Developing the Deductive Reasoning of BS Mathematics Students

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Abstract

This study was an attempt to develop the thinking and reasoning skills of Bachelor of Science in Mathematics students. It employed an instructional intervention that utilized nonroutine activities, designed to foster a constructivist-motivated environment. The focus of analysis on the students' thinking skills was on how they write definitions, how they draw conclusions, and their ability to reason out deductively. Phenomenographic analysis of the quality of the responses on the activities and journal entries of the students was performed to determine the level of deductive reasoning of the students. The learners were classified as reflective, transitional, or emergent thinker. The responses of the five students showed the unique thinking pattern that each one utilized, and the level of deductive reasoning of every participant. One of the participants was a reflective thinker; three students were on the transitional level and one in the emergent level. The results of the qualitative analysis revealed that four out of the five students developed their ability to prove the validity of arguments using mathematical inference.

Introduction

Educators are given the task to develop the students' ability to communicate mathematically and equip them with the proper command of the subject proportionate to the needs and demands of the time. This is one of the many challenges that mathematics educators are confronted with, to meet the needs of the students with a complete, balanced curriculum and effective instructional strategies. Relative to this, Philippine Education Act of 1982, states that: "Every teacher shall be accountable for the efficient and effective attainment of specified learning objectives pursuant to national development goals within the limits of available school resources."

Thus, the researcher conceived the idea of introducing a constructivist based instructional intervention for some topics of symbolic logic contained in the math logic course. The use of the nonroutine activities in Math Logic will help teachers to develop the deductive reasoning skills of the students. It bridges the prior knowledge and skills of the students with the new knowledge that they have constructed. While facilitating students' activity, teachers enrich their own knowledge of the subject matter. It likewise improves their teaching competence and enables them to employ appropriate nonroutine activities that will develop in every student a reasonable degree of skills in the subject.

Objective

This study described the thinking and reasoning skills of students in a constructivist inspired learning environment. The focus of analysis on the students' thinking skills was on how they write definition, how they draw conclusions, and their ability to reason out deductively as manifested by their ability to make valid inferences in an argument using the rules of logic that lead to desired conclusion.

The study included arguments that can be easily symbolized and on which the rule of inference can be easily applied. It does not look into other reasoning skills like how to detect errors, how to detect biases, among others. It dealt with the direct application of the rules of replacement and inference.

Methodology

This study was a qualitative research which employed the descriptive method. Triangulation was used consisting of observation, analysis of written output, and interview.

Five students enrolled in the Math Logic course were made as participants of the study. The study used purposive sampling of students in the College of Science, University of Eastern

Philippines. The subjects involved were all Bachelor of Science in Mathematics students taking the Math Logic course in the first semester of school year 2004-2005.

The researcher used the syllabus for the course, the teaching materials, and nonroutine activities for some topics of symbolic logic developed and implemented by Limjap (1996) in a course in undergraduate discrete mathematics for computer science students. The course content was limited to the eight chapters on the fundamentals and algebra of logic in the instructional material of Limjap (2000).

The researcher presented the lessons as designed to the subjects using the teaching materials that promote the constructivist theory of learning. The course included a thorough presentation of the logico-deductive rules from given premises. Nonroutine activities on defining terms, and making valid inferences were given to the subjects. Students were grouped to work with one another during the activities. Journal writing was used after the nonroutine activities to reinforce the concepts formed by the students. Interviews were conducted after the nonroutine activities to allow the students to clarify, explain and discuss his/her answer to each activity. Data were recorded in the activity sheets and tape recorder. Interviews with the students were voice recorded. Some answers to questions asked were written immediately on the activity sheets.

A qualitative analysis of the activities and journal entries of the students was conducted. Phenomenographic analysis of the quality of the responses on the activities was performed to determine the level of deductive reasoning of the students. Analyses of data were made on three aspects: the participant's ability to define terms, to reason out deductively, and prove arguments as manifested by being able to make valid inferences in an argument using the rules of logic that lead to desired conclusion.

Based on the results of the qualitative analysis, the learners were classified as reflective, transitional, and emergent thinkers (Limjap, 1999). Reflective thinkers are those who were able to make valid inferences in an argument using the rules of logic and lead to desired conclusion. The strategy used in making a proof is clear and direct to the point. Consequently, they were always successful in proving arguments. Transitional thinkers were able to use the rules of logic correctly most of the time but were not always successful in obtaining valid conclusions. The correct proofs they make were usually long because of some unnecessary steps. They were not always successful in proving arguments. . The emergent thinkers are those who may make correct initial inferences from given premises but were not able to proceed because of the absence of a clear strategy in arriving at a conclusion. They force to make it appear that the inferences lead to the desired conclusion. They were never successful in proving arguments.

Results and Discussion

There were 20 activities, which included exercises and journal writings. Responses to the activities were analyzed and categorized according to a rubric. The activities were not given scores rather phenomenographic analysis was done to come up with the proper categories for the responses. Misconceptions were identified as well as the patterns that emerged from the solutions. The researcher also noted down the insights gained by each participant from every activity.

The table for nonroutine activities performed by the five participants shows the fictitious names given to the students. The first activity was on defining geometric terms. Students were asked to define every geometric term based on the given group of figures. The responses were categorized in the following manner:

The students had difficulty in identifying similarities and differences among the items using identified characteristics. They also had difficulty in grouping items into definable categories based on their attributes. Thus, they could not describe the geometric figure and give an accurate

definition of the term. It was observed that the students had written definitions with incorrect grammar.

Table 1. Types of Responses for Activity 1

Item	Exemplary	Proficient	Emerging	Beginning
I	none	Rey, Reg	Bill	Roy, Joy
II	none	Rey, Bill	none	Roy
III	none	Rey	Bill	Roy
IV	none	Rey	Bill, Reg	Roy

The ability of the students to draw inferences can be seen in activities 3-7. Table 2 shows the nonroutine activities that made use of the ability to draw inferences. These are applications of the rules of logic to real life situations.

Table 2. Summary of Responses for Activities 3-7

Name	A3	A4	A5	A6	A7
Roy	M	B	B	P	M
Bill	B	B	B	B	B
Joy	M	B	B	M	B
Reg	M	B	M	P	M
Rey	P	B	P	P	M

X – Exemplary   **P** - Proficient   **M** - Emerging   **B** – Beginning

One student had the tendency to become proficient in drawing inferences. Rey could use logic to justify conclusions. He was somewhat careless, though, in the use of the rules in activity 7 and could not recognize the use of Disjunctive Syllogism in activity 4. Two students namely: Reg and Roy were on the emerging level. Both of them attempted to use logic and evidence to justify conclusions but demonstrated a limited ability to draw conclusions from the given information. Reg and Roy used intuition to arrive at their answers in activity 5. In activity 6, each of their proofs had more statements than what were required. Joy and Bill were on beginning level in their ability to draw inferences. Joy had the tendency to be on the emerging level except that he was careless in writing propositional form.

The next table shows the summary of responses of students for the nonroutine activities.

Table 3. Summary of Responses for the Nonroutine Activities

Name	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
Roy	B	M	M	B	B	P	M	M	M	P
Bill	M	B	B	B	B	B	B	M	P	P
Joy	B	M	M	B	B	M	B	P	P	X
Reg	M	M	M	B	M	P	M	X	M	P
Rey	P	M	P	B	P	P	M	X	X	X

X - Exemplary **P** - Proficient   **M** - Emerging   **B** - Beginning

The table shows that one student has gained expertise in deductive reasoning. Rey's proofs were always valid and accurate. He is therefore a reflective thinker. Three students namely: Joy, Reg, and Bill were on the transitional level. Joy had the tendency to become expert except that he was careless in writing propositional form. He gained expertise more quickly than Reg, and Bill. Bill was in the early stage of a transitional thinker. One student was an emergent thinker. Roy remained in the emergent level throughout the whole session.

The teacher saw the need to assist each student to (re)construct his/her mathematical networks so as to accommodate new ideas, and to restructure their views until their ideas were suitable to the given mathematical lessons. Misconceptions were identified and corrected.

The constructivist-based environment in the classroom helped the learners to reflect on their own thinking patterns. The students devotedly organized their ideas, coordinated their concepts and procedures and persisted on to gain the skills in making deductive proof. The difficulty in defining the terms was caused by their inability to identify and articulate similarities and differences of the figures illustrated. The translation of the arguments either partially or completely in Filipino or Ninorte Samarnon dialect enabled the students to understand better the structure of the arguments and gradually made valid conclusions. One of the participants was a reflective thinker; three were on the transitional level and one in the emergent level.

Conclusions

Based on the results of the qualitative analysis of the activities and journal entries of the students the following conclusions were drawn:

1. Students' difficulty to define terms and make valid inferences was caused by their inability to comprehend the structure of figures and arguments. Language played an important role in the thinking and reasoning skills of the students. The use of their home language helped them to understand better the structure of the arguments.
2. The reasoning skills of the students can be developed with the use of appropriate instructional intervention such as nonroutine activities that is constructivist-based.
3. Students' difficulty to apply deductive reasoning to problems on real life situation was caused by their inability to do higher order thinking skills.

In the light of the above conclusions, the following recommendations are suggested:

1 a. Efforts can be focused on activities that bridge the experience and prior knowledge of the students with the new concepts that they have constructed.
   b. Encourage the students to investigate into the mathematical concepts in which the procedures are based.
   c. Help them monitor and coordinate the strategies that they apply to accomplish mathematical tasks.
2. Mathematics educators should try to reconsider the language best suited to use in teaching mathematics. Students' difficulty in comprehending arguments written in a language they are not at home with, hinder their conceptual understanding of the structure of the argument and eventually their reasoning skills.
3. Further studies along this problem should include a wider scope such as larger number of respondents, and other reasoning skills like how to detect errors, how to detect biases, among others.

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