sat on the carpet in front of Virginnia to show her their work. She chatted quietly with them about what they had found until most of the class had joined them. Virginnia asked a few children to explain why they had drawn a rectangle and how they had then calculated its area. Most had multiplied length by width then had used half of the product.

Frank had gone ahead with drawing other rectangles and triangles, recording their areas accurately. He had realised independently that the triangles were half rectangles, so created many of them with ease. During the whole class discussion, he had been able to explain to Virginnia which sections of the triangle joined to make one square unit, and why. Virginnia said, “It’s like Dorothy’s way, isn’t it? Half the rectangle?” Jack responded, “Yes. You turn the black triangle around to make the rectangle. The black triangle and the white triangle are the same, and (pointing) these bits fit together.”

Of interest to us was the effect of the short verbal prompt that Virginnia gave when the Dorothy had met a barrier: “Try to use what you have been doing with the rectangles to help you”. This was possible because the tasks had been scaffolded with this connection in mind. The fact that it was followed by Dorothy’s very public “Oh!” had drawn more children’s attention to it. The children heard such prompts and worked with them: they were been engaged in a learning community.

By the end of the planned learning trajectory were all able to draw rectangles of a given area—the concept that was to be used in the following lesson.

CONCLUSION

We have found that carefully scaffolded open-ended tasks, with accompanying enabling and extending prompts readily at hand for use as needed, can facilitate the successful movement of whole classes of students along planned learning trajectories in mathematics lessons. Thus, the class remains a coherent learning community based on common mathematical experiences and ideas.

In our major research reported briefly here, we have found that it is possible to predict or identify aspects of mathematics pedagogy that are likely to create barriers for some children's learning of mathematics. We have implemented approaches that address the learning needs of a wide range of students, and have extensive evidence of student progress and teacher satisfaction. We have noted that the teaching model developed addresses the needs of particular student groups.

The concept of mentoring students through a zone of proximal development—whether it is an individual's ZPD or a whole class ZDP—has been a useful way of thinking about the creation of differentiated learning trajectories.

Reference

Bruner, J. S. (1996c). *The culture of education*. Cambridge, MA: Harvard University Press.

Christiansen, B., & Walther, G. (1986). Task and activity. In B. Christiansen, A. G. Howson & M. Otte (Eds.), *Perspectives on mathematics education* (pp. 243–307). Holland: Reidel.

Clarke, D. M. (1994). Ten key principles from research for the professional development of mathematics teachers. In D. B. Aichele & A. F. Croxford (Eds.), *Professional development for teachers of mathematics. 1994 Yearbook of the National Council of Teachers of Mathematics* (pp. 37–48). Reston, VA: NCTM.

Guskey, T. R. (1986). Staff development and the process of teacher change. *Educational Researcher, 15*(5), 5–12.

Middleton, J. A. (1995). A study of intrinsic motivation in the mathematics classroom: A personal construct approach. *Journal for Research in Mathematics Education, 26*(3), 254–279.

Simon, M. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26,* 114–145.

Sinclair, H. (1990). Learning: The interactive recreation of knowledge. In L. P. Steffe & T. Wood (Eds.), *Transforming children's mathematics education: International perspectives* (pp. 19–29). Hillsdale, NJ: Erlbaum.

Stephens, M., & Sullivan, P. (1997). Developing tasks to assess mathematical performance. In F. Biddulph & K. Carr (Eds.), *People in mathematics education. Proceedings of the 20th Conference of the Mathematics Education Research Group of Australasia* (pp. 470–477). Rotorua: MERGA.

Sullivan, P., Clarke, D. J., & Wallbridge, M. (1991). *Problem solving with conventional mathematics content: Responses of pupils to open mathematical tasks (Mathematics Teaching and Learning Centre Report No. 1)*. Oakleigh, Vic: Australian Catholic University.

Vygotsky, L.S. (1997). *Educational psychology*. Boca Raton, FL: St Lucie Press. (Original work written 1921–1923 was unpublished).

Vygotsky, V. (1978). *Mind in society*. Boston, MA: Harvard University Press.

Wertsch, J. V. (1985). *Culture, communication and cognition: Vygotskian perspectives*. Cambridge: Cambridge University Press.