A Mathematical Problem Solving Process Model of Thai Gifted Students

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Abstract

The purpose of this study was to examine, understand, and model the problem solving processes of gifted students when they solved non-routine mathematical problems. Five Thai gifted students participated and were selected from the Thai Mathematical Olympiad project and met specific selection criteria assuring a diversity of location, school, grade, gender, and age. Each student was required to use the think aloud method while individually solving three non-routine mathematical problems and were interviewed at the end of each problem. Data were analyzed and categorized using a constant comparative method to conceptualize a model of problem solving process. This model described the students' behaviors in each of four stages: *understanding*, *planning*, *executing*, and *verifying*.

Introduction

Research on problem solving has identified several factors that influence problem solving performance. Among these factors are knowledge, cognitive processes and strategies, individual differences in ability and dispositions, as well as external factors such as social context (Pretz, Naples, & Sternberg, 2003). Schoenfeld (1985) identified four categories of knowledge/skills needed: resources, heuristics, control, and beliefs. Various models related to mathematical problem solving have been suggested. For instance, Polya (1957) suggests a well-known model of problem solving that consists of four stages: understanding the problem, devising a plan, carrying out the plan, and looking back. Garofalo and Lester (1985) propose a metacognitive model for problem solving. There are four stages in this model: orientation, organization, execution, and verification.

Mathematically gifted students are identified as students who are able to do mathematics typically accomplished by older students. They are able to employ qualitatively different thinking processes in solving problems (Sowell, et al., 1990). They also display curiosity and creativity when assessing a problem situation and possess a high level of task commitment (House, 1987). Research indicates that gifted students prefer to solve non-routine problems because of the challenge of working with these problems (Garofalo, 1993). Non-routine problems are the type of problems where the students are not familiar with problem situations and they are not expected to have previously solved or have not met regularly in the curriculum. Researchers have also found that more difficult problems have potential to activate metacognitive functioning to the extent that good problem solvers consciously regulate and control their cognitive processes (Montague & Applegate, 1993). Thus, non-routine problems are more likely to activate gifted students to demonstrate their high ability in problem solving.

Many researchers have studied how secondary gifted students solve non-routine mathematics problems (Garofalo, 1993; Lawson & Chinnappan, 1994; Montague, 1991; Sriraman, 2003). Results indicate that gifted students spend much time rereading and translating the problems into their own words (Garofalo, 1993; Montague, 1991; Sriraman, 2003). This paraphrasing ability supports them in understanding the problem and indicates one way they differ from other students in problem solving. They are more verbal than other students and their verbalization increases when they are confronted with more difficult problems (Sriraman, 2003).

They recall theorems for generating given information, apply prior knowledge in the problem and use it to access further relevant knowledge (Lawson & Chinnappan, 1994; Sriraman, 2003). Gifted students identify the assumptions in the problem, frequently set up an equation or algorithm after reading, and generally divide the problem into sub-problems. They identify a goal before developing their solution plans. They solve the problems systematically, and use efficient strategies. They redo the problems by working through the whole problems, rereading them, redoing computations, and checking steps and processes (Montague, 1991; Sriraman, 2003).

Considering research on the mathematical problem solving of Thai gifted students, only two related studies were identified (Klaimongkol, 2002; Thipatdee, 1996). These studies focused on the development of an enrichment program, rather than understanding the problem solving process of gifted students. Consequently, understanding in problem solving processes of gifted students would benefit teachers in helping to improve all their students' abilities through adjustment of the classroom instruction and expectations. The purpose of this study was to examine, understand, and model the problem solving processes of Thai gifted students when they solved non-routine mathematical problems.

Participants

This study was conducted while 24 gifted students, who were eligible for the Thailand Mathematical Olympiad (TMO) training camp, were participating in the TMO training camp. The researcher used purposeful sampling to select participants out of the pool of 24 gifted students within the camp. Students were selected if they a) had similar scores on the second round of the entrance examination to the TMO project, b) did not participate in the training camp in the previous year, and c) were in grade 10 or below. Five of the seven gifted students who matched the selection criteria were selected to assure a diversity of school, grade, gender, and age. Voluntary participation was emphasized with no pressure for the students to be involved.

Problem Selections

The researcher developed a pool of problems that consisted of 13 problems: 6 number theory problems, 3 combinatorics problems, and 4 geometry problems. These problems were selected and modified from a variety of sources, including mathematical journals, textbooks and examination contests (ApSimon, 1991; Covington, 2005; Gardiner, 1987; Krantz, 1996; Posamenteir & Schulz, 1996; Posamenteir & Salkind, 1996; Schoenfeld, 1985). After modifying problems according to the experts' suggestions, one problem was selected for the study from each mathematical area (number theory, combinatorics, and geometry). The three problems used in the study are presented below:

Problem One: *Does a Friday the 13th occur every year? Explain your reasons.*
Problem Two: *In a tournament, there are 15 teams. Each team plays with every other team exactly once. A team gets 3 points for a win, 2 points for a draw, and 1 point for a loss. When the tournament finishes, every team receives a different total score. The team with the lowest total score is 21 points. Explain why the highest total score team has at least one draw.*
Problem Three: *Let ABC be an isosceles triangle with AB = BC. Angle ABC equals to 20 degrees. Point D is on AB such that angle ACD equals to 60 degrees. Point E is on BC such that angle EAC equal to 50 degrees. Find the value of angle CDE.*

Data Collection

Data were collected in a one-to-one setting between the participant and the researcher. Each participant made three appointments for solving the problems with the think aloud method followed by individual interviews. Before solving Problem One, each student practiced the think aloud method around 15 minutes. After receiving Problem One, the participants began by reading the problem aloud. They asked any questions they had to make sure they understood the wording in the problems before beginning to work on the problem. Participants spoke aloud describing their thinking while also writing their solutions on the paper. They used as much time as they needed in solving each problem. This unlimited time decreased pressure and motivated students to make a more complete solution. On average, the participants took around 20 minutes per problem, followed by a 15-minute interview. Before beginning the interview, the participant and the researcher had copies of the participant's solution paper. This same procedure was used for Problem Two and for Problem Three.

Results

The researcher used two models (Garofalo and Lester's model (1985) and Montague and Applegate's model (1993)) as frameworks when coding the students' responses. The researcher also considered the nature of participants' responses when identifying the students' problem solving processes. Finally, students' responses were identified and categorized in four stages: understanding, planning, executing, and verifying. These stage names were modified from the study of Garofalo and Lester (1985). In each stage, the term *self-evaluation* from Montague and Applegate (1993) was used to label when the problem solvers monitored their thinking and efforts and demonstrated affective behaviors as they worked in solving the problem. In other words, self-evaluation affected the participants' actions in each of the stages of the model.

<u>Stage: Understanding</u>. During this stage, the students identified the specific problem. They started by reading the problem aloud. They looked for the question or stated some words in the questions to make sure of what was asked or they restated the questions in their own words. Typically, they first stated the given information and then restated the information in their own words. The students analyzed the problem by representing the given information with pictures or tables. Then, they reread the problem to ensure they made the correct representations. They used their prior knowledge in mathematics for interpreting the given information and referred to relevant mathematical concepts that might be used in the problem. They also connected the problem with their prior experience or the current situation. The students organized the given information into a systematic format before attempting to develop the solution plan. They reflected on the problem in terms of familiarity and difficulty of the problem.

<u>Stage: Planning</u>. During this stage, the students developed their plans by selecting given information and generating new information. They also represented the information with pictures, symbols, or tables before entering them in a solution plan. When assessing a plan, the students applied relevant prior knowledge from number theory, basic of counting and geometry in solving the problems. They set the conditions and stated a formula relying on their prior knowledge. The students solved the problems using efficient strategies, such as drawing pictures, making tables, or looking for patterns. They demonstrated their understanding in mathematical knowledge and strategies that had helped them in solving problems. Students rechecked their ideas to determine the plan in terms of making sense. They changed their plans or looked for other plans as their original plans were unsuccessful.

<u>Stage: Executing</u>. During the stage labeled *executing*, the students were directed by their goal to find the final answer. They applied mathematical formulas and carried out computations as called for in their solution plans. They made logical mathematical statements to support their plans. They also stated the conclusions or the final answer.

Stage: Verifying. During the final stage, *verifying*, the students sometimes checked what was done to make certain that the solution made sense. They usually revised the solution plans when the plans did not work. The students rechecked what was done and were able to explain reasons for their solutions. When they verified the solutions, they reread the problem and examined all their written responses in cyclical processes as they attempted to verify local plan as to its usefulness for solving the problems.

Self-Evaluation. In addition to each of the above stages, students frequently exhibited self-evaluative statements that helped them continue working on the problem until they had finished the tasks. For this study, self-evaluative statements were divided into two types. First, students demonstrated self-monitoring as they monitored their work on the problems until they got complete solutions. For instance, the participants usually asked themselves with the sentences such as: “What’s next?” or “What am I going to do?”. Second, the students demonstrated affect statements as they evaluated themselves as problem solvers in terms of how much confidence they had, their difficulties and frustrations, and their efforts while solving the problems.

The Thai students’ behaviors discussed through the four stages were conceptualized into a four-stage model and presented in Figure 1. The stage description of the model is presented in Figure 2. As students were engaged in solving the problems, their thinking processes did not proceed in a strictly linear order from the *understanding* stage to the *verifying* stage. Thus, an important result for this study was that this model is not linear. Furthermore, the *self- evaluation* aspect identified in each of the stages may have been the activity that initiated them in vacillating among the stages as needed to work on the problem.

Figure 1: A Mathematical Problem Solving Process Model

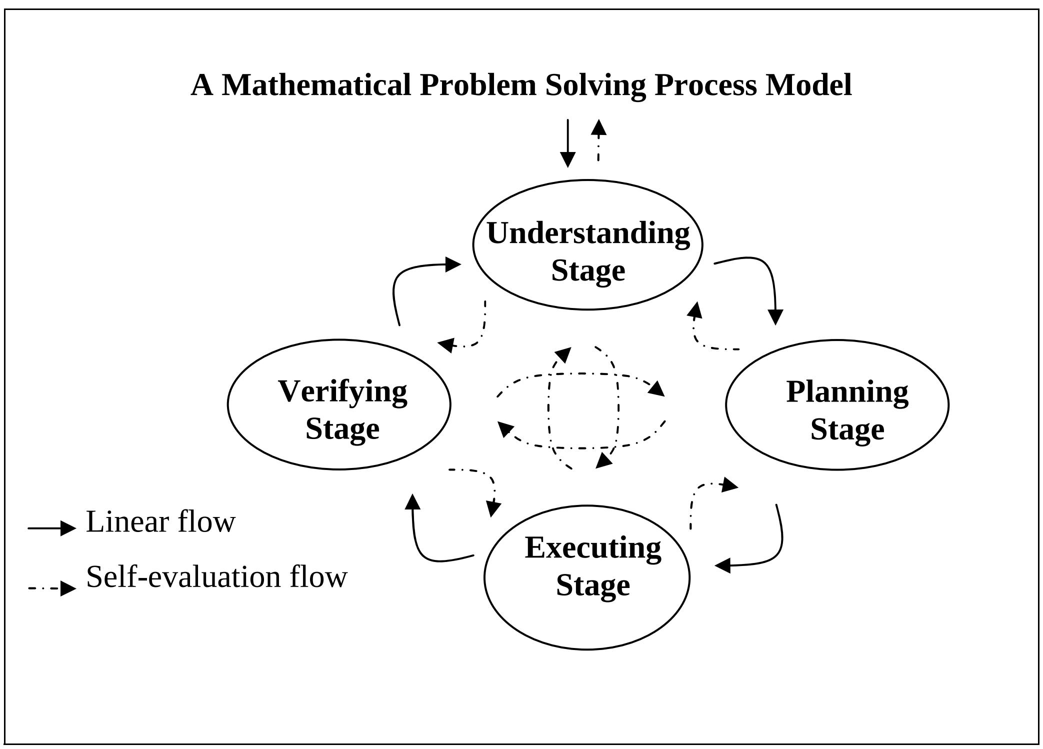

Figure 2: Students' Problem Solving Stage Description

Students' Problem Solving Stage Description

Stage: Understanding
- (1) Identify the problem
  - Read/reread/restate the problem, the given information, and the question
- (2) Analyze the problem
  - Represent the problem with pictures or tables
  - Clarify/interpret/organize the given information
  - Connect with prior experience
  - Reflect on the problem
- (3) Self-evaluation

Stage: Planning
- (1) Devise a plan
  - Manipulate the given and generating information
- (2) Assess a plan
  - Apply prior knowledge/ mathematical concepts/ theorems
  - Use strategies (look for pattern/make a table)
  - Predict possible answers/use estimation
- (3) Revise a plan
  - Determine the plan makes sense
  - Change the plan if it is not working
- (4) Self-evaluation

Stage: Executing
- Carry out computations
- Make logical mathematical statements
- State the conclusion/the answer
- Self-evaluation

Stage: Verifying
- Check results for reasonableness
- Reread the problem and solutions for checking
- Move to a new plan based on verifying results
- Self-evaluation

Conclusions

The purpose of this study was to examine, understand, and model the problem solving processes of Thai gifted students when they solved non-routine mathematical problems. The findings generated a model of problem solving process that detailed the students' behaviors in each of four stages: *understanding*, *planning*, *executing*, and *verifying*. Their behaviors described according to this model not only demonstrated the complex processes but also helped in understanding student actions during the thinking processes. The study results may be useful for the Thai Mathematical Olympiad project in developing future training programs and teaching methods in order to improve the high abilities of Thai gifted students. Because not much research has examined the problem solving processes of Thai gifted student, this research serves as initial evidence to motivate Thai educators in creating enrichment programs or learning materials for gifted students. The findings are also important for guiding directions for researchers to extend future research on student thought processes.

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