The Mathematics Education into the 21st Century Project
The University of North Carolina Charlotte
"Mathematics Education in a Global Community"
Charlotte, USA, September 7 – 12, 2007

Using the ClassPad300Plus in Analysis to Solve a System of Linear Differential Equations

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Abstract:

In real life situations quantities and their rate of change depend on more than one variable. For example, the rabbit population, though it may be represented by a single number, depends on the size of predator populations and the availability of food. In order to represent and study such complicated problems we need to use more than one dependent variable and more than one equation. Systems of differential equations are the models to use. The nonlinear systems are very hard to solve explicitly, but qualitative and numerical techniques may help us to get some information on the behaviour of the solutions.
Let us consider the ClassPad300Plus (with the new operating system OS 03.01) and discuss on some new exercises in analysis, e.g. solving a linear system of differential equations.
We know several ways to get a solution. The techniques for studying systems fall into the following three categories: ***analytic***, ***graphic*** and ***numeric***.
We can transform a system of equations in one equation of higher order and we have for linear systems with initial conditions the possibility to use the Laplace transformation.
On the other hand we can transform a system of differential equations in a system of difference equations, i.e. sequences of numbers given by the help of recursive equations. These sequences are used as a discrete mathematical model for differential equations.
The ClassPad300 has the **dSolve-** and the **rSolve-**function to study systems of differential and difference equations respectively and additionally the Laplace and inverse Laplace trans-formation. Finally we have the possibility to generate large **dSolve-** or **rSolve-**terms by the help of commands for strings and characters. Thus the calculator can generate the large syntax for the used **dSolve-** and **rSolve-**function. This is a convenient method to input a long command row not manually but by the help of a program.

By the help of several examples the interactive work with the ClassPad300Plus is considered. The student can solve difficult exercises of practical applications step by step using the symbolic calculation and the graphic possibilities of the calculator. Sometimes several fields of mathematics are combined to solve a problem.

References:

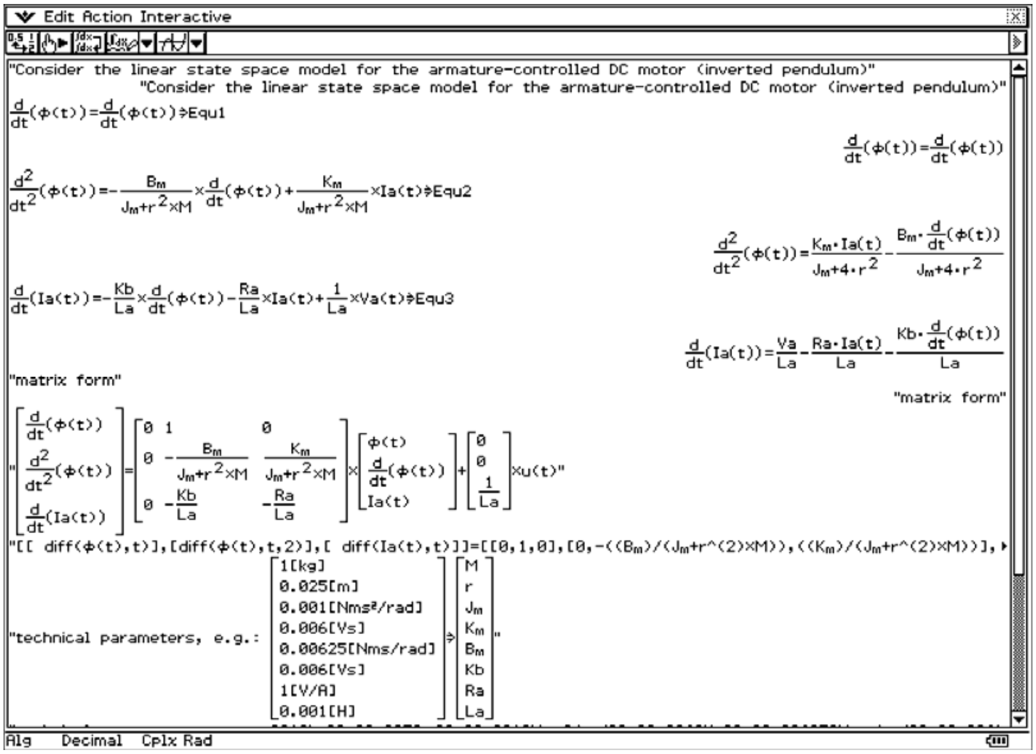
http://www.sosmath.com/diffeq/diffeq.html
http://www.informatik.htw-dresden.de/~paditz/Pendulum_Program.pdf
http://www.informatik.htw-dresden.de/~paditz/LaplaceTransf2006.pdf

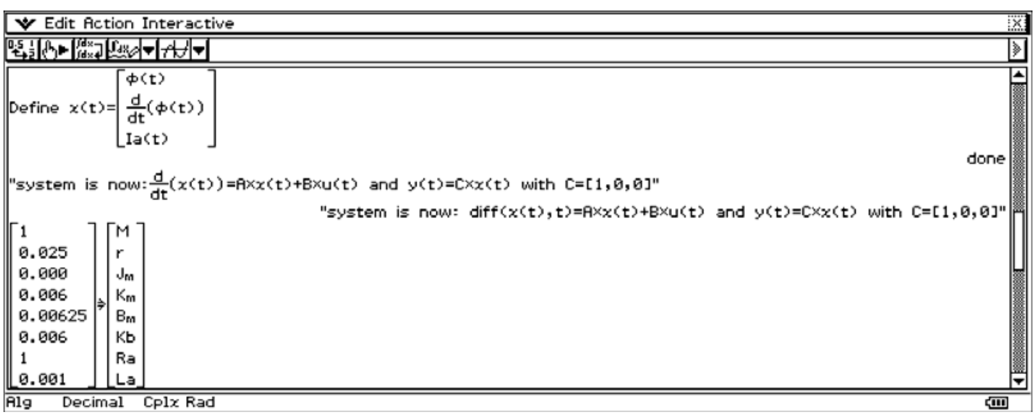
Example of finding the mathematical model and several ways of solution:

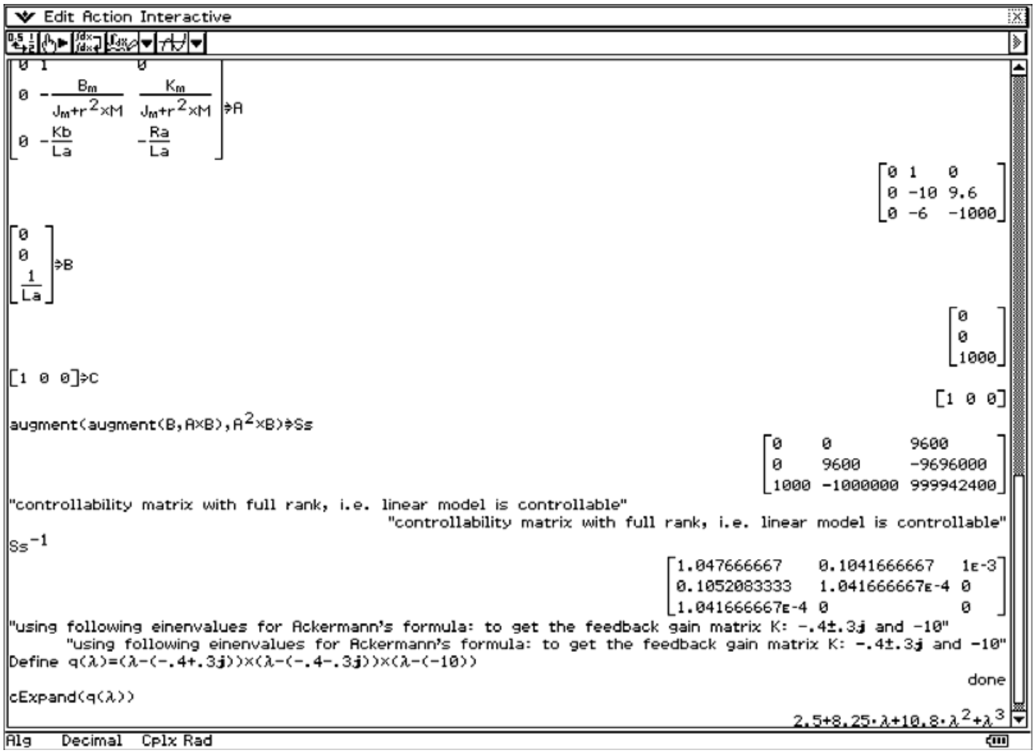
The following mathematical model due to an inverted pendulum, cp.
http://www.fh-kempten.de/deu/hochschule/fachbereiche/fbe/labore/digital/homepage/swpr/ss98/Staude_Sommer/Pendel/Pendelengl.htm
http://instruct1.cit.cornell.edu/courses/ee476/FinalProjects/s2003/es89kh98/es89kh98/Inverted_Pendulum_Balancer.mov
http://www.htw-dresden.de/~kaestner/www/pa/pendel/simulation14/InversesPendel.html
http://www.htw-dresden.de/~kaestner/www/pa/pendel/pendel_modell.htm

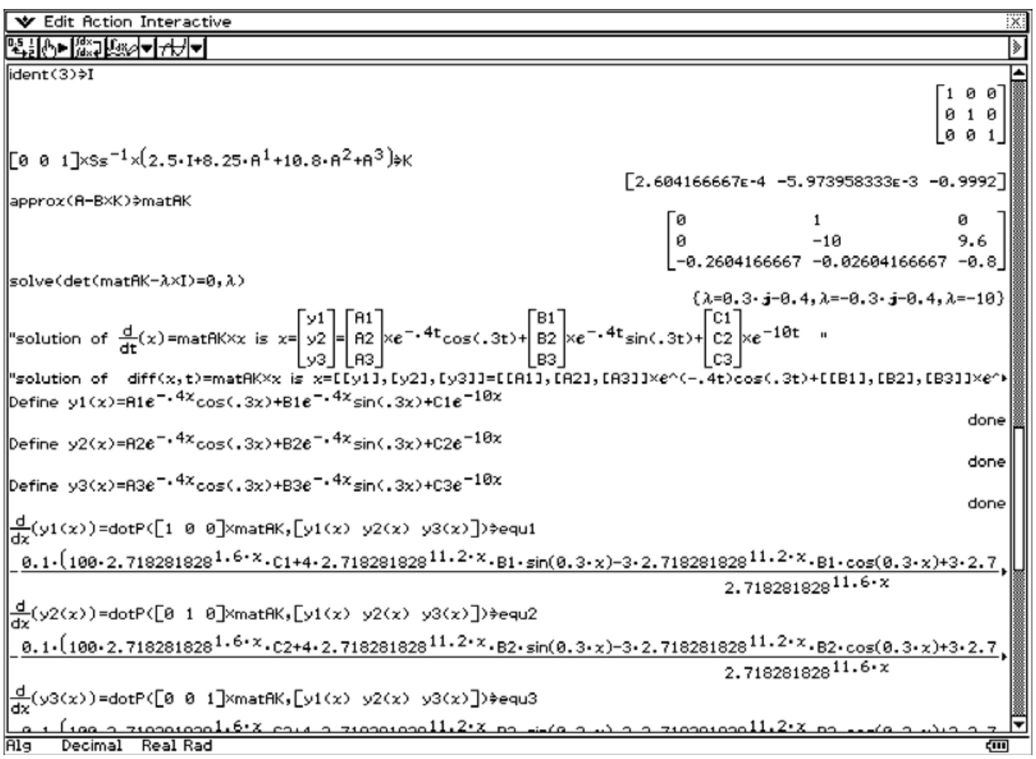
http://www.htw-dresden.de/~kaestner/www/pa/pendel/pendel_text.htm

A complete analytic model of the inverted pendulum controlled by a DC motor is derived in three parts, the pendulum-cart dynamics, the friction model, and the motor dynamics. Here we will study the dynamics of the DC motor by the following equations, cp.
http://www.sei.cmu.edu/pub/documents/99.reports/pdf/99tr023.pdf
http://ieeexplore.ieee.org/iel5/41/33886/01614147.pdf

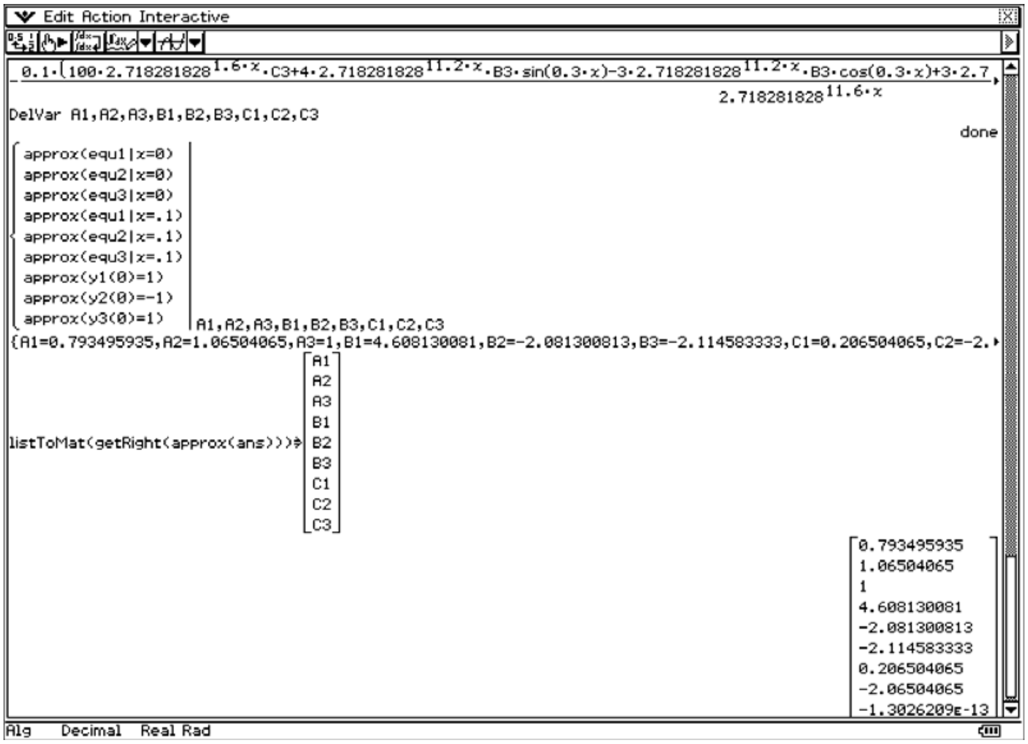


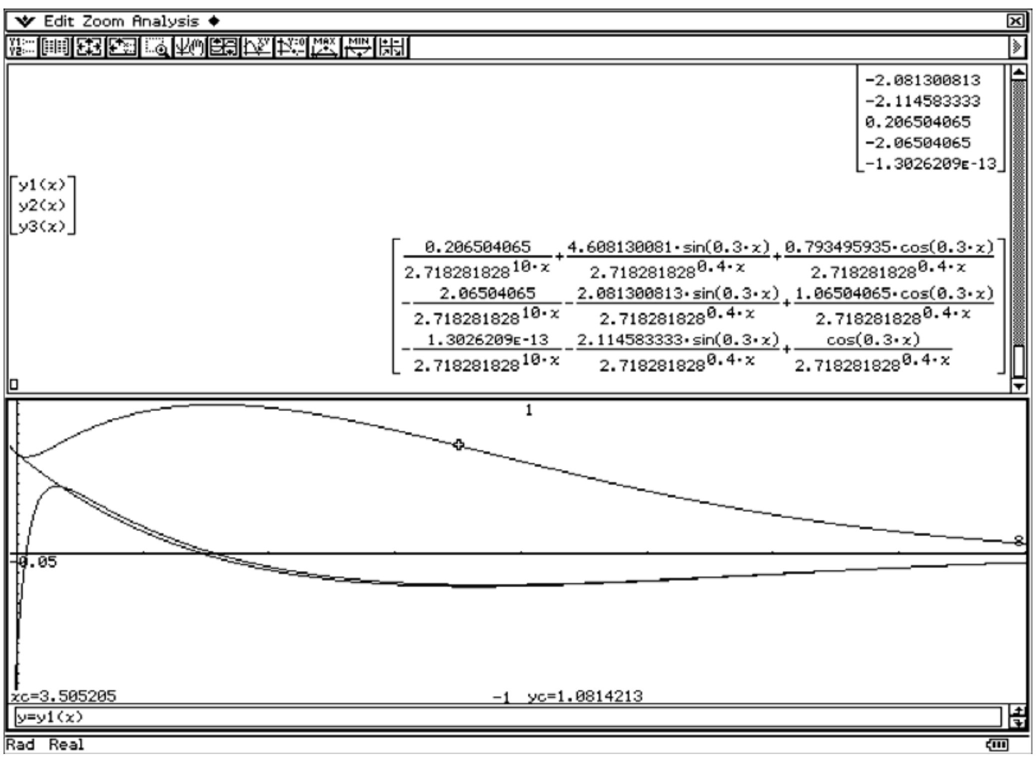




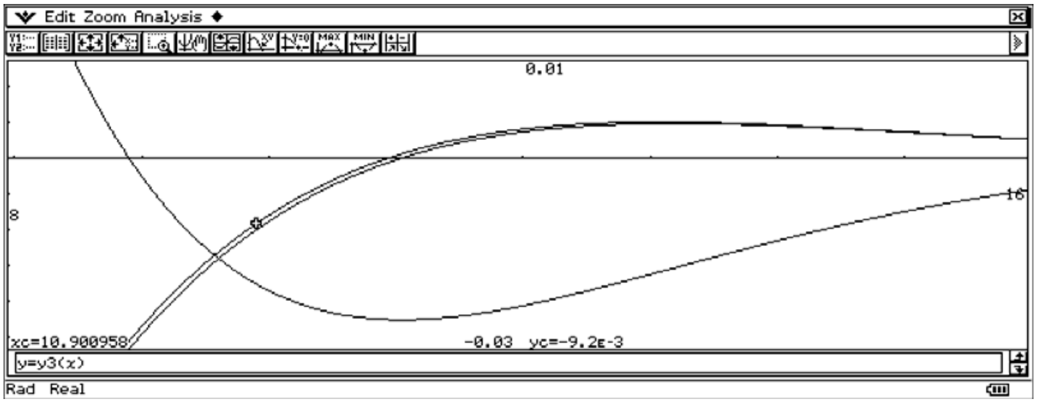


Solution of the linear system with unknown coefficients *A*1, *A*2, …, *C*2, *C*3.
Determination of the coefficients with the initial conditions *y*1=1, *y*2=–1, *y*3=1 for *x*=0:



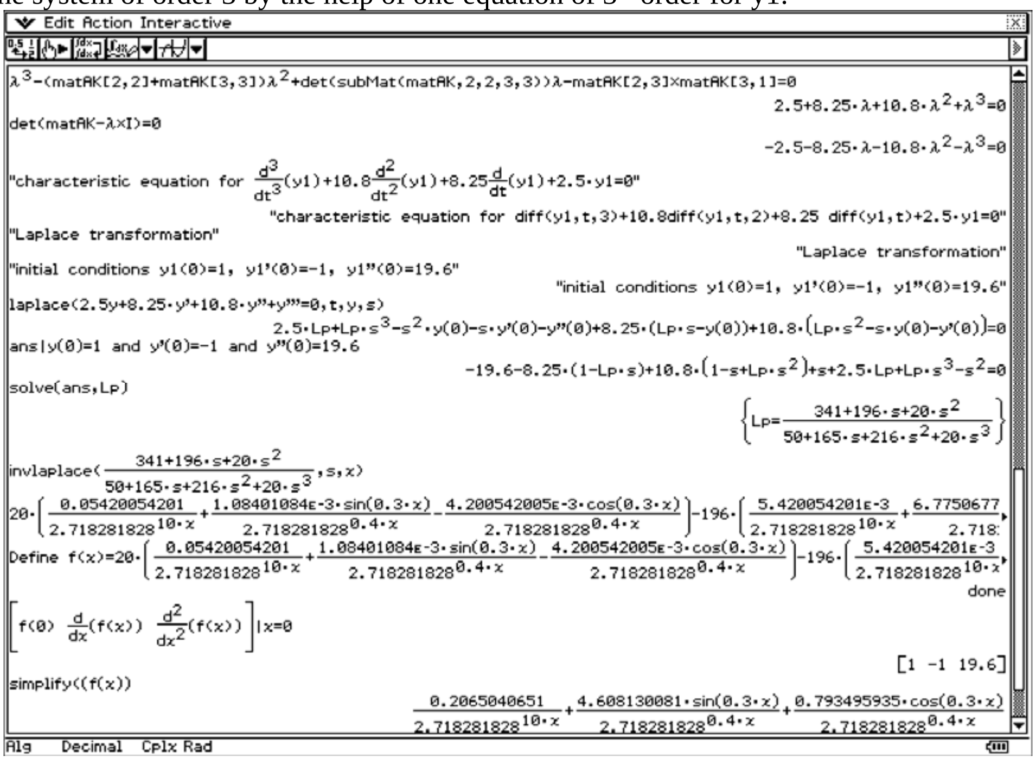


View window: $-0.05 < x < 8$ and $-1 < y < 1$
and graphical representation of $y1$, $y2$, $y3$



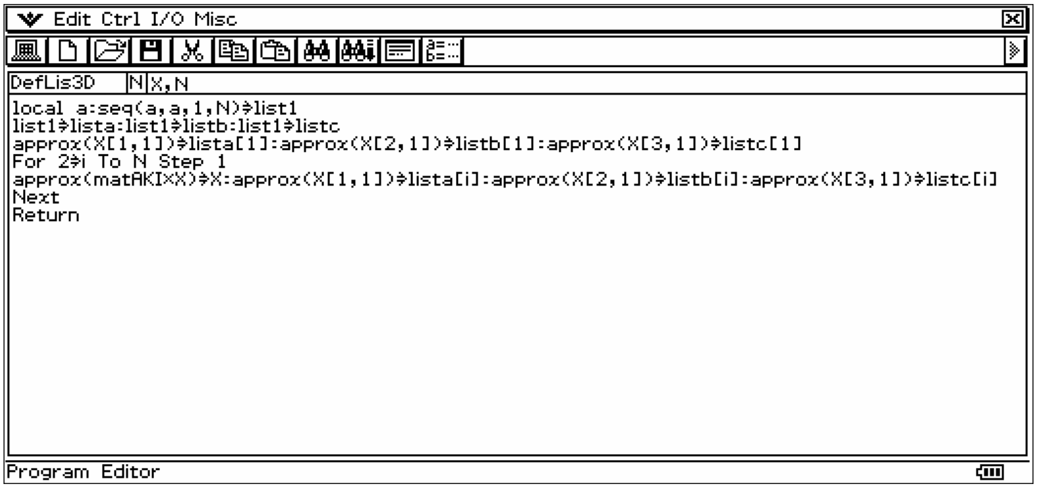
View window: $8 < x < 16$ and $-0.03 < y < 0.01$

Solving the system of order 3 by the help of one equation of 3rd order for y1:

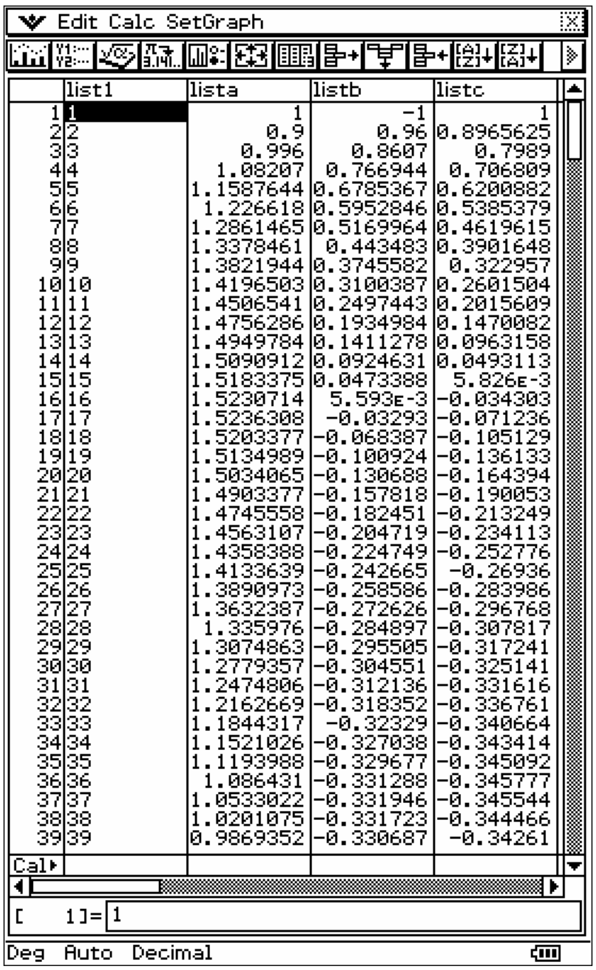


For the Laplace transformation again used the initial conditions $y1=1$, $y2=-1$, $y3=1$ for $x=0$.

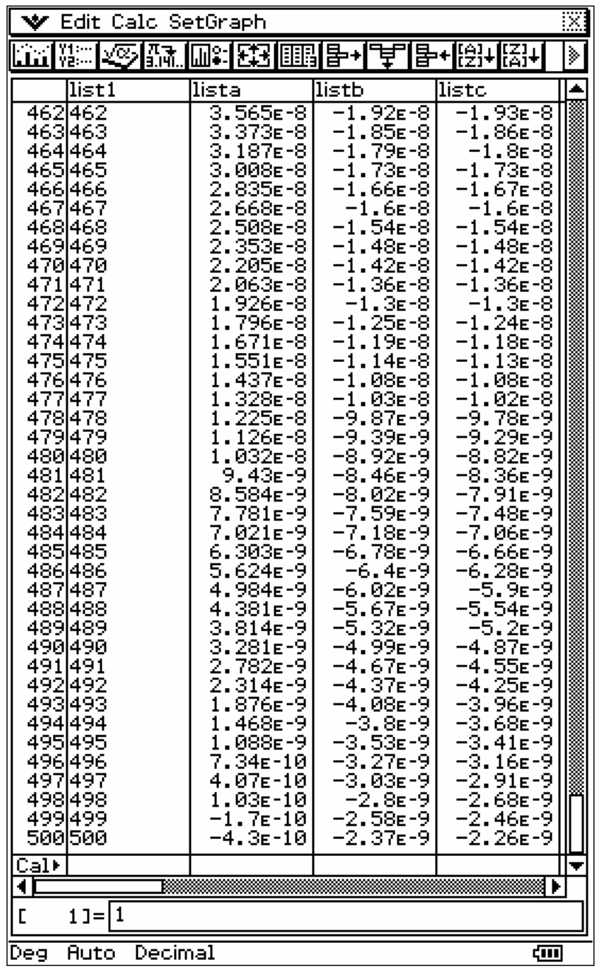
Finally another way of solution is the transformation in difference equations:
$y'(t) = (y(t+T) - y(t))/T$ for small T, say $T=0.1$.
Now the new system is $x(t+T) = x(t) + T * \text{matAK} * x(t) = (I + T * \text{matAK}) * x(t)$.
We use the fixpoint iteration $x_{k+1} = (I + T * \text{matAK}) * x_k$ with $x_0 = [1, \quad -1, \quad 1]^T$ and create 3 lists.
Here $\text{matAKI} = I + T * \text{matAK}$.
The program DefLis3D creates the lists.



```
DefLis3D    N  X,N
local a:seq(a,a,1,N)⇒list1
list1⇒lista:list1⇒listb:list1⇒listc
approx(X[1,1])⇒lista[1]:approx(X[2,1])⇒listb[1]:approx(X[3,1])⇒listc[1]
For 2⇒i To N Step 1
approx(matAKI×X)⇒X:approx(X[1,1])⇒lista[i]:approx(X[2,1])⇒listb[i]:approx(X[3,1])⇒listc[i]
Next
Return
```

	list1	lista	listb	listc
1	1	1	-1	1
2	2	0.9	0.96	0.8965625
3	3	0.996	0.8607	0.7989
4	4	1.08207	0.766944	0.706809
5	5	1.1587644	0.6785367	0.6200882
6	6	1.226618	0.5952846	0.5385379
7	7	1.2861465	0.5169964	0.4619615
8	8	1.3378461	0.443483	0.3901648
9	9	1.3821944	0.3745582	0.322957
10	10	1.4196503	0.3100387	0.2601504
11	11	1.4506541	0.2497443	0.2015609
12	12	1.4756286	0.1934984	0.1470082
13	13	1.4949784	0.1411278	0.0963158
14	14	1.5090912	0.0924631	0.0493113
15	15	1.5183375	0.0473388	5.826E-3
16	16	1.5230714	5.593E-3	-0.034303
17	17	1.5236308	-0.03293	-0.071236
18	18	1.5203377	-0.068387	-0.105129
19	19	1.5134989	-0.100924	-0.136133
20	20	1.5034065	-0.130688	-0.164394
21	21	1.4903377	-0.157818	-0.190053
22	22	1.4745558	-0.182451	-0.213249
23	23	1.4563107	-0.204719	-0.234113
24	24	1.4358388	-0.224749	-0.252776
25	25	1.4133639	-0.242665	-0.26936
26	26	1.3890973	-0.258586	-0.283986
27	27	1.3632387	-0.272626	-0.296768
28	28	1.335976	-0.284897	-0.307817
29	29	1.3074863	-0.295505	-0.317241
30	30	1.2779357	-0.304551	-0.325141
31	31	1.2474806	-0.312136	-0.331616
32	32	1.2162669	-0.318352	-0.336761
33	33	1.1844317	-0.32329	-0.340664
34	34	1.1521026	-0.327038	-0.343414
35	35	1.1193988	-0.329677	-0.345092
36	36	1.086431	-0.331288	-0.345777
37	37	1.0533022	-0.331946	-0.345544
38	38	1.0201075	-0.331723	-0.344466
39	39	0.9869352	-0.330687	-0.34261
Cal▸				

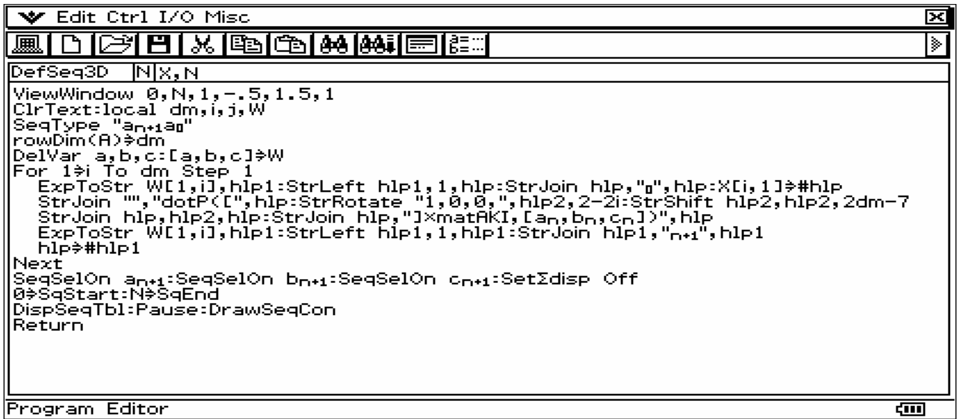


	list1	lista	listb	listc
462	462	3.565E-8	-1.92E-8	-1.93E-8
463	463	3.373E-8	-1.85E-8	-1.86E-8
464	464	3.187E-8	-1.79E-8	-1.8E-8
465	465	3.008E-8	-1.73E-8	-1.73E-8
466	466	2.835E-8	-1.66E-8	-1.67E-8
467	467	2.668E-8	-1.6E-8	-1.6E-8
468	468	2.508E-8	-1.54E-8	-1.54E-8
469	469	2.353E-8	-1.48E-8	-1.48E-8
470	470	2.205E-8	-1.42E-8	-1.42E-8
471	471	2.063E-8	-1.36E-8	-1.36E-8
472	472	1.926E-8	-1.3E-8	-1.3E-8
473	473	1.796E-8	-1.25E-8	-1.24E-8
474	474	1.671E-8	-1.19E-8	-1.18E-8
475	475	1.551E-8	-1.14E-8	-1.13E-8
476	476	1.437E-8	-1.08E-8	-1.08E-8
477	477	1.328E-8	-1.03E-8	-1.02E-8
478	478	1.225E-8	-9.87E-9	-9.78E-9
479	479	1.126E-8	-9.39E-9	-9.29E-9
480	480	1.032E-8	-8.92E-9	-8.82E-9
481	481	9.43E-9	-8.46E-9	-8.36E-9
482	482	8.584E-9	-8.02E-9	-7.91E-9
483	483	7.781E-9	-7.59E-9	-7.48E-9
484	484	7.021E-9	-7.18E-9	-7.06E-9
485	485	6.303E-9	-6.78E-9	-6.66E-9
486	486	5.624E-9	-6.4E-9	-6.28E-9
487	487	4.984E-9	-6.02E-9	-5.9E-9
488	488	4.381E-9	-5.67E-9	-5.54E-9
489	489	3.814E-9	-5.32E-9	-5.2E-9
490	490	3.281E-9	-4.99E-9	-4.87E-9
491	491	2.782E-9	-4.67E-9	-4.55E-9
492	492	2.314E-9	-4.37E-9	-4.25E-9
493	493	1.876E-9	-4.08E-9	-3.96E-9
494	494	1.468E-9	-3.8E-9	-3.68E-9
495	495	1.088E-9	-3.53E-9	-3.41E-9
496	496	7.34E-10	-3.27E-9	-3.16E-9
497	497	4.07E-10	-3.03E-9	-2.91E-9
498	498	1.03E-10	-2.8E-9	-2.68E-9
499	499	-1.7E-10	-2.58E-9	-2.46E-9
500	500	-4.3E-10	-2.37E-9	-2.26E-9
Cal▸				

By the help of these lists we get the same graphical representations of *y*1, *y*2, *y*3.
Finally we use the sequence menu to create the sequences given in *lista*, *listb*, *listc*.
The file for the classpad manager you can download here:
http://www.informatik.htw-dresden.de/~paditz/paper_charlotte_2007.vcp

The program DefSeq3D creates the equations for the sequence menu.

Contact: paditz@informatik.htw-dresden.de



```
Edit Ctrl I/O Misc
DefSeq3D   N X,N
ViewWindow 0,N,1,-.5,1.5,1
ClrText:local dm,i,j,W
SeqType "an+1a0"
rowDim(A)⇒dm
DelVar a,b,c:[a,b,c]⇒W
For 1⇒i To dm Step 1
  ExpToStr W[1,i],hlp1:StrLeft hlp1,1,hlp:StrJoin hlp,"0",hlp:X[i,1]⇒#hlp
  StrJoin "","dotP([",hlp:StrRotate "1,0,0,",hlp2,2-2i:StrShift hlp2,hlp2,2dm-7
  StrJoin hlp,hlp2,hlp:StrJoin hlp,"]×matAKI,[an,bn,cn])",hlp
  ExpToStr W[1,i],hlp1:StrLeft hlp1,1,hlp1:StrJoin hlp1,"n+1",hlp1
  hlp⇒#hlp1
Next
SeqSelOn an+1:SeqSelOn bn+1:SeqSelOn cn+1:SetΣdisp Off
0⇒SqStart:N⇒SqEnd
DispSeqTbl:Pause:DrawSeqCon
Return
Program Editor
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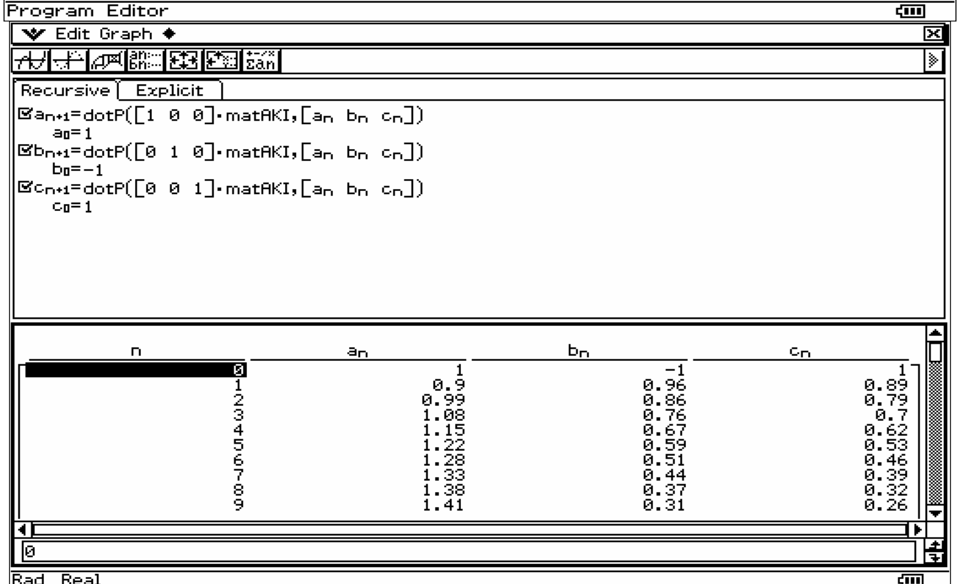

Edit Graph ◆

Recursive | Explicit

☑ a_{n+1}=dotP([1 0 0]·matAKI,[a_n b_n c_n])
a_0=1
☑ b_{n+1}=dotP([0 1 0]·matAKI,[a_n b_n c_n])
b_0=−1
☑ c_{n+1}=dotP([0 0 1]·matAKI,[a_n b_n c_n])
c_0=1

n	a_n	b_n	c_n
0	1	−1	1
1	0.9	0.96	0.89
2	0.99	0.86	0.79
3	1.08	0.76	0.7
4	1.15	0.67	0.62
5	1.22	0.59	0.53
6	1.28	0.51	0.46
7	1.33	0.44	0.39
8	1.38	0.37	0.32
9	1.41	0.31	0.26

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