# A Cognitive Gap Between Formal Arithmetic And Visual Representation In Fractional Operations

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**Abstract**

The purpose of this study was to investigate the achievement of fourth grade students on fractions via formal arithmetics and visual representational point of view, according to gender and to obtain the relationship between formal arithmetics and visual representation of fractional operations. According to findings, it was observed that boy students were more successful in formal arithmetical view of fractions whereas there was no meaningful difference in girl students' success in fractions from formal arithmetical and visual representational point of view. According to this investigation, it was found that, there was a cognitive deficiency between formal arithmetics and visual representation on fractions. It was seen that, the success level of boy students and girl students is extremely low in fractions, and there is no meaningful difference between their success levels.

## Introduction

The concept of fractions is one of most difficult mathematical concepts for elementary school students. The former studies (Hart, 1981; Aksu, 1997; Lachance, Confrey, 2002; Davis, 2003 ) determined that the students have some difficulties on understanding the concept of fraction at every level of class. The main cause of these difficulties is the structure of fractions and teaching them. (Aksu, 1997). Extending the ideas concerned with the natural numbers, students work hard to learn the rational numbers and fractional numbers (Olive, 1999; Ni, Zhou, 2005). But they don't use their knowledge and experience on natural numbers while learning fractional numbers. It is known that the concept of natural numbers was derived from counting and the concept of fractional number was derived from measuring. Natural numbers and counting process support each other, contrary to this fact, fractional numbers are dense. The uniqueness on visualisation of natural numbers can't be seen in fractional numbers. These properties make hard to understand the order and equivalence of fractional numbers (Steiner, Stoecklin, 1997; Steencken, Maher, 2003; Stafulidou, Vasniadou, 2004; Ni, Zhou, 2005). Most suitable methods on teaching fractional number are to use the rules or to make visual representations. A popular mistake is to have the students make calculations with fractional numbers before their background reaches to some extend (Mack, 1990; Aksu, 1997). However, rules of fractions become the focus of easy learning result an artificial success. Focusing to the rules of fractions does not always mean that students learn the logic of the calculations. Many elementary or high school students could easily divide two fractions but it is not understandable for them why they multiply with the inverse of the second fraction. Most recent progresses on mathematics education cover teaching mathematics via forming the concepts by students instead having them memorised (Arcavi, 2003)

Many studies verify problems on learning fractions encountered by students, especially, when fractions and fractional operations aren't related to concrete experiences (Keijzer, Terwel, 2003; Garofalo, Sharp, 2003). To investigate the meaningless rules in calculations with fractions Hart, (1987) applied formal arithmetic rules concerned with fractions on a large group of elementary school students. The study showed that there exists a great gap between rules of calculation concerned with fractions and their applications.

The aim of this study is to introduce forth grade student's special tendencies on fractional operations. Besides, the other aim is to investigate the relationship between formal arithmetic and visual representations.

Mathematical comprehension is considered as an important target of education. It was known from many studies that students have difficulties applying the mathematical comprehension on mathematical modeling and formal mathematics. The transition from formal mathematics to visual representations and vice versa could not be easily managed by the students. By the help of modeling fractions, the transition from concrete experiences to reasoning gets simpler. This situation could be related to the use of circular, rectangular models, number lines and etc., on the visual representations of fractional numbers (Streefland, 1990; Steiner, Stoecklin, 1997).

Bright, Behr, Wachsmuth (1988) investigated the exact indication ways of fractions on the number line and this study showed that this indication is the superior one. Besides, Streefland (1990) showed that usage of number line as a model for fractions wasn't suitable neither on formation of language for fractions nor on investigation of fractions as an illustration. The students need to form a connection between formal arithmetic and visual representations. Full attention to the rules of fractions is not suitable due to the application skills to be lost quickly. (Aksu, 1997). Learning fractions, limiting to mathematical concepts as piece-total, division-ratio, operator-measuring and basic concepts form informational complexity for students ( Kieren, 1993; Olive, 1999;). All these difficulties cause students to make operations instead of understanding mathematical concepts and operations of fractions (Steiner, Stoecklin, 1997). Memorizing rules, concepts and lack of knowledge of basic concepts brings the diffulties in using the knowledge.

**Method**

The sample of this research was obtained from forth grade classes at different primary schools in Eskisehir on autumn semester at 2005-2006 educational year. 73 students (41girls and 32 boys) participated in this research. 30 open ended questions were asked to the students. These questions concern with operator, ratio, part whole, equivalent, and measure. These questions were directed to determine the success of students in learning fractions according to formal arithmetic and visuality. Students were to reply these questions and to write how they find the results of questions concerning with visuality. All answers and all the operations were examined. Some of the questions investigated can be seen below.

A.Operator

1. Explain whether the following statement is correct " if we divide a number by five and then multiply the result by 2 we are going to get the same result we would get if we multiplied this number by $\frac{2}{5}$".

2. Find $\frac{2}{5}$ in below..

5. $\frac{2}{3}+\frac{3}{5}=?$ Find the result of this operation

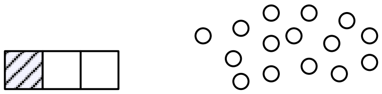
6. $3.\frac{2}{5}=?$ Find the result of this operation.

B. Equivalent fractions

Point to the equivalent fractions

8. $\frac{3}{4}=\frac{?}{8}$, $\frac{2}{?}=\frac{10}{15}$

9. Use the diagram on the right to represent an equivalent fraction to the one presented on the left.



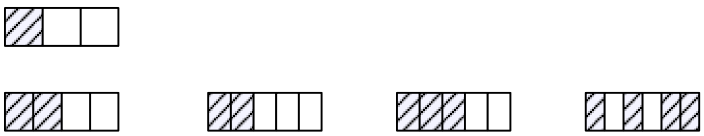
C. Ratio

10. $\frac{1}{2}, \frac{4}{5}, \frac{100}{500}$ Which fractions is the biggest?

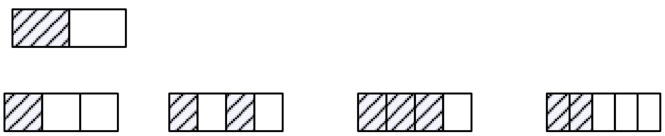
11. 5 girls share 4 apples and 3 boys share 2 apples. Who gets more apples, a girl or a boy?

D. The part-whole

17. Which of the followings correspond to the two times of shaded part ?
Please explain your answer.



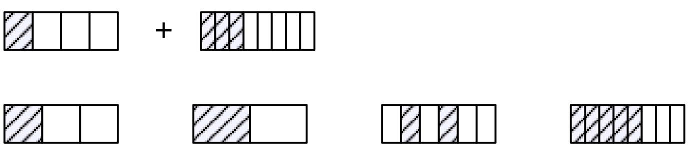
22. Which part is equal to shaded part below? Please explain your answer.



24.

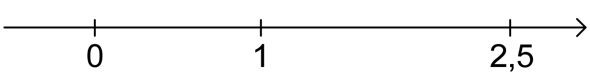
o o
o o  This is $\frac{2}{3}$ of a set of balls. Draw the set of balls.

27. Which is the addition of shaded parts ? Please explain your answer.



E. Measure

29. Show $\frac{3}{2}$ on the number line below.




30. Which of the followings correspond to $\frac{3}{4}$ ?

## Findings

Generally, the questions with four operations in formal arithmetic were answered by girl and boy students. Partially, successful results were achieved in addition and subtraction of fractions which have equal denominators. But in addition and subtraction of fractions with different denominators, adding their dominators and their denominators or subtracting them were examined. Lamon (1999) claimed that fractions can not be seen as numbers as they are not part of a counting sequence. The resistance to accept the fractions as numbers leaded the students to conceptualize fractions as two different whole numbers which concluded a misconception that often results in computational bugs, such as $\frac{2}{3}+\frac{3}{5}=\frac{2+3}{3+5}=\frac{5}{8}$. In this type of questions, success rate of girl students was 26% where as success rate of boy students' was 41%.

Naturally, the most important concept in addition and subtraction of fractions is their denominator's equality. It was observed that this is the part which the students had difficulties to understand and seems to be the origin of the mistakes. Researches related to this subject verified this result (Streefland, 1990; Howard, 1991; Nowlin, 1996; Steencken, Maher, 2003; Davis, 2003; Şiap, Duru,2004 ).

The most usual mistake is the multiplication of a fraction with a whole number:

The answer of the question $\frac{2}{5}.3=\frac{2.3}{5.3}=\frac{6}{15}$. The success rate of this question was 13.8 % for girl students, 20.3 % for boy students.

Where as the correct answer of the question $\frac{3}{5}.\frac{2}{4}=\frac{6}{20}$ was found 100 % by all students.

According to this result, the transition from natural numbers to fractional numbers can not be fully maintained by the students.

Another type of problem which students have difficulties is finding out the greatest and the smallest fraction in a given set:

$\frac{1}{2},\frac{4}{5},\frac{100}{500}$ Which fractions is the biggest?

$\frac{100}{500}$ was the most popular answer . "since 100 is greater than 1 and 4, $\frac{100}{500}$ is the biggest fraction", "since $\frac{100}{500}$ has more parts than the others, $\frac{100}{500}$ is the biggest fraction" or "$\frac{100}{500}$ must be biggest because it has larger numerator and denominator than $\frac{1}{2},\frac{4}{5}$" or "between the 100 and the 500 in $\frac{100}{500}$ gap is 400, while $\frac{1}{2}$ and $\frac{1}{4}$ have a gap of 1, and 3 making $\frac{100}{500}$ the biggest", or "$\frac{100}{500}$ is the biggest fraction was marked or not any explanation, or no answer was given. Another typical mistake was $\frac{1}{2}$

which was because of the sense “since its denominator is the smallest, $\frac{1}{2}$ is the biggest fraction”. Ratio of right answers was 17% for girl students, 21% for boy students.

The wrong answers to this question were originated from the absence of the concept of fractions in students.

In the questions connected with visuality, it was observed that the students marked the answers without any explanations. Main cause of this situation is the multiple choice exam type which can be seen in every grades in Turkish schools. In this group, other cause is that the student does not find the solution by a geometrical representation instead the result is achieved by making arithmetical operations. The research by Lehrer, Strom, Confrey (2002) verifies this result.

While data analysis, reliability modules, dependent t tests, matched t tests of SPSS were used. For girl students, formal arithmetic average was 33.16, visuality average was 33.66, t=-0.25 was located between lower limit -4.67 and upper limit was 3.67 with the confidence interval 0.95. According to the results of girls, no meaningful difference could be found between formal arithmetic average and visuality average from statistical point of view.

For boy students, formal arithmetic average was 34.61 and visuality average was 18.46, t=10.09 was located between lower limit (12.85) and upper limit (19.45) with confidence interval 0.95. According to the results of boys, a meaningful difference could be found between formal arithmetic average and visuality average from statistical point of view.

According to the results obtained, when the success of students in fractional operations was evaluated from formal arithmetical and visual points of view, it was statistically shown that the arithmetical grades mean was greater than visual grade mean at confidence of 95 %. A positively directed statistically meaningful relation between these two means was examined. In another words, any student with a higher formal arithmetical grade in fractional operations had a higher visual grade.

**Results and Suggestions**

The students in elementary schools have more difficulties in mathematics after they begin to learn fractions after natural numbers. This situation negatively affects the students’ academical grades in mathematics and cognitive developments. Difficulties in learning the fractions lead failures in other mathematics topics, too. For this reason, in the beginning all basic concepts should be taught exactly. Thus, before the beginning of the fractions subject, each student must be informed where he will meet the fractions, how he is going to use fractions in his daily life. So as, each student must see that subject of fractions has an important role in daily living (Saenz-Ludlow, 1995; Aksu, 1997; Sharp, Adams, 2002). These preparations can make learning of the fractions easier. The students must achieve success about the transformation of formal arithmetic concerned with fractions to visuality in teaching process. These two concepts, i.e. formal arithmetic and visuality, should not be separated. Otherwise, conversion of mathematical knowledge and skills of students to expressions is hardly achieved. The lack of using both concepts at the same time lead to no answers to the questions or marking the results without any operations. Glancing over all this research, no significant difference between girl and boy students according to their ratios of success is noticed and both of them are low. Especially the students have difficulties in sorting the fractions in a set of fractions, multiplication of a fraction with a natural number, finding the matching fraction in the visual expressions of the fractions set, etc. Recently, many researches about similar subjects verify these difficulties. When we analyze all these difficulties, shifting the subject of fractions from the third class of elementary school to the sixth class of elementary school will be appropriate as like Hart (1981) suggested.

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