Fractions Without Pain!

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Abstract
One of the most important challenges in many countries is: to achieve their people not only learn, but understand mathematics. The Fractions With Out Pain! Workshop, will present the desquebra/2 model. 8 multicolored cubes used for accelerating fractions understanding including their four arithmetic operations, here you will know the convenience of start teaching fractions with division operation. What is the conceptual difference between division and multiply by the inverse process. This workshop, suggests an innovative strategy that uses concrete educational material, a graphic representation and its symbolic translation for teaching fractions, in order your students can understand them while they play, enjoy and stimulate their imagination

Desquebra/2 are highly recommended to be used in kinder garden, elementary and high school.

The problem
Most people in Mexico and in many other countries, have problems for learning and teaching fractions, the low understanding about what fractions are, how to handle and operate them, makes a lot of people hate them, When they can not understand them, they feel on their own frustrated and few intelligent. Unfortunately most of the Mexican people decide their University studies trying to avoid Math and many others decide not to study any more. In 1994 The Ministry of Education in Mexico reformed the basic Math education programs. The teaching of division and multiply operations with fractions now are taught in high school and not in the elementary school as we used to do it

The major proposal of this workshop is to show a revolutionary way, for teaching fractions and their four arithmetic operations. For getting it, we need to break the next five paradigms:

First "Fractions are difficult" If we continue saying fractions are difficult, any body will try to avoid them. We need to transmit the idea fractions are easy and may become funny.

Second. "We must teach arithmetic fractions operations in the same order, we teach arithmetic with natural numbers." For teaching arithmetic operations with fractions, we should start with division, besides once our kids know the numbers, we should introduce division and multiply operations in the kinder garden

Third, "Fractions teaching is founded in algorithms", we need to teach fractions as game, we need to attract our students to math, we need our students play fractions and operate concepts, in order to empower them and helping them they get on their own self confidence

Fourth "You don't need to learn how to divide fractions, while you are able to transform the second fraction for its inverse and then multiply both fractions" In fact, even this let you get the same numeric result, in both operations, the meaning of the result in each case is completely different.

Fifth "People should learn mathematics", mathematics must be understood and then applied not only learned

Introduction to the desquebra/2 model.
The desquebra/2, is a set of eight multicolored cubes, all of them are equal and their faces shows the following colors and forms: green, white, red, blue, an orange triangle and small purple triangle:

As this paper was not able to be printed in color, you must consider the next convention:
w=white, g=green, r=red, b=blue, the light gray triangle should be orange, the dark gray triangle should be purple.
I encourage you, please add right colors in your document

We will use Desquebra/2 for teaching fractions and their four arithmetic operations, as I said earlier we can teach our kids in kinder garden to divide fractions, but before starting we will remember the algorithm for dividing fractions:

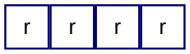
1 Multiply the numerator of the dividend by the denominator of the divisor and the product will be the numerator of our result
2. Multiply the denominator of the dividend by the numerator of the divisor and the product will be the denominator of our result

3.If you can simplify the result, do it! So, in this way if we take 3/4 1/ 4 = 12 / 4 = 3.

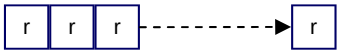
Any body can learn that process but the important issue here is: What does the number three mean? Most people will answer "Three Units", and it is a misunderstanding. May be, this misunderstanding is dispersed for many countries and I consider it, one of the main problems for understanding fractions.

Let's analyze how the desquebra/2 model, it is being used in some kinder gardens, elementary and high schools in Mexico for teaching fractions

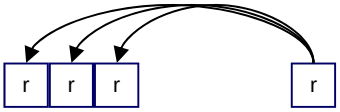
Using Desquebra/2 for teaching fractions at kinder garden level.
We are showing to our kids the next figure (a rectangle) created with desquebra/2. Then we question our kids. How many parts does the rectangle have?, kids answer four.



So we ask them please take one of those parts, when the kids take one of the desquebra/2 (in the draw one of the squares) we tell them they have in their hands one part of a rectangle that had 4, so they left 3 parts of the rectangle that had 4, then we teach them that they can write 1/4 for describing one of four (parts), at this level we do not teach them the term "one quarter" because they are starting to know numbers.



Then we question them How many times does the desquebra/2 they have in their hands (1/4) can be placed over the desquebra/2 they left (3/4). And their answers are three!



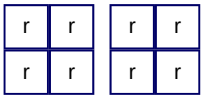
What they have done it's a division of fractions, we do not need to tell kinder garden kids, they are dividing fractions, we do not need to tell them, that one connotation of division is to know how many times does a number fits in other like 21 7 = 3, it means that seven fits 3 times in twenty one. At kinder garden level, the only important thing for us and our kids is: they must be able to allocate as many times a desquebra/2 over the rest. In this, way we are helping our kinder garden kids to develop their space perception.

When we teach our high school students/teachers and our elementary school students/teachers, we need to recall them, that a unit is composed of 4/4 (four quarters), so if we divide 3/4 between 1/4 it's impossible to get 3 units, then we explain them that even they get a 3 as an answer using traditional methods for dividing fractions, the important issue is, how they understand that number, it must be understood as the number of times that the divisor fits in the dividend.

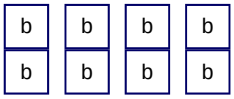
Using Desquebra/2 for teaching fractions at elementary school level.
For teaching fractions at elementary level, we explain them the next conventions

If we place eight desquebra/2 with their white face up, we form a unit

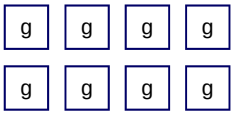
Four desquebra/2 with their red face up, will represent a half (1/2) a unit has 2 halves



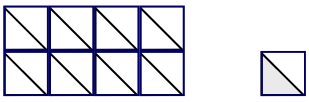
Two desquebra/2 with their blue face up, represent a quarter (1/4) a unit has 4 quarters



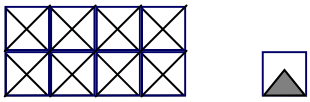
If we Place one desquebra/2 with their green face up, it represents 1/8, a unit has 8/8



You have noticed that with desquebra/2, the unit is formed with 8 of them, so if we divide the white face diagonally, our original rectangle will have 16 parts, so if a desquebra/2 shows its face with the orange triangle up, it represents 1/16



If we would divide the 8 white squares with two diagonals, our original rectangle would have 32 parts. So, if we put a desquebra/2 with its purple triangle face up, it represents 1/32



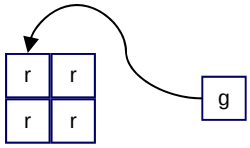
These conventions are very important. If a teacher is replaced, every body will be able to understand that if you want to represent 1/8 allocating a desquebra/2 with its blue face up, every body will tell You, You have made a mistake, because 1/8 is represented with a desquebra/2 showing its green face up.

I recommend you teach this conventions to students of the second grade, You are the best judge for knowing if you teach your students up to 1/8's, 1/16's or 1/32's

Once every body knows conventions for working fractions with desquebra/2, you can demonstrate that a bigger number in the denominator makes a fractions smaller, also you can teach the concept of equivalent fractions and show that 1/4 = 2/8, 2/4 = 1/2, 1/16 =2/32, 4/32 = 1/8, etc.

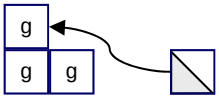
Dividing fractions with desquebra/2.
Once our kids know the conventions, we are ready for introduce them in the concept of division. Try to do it playing, so if you pretend to divide:

1.*1/2 1/8* We should ask: How many times 1/8 can be allocated over 1/2?



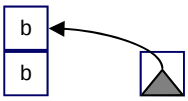
Yes, the answer is 4

2. *3/8 1/16* We should ask: How many times 1/16 (the orange triangle) can be allocated over 3/8?



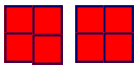
Yes, the answer is 6

3. *1/4 1/32* We should ask: How many times 1/32 (the purple triangle) can be allocated over 1/4?



Yes, the answer is 8

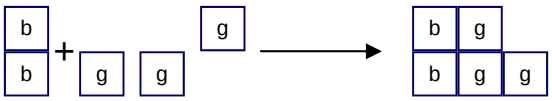
When kids realize fraction division utilizing concrete material and you noticed they have understand the process, they are ready for learning how to multiply, add or subtract fractions.



Adding fractions with desquebra/2.

We have to be sure kids understand the addition process, if they don't. We will need to recall them that an addition is to put things together, to join them. So if we would like to add

1. *1/4 + 3/8* We should represent



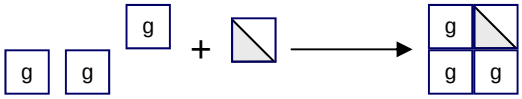
We will start playing for making feel our students they are intelligent or that fractions are easy, we will ask them in each case *What is the smallest area represented?*

In this case the smallest represented area is the 1/8

Then we will ask them *How many times the smallest represented area can be allocated in all the colored area?*

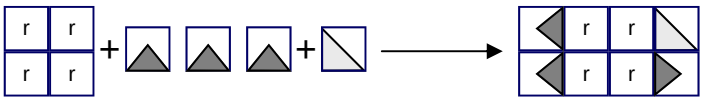
The 1/8 can be allocated 5 times [2 in the blue area (1/4), plus one in each green square (1/8)] So as we are talking about 1/8's. Then 1/4 + 3/8=5/8

2. *3/8 + 1/16 We should represent*



In this case the smallest represented area is the 1/16. The 1/16 can be allocated 7 times [6 times in the green square (3/8), plus 1 in the orange area (1/16)] As we are talking about 1/16's then:
3/8 + 1/16 = 7/16

3. *1/2+3/32 +1/16For adding this fractions we can represent*



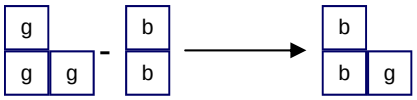
In this exercise, the smallest represented area is the 1/32. Here the 1/32 can be allocated 21 times [16 times in the red area (1/2), plus three times in the purple area (3/32), plus 2 times in the orange area that represents 1/16] and as we are talking about 1/32's
1/2+3/32 +1/16=21/32

Once our students play and they can notice adding fractions is so easy, then we can explain them that we were playing, and that in fact we can't add fractions when they have different denominators, so we need to find equivalent fractions. In the first case for example if we add 1/4 + 3/8, what we need to do is to find how many times can be allocated a 1/8 over 1/4. Right 2 times, then 1/4=2/8, now we can add 2/8 plus 3/8 then we get 5/8. With this procedure we teach the common denominator concept.

Then once we notice our kids add with out problems those fractions with denominators 4,8,16 and 32. And that they can build a mental image of the operation they are performing, We can teach them the algorithm a/b+c/d=[(ad)+(bc)]/(bd)

Subtracting fractions with desquebra/2.
For teaching how to subtract fractions, you should be sure your kids understand the subtract process, if they don't, we need to recall them Subtract mean to take off or to identify how much you need for getting a whole, so for subtracting we need to put the fractions that represents the minuend and over it the subtrahend.

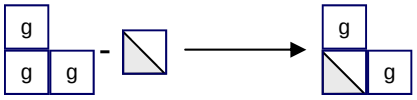
1. *3/8 – 1/4* We represent



As we are subtracting, we will ask them How many times the smallest represented area can be allocated in all uncovered area of the minuend or over the white area that covers the minuend

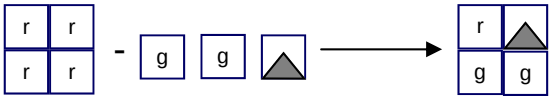
The 1/8 can be allocated only 1 time in the green uncovered area, so 3/8 – 1/4 =1/8

2. *3/8 - 1/16* For performing this subtraction we put



In this case the smallest would be 1/16. The 1/16 can be allocated 5 times (4 times in the green uncovered area, plus 1 in the white area that covers the minuend) then 3/8-1/16=5/16

3. *1/2-9/32*



Here the smallest represented area, would be 1/32. (notice we use 2/8 because each of them contains 4/32 and we wanted to show 9/32) The 1/32 can be allocated 7 times [4 times in the red uncovered area, plus three times in the white area that covers the minuend] So 1/2-9/32 =7/32

Once our students play and they can notice subtracting fractions is also easy, then we can explain them that we were playing, and that in fact, we can't subtract fractions when they have different denominators, so we need to find equivalent fractions. In the first case for example if subtract 3/8 – 1/4, what we need to do is to find how many times can be allocated a 1/8 over 1/4, right 2 times, then 1/4=2/8, now we can subtract 3/8 minus 2/8 then we get 1/8. With this procedure we review the common denominator concept.

Then once we notice our kids subtract with out problems those fractions with denominators 4,8,16 and 32. And that they can build a mental image of the operation they are performing, We can teach them the algorithm a/b-c/d=[(ad)-(bc)]/(bd)

Multiplying fractions with desquebra/2.
For many students, it is difficult to understand, why when we multiply natural numbers, products increase, but when we multiply fractions their products are reduced. This is easy to explain when we translate the "X" instead as by, as "*of*" so when we find the expression 1/2 X 1/2, we should read it as 1/2 of 1/2, then it's easy to understand why you get 1/4.

If we follow the next samples will be easy to understand how to multiply fractions with the desquebra/2 model.

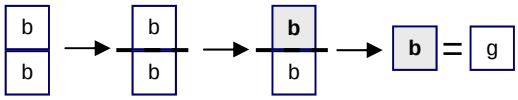
1. *1/2 X 1/4* We must translate it as 1/2 *of 1/4*

2. 3/8 X ½ We must translate it as 3/8 *of 1/2*

3. 3/4 X 4 We must translate it as 3/4 *of 4* (4/1)

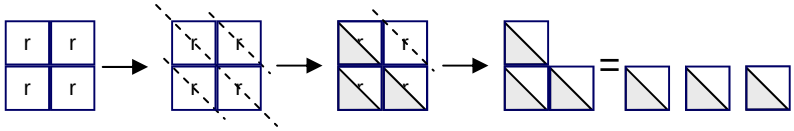
Once we have translated the "X" symbol as "*of*" We understand what are we talking about, then denominator of first factor will tell us in how many parts we should portion the second factor, and numerator of the first factor will tell us, how many of those parts must be considered for getting the product.

1. 1/2 X 1/4 We should translate it as 1/2 *of 1/4*, as we are talking about a quarter. We will represent the quarter, it should be portioned in two parts and we should shadow one of its two parts, the product will be the equivalent of shadowed area



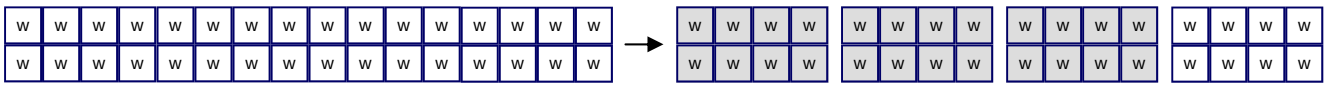
In this case 1/2 X 1/4 = 1/8

2. 3/8 X 1/2 Here we are talking about a1/2, it must be portioned in 8 parts, we will consider 3 of 8 parts. So let's shadow them



We notice that shadowed areas are equivalent to 3/16, then 3/8 X 1/2 = 3/16

3. 3/4 X 4 Here we are talking about 4 Here we are talking about 4 units, then we will portion the 4 units in 4 and we will shadow 3 of those four



In this case 3/4 X 4/1 = 3 (three units)
Again, once our students have understood the multiply process, they area ready for learning the algorithm a/b X c/d = ac / bd

The inconvenience of teaching division of fractions operation as a multiply by the inverse of the second factor method
Of course you get the same number, but the meaning of the result is completely different

If We divide 3/4 1/4 = 12 / 4 = 3, this three means 1/4 can be allocated 3 times in 3/4, while if you use the method of multiply by the inverse of the second factor saying 3/4 1/4 = 3/4 X 4/1 (*it means 3/4 of 4*) = 12/4 = 3 you get the same numeric result but you must understand that this last three means *3 units, because you wouldn't be operating with fractions, but a fraction and 4 units.*

Benefits using desquebra/2 for teaching fractions.
Our students have felt more competent by them selves, also our teachers felt more relaxed on their own when they teach fractions. Teaching fractions starting with division operation, accelerates the teaching and understanding processes. Our students are able to build a model in their minds when they practice enough with desquebra/2. This way they are able to perform mental calculus of fractions with no problem. The desquebra/2 model has been widely accepted by students and teachers, because they think they are playing, their classes don't have to be taught in the classroom, they can take place in the yard. Once our kids have played and understand the concepts of what are the operations they are performing, it has been easier teach them the algorithms, they require for working with any kind of common fractions (1/3's, 1/5's, etc.)

We also have noticed that when our students understand how multiply process with the desquebra/2 model. It is easier for them to understand percentages.

I hope this alternative can help you in your teaching process, I also hope this alternative help you and your students ***To enjoy mathematic and live it with out no pain!***