

Structures, Journeys, and Tools: Using Metaphors to Unpack Student Beliefs about Mathematics

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Abstract

This paper reports on the beliefs that ninth and tenth grade students have about mathematics. These beliefs were revealed using contemporary metaphor theory. An analysis of the students' metaphors for mathematics indicated that students had well developed and complex views about mathematics including math as: an Interconnected Structure, a Hierarchical Structure, a Journey of Discovery, an Uncertain Journey, and a Tool. The other common themes that were revealed were that perseverance is needed for success in mathematics and that some students view their role in learning math as active while others view their role as passive.

Introduction

Students have broad views about the nature of mathematics and about their role in the learning of mathematics. These views are linked to interpretations of how mathematics is played out in their own lives, both within and outside of school. Contemporary metaphor theory provides a means of unpacking the beliefs that students have about mathematics. Identifying and understanding students' metaphors for mathematics will provide mathematics professionals with a window into students' internalized views about mathematics. Such understanding informs how beliefs influence students' acquisition and practice of mathematics (see Leder, Pehkonen, & Torner, G., 2002; Pajares, 1992; Posner et. al., 1982). The purpose of this study is to determine students' beliefs about mathematics using metaphor theory. The study was guided by two main research questions:

What metaphors do ninth and tenth grade students use to describe mathematics?

What do these metaphors reveal about students' beliefs about mathematics?

Literature Review

Metaphors allow students to work with novel or abstract ideas by mapping them into strong, meaningful images that were originally developed in a different context (Davis, 1984). Ashton (1994) stated that: "An essential feature of metaphor is that it demands the interpreter becomes actively involved in searching for meaning. This is done by seeking for elements that the two parts of the metaphor have in common in order to share insight."(p. 358) Metaphors are "very private, personal, and ripe with meaning for an individual (Presmeg, 1997, p. 277). According to Lakoff and Johnson (1980), metaphor is of fundamental importance to meaning making. That is, how we think is fundamentally metaphorical. Creating metaphors helps us to structure our experiences.

Contemporary metaphor theory is situated within the psychology of embodied cognition, which emphasizes the social context and origins of knowledge and thus critiques mathematical absolutism (i.e. denies the independent existence of mathematics). In the subjectivist embodied cognition framework, the main question of interest is: "What does each student think mathematics is really about?" With contemporary metaphor theory, poetic metaphors such as "Death is the mother of beauty" (Stevens, 1954) are considered specific cases of the ordinary, everyday metaphors that are pervasive in our daily lives (Lakoff, 1993). Within this framework "...the locus of metaphor is not in language at all, but in the way we conceptualize one mental domain in terms of another" (Lakoff, 1993, p. 203). In this study, we will use the following operational definition of metaphor. A metaphor is the recursive movement between a source and a target that are structurally similar, both changing through the dynamic process of learning (Lakoff & Johnson, 1980, Lakoff & Nunez, 1997).

In Latin *metaphora* denotes something that is carried somewhere else. Lakoff (1993) uses “metaphor” as the root, underlying mapping of two conceptual domains, and “metaphoric expressions” as the individual linguistic expressions of this metaphor. For instance, “It’s been a long and bumpy road” is a metaphoric expression of the root metaphor Love as Journey (Lakoff, 1993). This metaphor “maps knowledge about journeys onto knowledge about love” (Lakoff, 1993, p. 207). In our study, this will inform our view of metaphors such as Math as Journey, which will map knowledge about journeys onto knowledge about math. In this way, a metaphor is a way to link two meanings, to transport the meaning from one semantic sphere to another.

Since metaphors allow students to map, understand, and express a relatively abstract subject matter in terms of a more concrete subject matter, they are helpful in the exploration of an abstract concept such as beliefs about mathematics. There have been a number of studies using metaphor to explore teachers’ thinking (e.g. Bullough, 1991; Kelly & Oldham, 1992; Noyes, 2006). For instance, Noyes (2006) used metaphor theory to explore pre-service mathematics teachers’ beliefs about mathematics as well as its teaching and learning, identifying four root metaphors from the analysis of his data: Mathematics as structure, language, journey and toolkit. Noyes (2006) reasoned that metaphors could be used to “provoke increased teacher reflexivity and to provide a catalyst for shifts in belief and practice”(p.1) One participant wrote: Before the exercise I had my views on maths, what it was and what it meant. I never realized the different views people had on maths and how strongly I disagreed with other peoples’ views. I decided that I need to be more open to other views.”(p. 906)

Studying images of mathematics of primary school teachers, Kelly and Oldham (1992) found that the “racecourse” metaphor seemed to best fit their data, where students and teachers had to gallop and jump hurdles. Previous studies using metaphor theory to unpack beliefs encountered some of the metaphors encountered in this study. For instance, Noyes (2006) categorized the metaphors by pre-service teachers into math-as-journey, math-as-structure, math-as-tool, and math as-language. Sam (1999), who studied adults’ images of mathematics categorized his findings into math-as-journey, math-as-skill, and math-as-game/puzzle. Sam had responses that included learning math is like ‘finding your way through a maze’, ‘brick laying’, and ‘jigsaw puzzle’.

The word “belief” has many meanings in the mathematics education literature. However, the underlying notion is that epistemological beliefs influence knowledge acquisition and interpretation, task definition and selection, interpretation of course content, and comprehension monitoring (Pajares, 1992). Included in “student beliefs” are beliefs about the nature of mathematics, the learning process, the role of schools in society, epistemology, and students themselves, such as self-efficacy (e.g. Leder, Pehkonen, & Torner, 2002). For instance, a student may believe that learning math is a function of drilling facts and procedures or that mathematics is “playing with numbers”. The literature on student beliefs suggests a strong link between beliefs and practice (e.g. Leder, Pehkonen, & Torner, 2002; Posner et. al., 1982). To make the case for the importance of studying belief, Pajares (1992) cites Posner et. al. (1982) who found that “students rejected new information, considered it irrelevant, and compartmentalized their conceptions to prevent it from conflicting with existing beliefs.”(p. 321) Schommer (1990) indicated that beliefs could cause the perseverance phenomena in which information is distorted in order to maintain self-consistency. In these ways, beliefs play a key role in learning.

Method

Participants

The participants were 9th and 10th graders at a large, independent, urban school, enrolled in the regular level of Geometry or the accelerated level of Algebra II. Of the 60 students that were invited to participate in the study, 34 chose to allow their metaphors to be included in the study data. Of the 34 participants, 9 were female, 20 were male, and 5 students provided no demographic data.

Data Collection

For *Phase I* of the study, students studied the concept of metaphor during part of a normal mathematics class period. A language arts teacher conducted activities designed to help students practice creating and developing metaphors. Select metaphors from literature were discussed during class, and for homework, students wrote their own metaphor for “friendship” which was discussed during the follow-up lesson.

For *Phase II*, students were asked, for homework, to (1) identify a metaphor for mathematics and (2) describe how the metaphor represents mathematics. All students in the two classes completed the assignments as part of the standard class work, but only the work of those choosing to participate were included in the study data. Students were asked to provide demographic information (gender, race, age, current math course, and previous math courses) and to provide responses to follow-up questions that provided further insight into the student's metaphor.

Data Analysis

Each metaphor submitted for inclusion in the study was coded by the researchers. Analyzing data segments based on their homogeneity and heterogeneity allowed for the emergence of the coding scheme. First, ten of the metaphors were coded by two of the researchers together until consistency in coding was established. The remaining metaphors were then coded by one of two researchers, but a third researcher used the metaphor representations that emerged from the initial coding and analysis to identify primary and secondary metaphors for a random sample of 14 of the responses. The agreement rate was 100% on the identification of both primary and secondary themes contained in the students' metaphors.

From the 34 metaphors, 182 total codes were found. From the codes, multiple themes became apparent. From the common themes and dominant codes, metaphors were chosen that were representative of the codes, themes, and student metaphors. Codes were permitted to fall into multiple themes and representative metaphors. This approach was used to allow the complex nature of the student metaphors to be revealed. 22 of the 34 student metaphors were categorized under multiple representative metaphors. The Underground Sewer System metaphor is a good example of a metaphor that fell into several themes. This metaphor exemplifies the complex nature of many of the student metaphors and why categorization into themes alone failed to properly contextualize student codes. The student describes a journey through a sewer system in which he is lost while also describing the way the system is interconnected, making this metaphor an example of math as Interconnected Structure and math as an Uncertain Journey.

“Mathematics = Underground Sewer System. It's huge, seemingly never ending, with so many different ways to go. It's dark and uncomfortable, and unless someone else is leading me, I can never see much further than my small flashlight beam in front of me.[...] I know that it is all joined together and everything works together as one giant system.”

Results

The key themes that were revealed by the codes were Perseverance and the Role of the Student. Perseverance was represented by codes referring to math being challenging, requiring effort to overcome, and mathematics being rewarding. The Role of the Student was viewed as being Active or Passive. The remaining themes that were revealed were best depicted using representative metaphors for Math as Structure, Math as Journey, and Math as Tool. With the exception of two outliers, every student metaphor was classified into at least one of these representative metaphors.

Themes

The theme most prevalently coded was Perseverance which included codes referring to mathematics being challenging, requiring effort, and being rewarding. 31 of the 34 student metaphors

included codes that fell into the Perseverance theme. Of the 23 student metaphors that included the code that mathematics is challenging or requires effort, 13 of them also included the code that math is rewarding. Roles of the Student refer to the ways in which students viewed their interaction with mathematics. 10 of the 34 metaphors contained codes indicating an active view toward mathematics much like choosing to hike a mountain. However, 3 of the 34 metaphors contained codes revealing a passive role toward learning mathematics. These students felt that they were passively thrown into mathematics like being taken for a ride or being blindfolded and thrown into a maze.

Math as Structure

Math as Structure was the most prevalent representative metaphor with 28 of 34 student metaphors containing codes referring to math as a structure. Math as a Structure was subdivided into Math as an Interconnected Structure and a Hierarchical Structure. A Jigsaw Puzzle was used as the representative metaphor for the Interconnected Structure and A Video Game Series was used as the representative metaphor for the Hierarchical Structure. The Active Role of the Student and Perseverance themes were often found in student metaphors categorized as Math as a Structure.

Math as an Interconnected Structure is represented by Puzzle metaphors but also included The Rubik's Cube, Castle of Cards, and Duct Tape. This representative metaphor was the most frequently identified within student metaphors with 15 of the 34 metaphors containing this code. The key aspect of Math as an Interconnected Structure is that math is viewed as many separate pieces that are connected together by the student. Students indicated that the whole structure of mathematics or of a particular problem can only be seen by putting all the pieces together. Students who described mathematics as an Interconnected Structure tended to emphasize 'connections'. They saw mathematics as frustrating and challenging, but satisfying and fun nonetheless. Much of the satisfaction came from completing the puzzle, being able to "see the picture" or "figuring it out". "Missing pieces" in puzzles meant being deprived from seeing the "whole picture".

"As more small pieces are found and connected, the greater the overall image."

"Math is made of many different components that are woven together like duct tape".

Math as a Hierarchical Structure was the second most frequently identified representative metaphor with 14 of the 34 student metaphors containing codes referring to Math as a Hierarchical Structure. The representative metaphor used to convey Math as a Hierarchical Structure was the Video Game. The key aspect of this metaphor is that mathematics must be created and understood one level at a time and that skipping levels would expose the student to danger they wouldn't be prepared for. This representative metaphor spanned a broad range of student metaphors including: a Video Game Series, Castle of Cards, Brick Building, Growing a tree, Race, and Life.

"Math is a video game. [...] as the game continues, the levels become harder and harder."

"Mathematics is a castle made out of cards. If you forget to put one card in place or place it incorrectly, it is impossible to build the castle."

Math as Journey

Math as Journey occurred less frequently than Math as a Structure. 12 of the 34 student metaphors contained codes referring to Math as a Journey. Math as a Journey was further subdivided into two types of journeys: a Journey of Discovery and an Uncertain Journey. Journey of Discovery was represented by Hiking and Math as an Uncertain Journey represented by Being Lost in a Maze. The key aspects of Hiking are that the student views this journey through mathematics as an enjoyable trip that requires effort but can be very rewarding. The journey can be boring at times and the student determines the pace at which they progress along their own path. The student sometimes encounters unknown barriers and must work hard to get over them. This journey sometimes has an end goal in mind such as the top of the mountain or end of a race. Journey of Discovery contained in 9 of the 34 student metaphors included the student metaphors of Rock-climbing, Backpacking in Alaska, and A

Race. This representative metaphor often reflected an active Role of the Student and emphasized the rewarding component of perseverance.

“Math is backpacking in Alaska. Although math is hard and takes considerable amount of effort, the reward is immense. Math constantly amazes me with the complexity of its structure like the scenery of Alaska amazes me with its complex beauty.”

The representative metaphor for Math as an Uncertain Journey is being lost in a maze. This metaphor frequently reflected a somewhat negative view toward mathematics and was found in 6 of the 34 student metaphors. The Uncertain Journey included the student metaphors of Wearing a Blindfold, Floating in the Ocean, and the Stuff under my Bed. The key aspect of this metaphor is that students feel they are thrown into the mathematics journey. They’re not sure where they’re going or how they’re going to get there. Getting out of the uncomfortable maze by ‘figuring out’ mathematics is the goal. Having a guide (the teacher) was mentioned in 3 student metaphors as being a helpful part of navigating the uncertain journey. Effort was a key component of metaphors with the Uncertain Journey classification. The majority of codes for the passive role of the student occurred in metaphors classified under the Uncertain Journey.

“Understanding mathematics is floating in the ocean; The shallows are rough and many new waves knock you over, but the deeper you wade the smoother the water becomes.”

“Mathematics is a maze. [...] In mazes, you can get trapped into a corner. [...] you can make a wrong turn, meaning that you can make an error that will lead you in the wrong direction.”

Math as a Tool

The final representative metaphor was Math as a Tool. This representative metaphor occurred in both positive and negative ways for students. 13 of the 34 student metaphors contained codes referring to mathematics as a tool. Math as a Tool included metaphors like Duct Tape, Instruction Manual, Friends, but also a Public Restroom and a Pencil. Students viewed this tool is useful in daily life but also to society as a whole in serving science and to understand of the word. One student compared mathematics to an instruction manual:

“Mathematics gives, whoever indulges in it, an explanation for how things work. Mathematics gives people step-by-step processes for completing certain tasks”

Some students resented the fact that they had to use this tool, although they recognized its usefulness and overall value.

“Sometimes, math is a public restroom. You don’t really want to use it, but you find yourself in a position where you have to.”

Conclusion

The purpose of this study was to determine students’ beliefs about mathematics using metaphor theory. Our findings indicate that students have complex but well developed views about mathematics. Students view math as a collection of concepts they have to connect together. Sometimes, to progress, these concepts must be mastered one at a time. Mathematics is also viewed as a journey during which students discover new things but sometimes a maze that students are forced to navigate. However, students also viewed mathematics as a useful tool that is helpful to individuals and society as whole.

Perseverance was a prevalent theme that pervaded diverse views towards mathematics. Students consistently expressed the belief that math is challenging and requires effort, but half the students that found math challenging found it rewarding as well. These finding indicate that these students recognize the important link between perseverance and success in mathematics. Another important theme that emerged is that students may view their role as passive or active. The metaphors containing more active codes more usually more positive and contained codes indicating self-direction, intrinsic motivation, and self-reliance. However, the metaphors with passive codes were frequently more negative with codes indicating a lack of confidence and a lack of motivation.

The depth to which the metaphors were developed indicates that the students participating had well developed views toward mathematics. The broad range of views toward mathematics indicates that students have widely varying experiences with mathematics. Many of the student metaphors were similar in theme to metaphors identified in other studies with adult participants and mathematics educators, though the emergence of the Video Game representative metaphor might indicate a significant difference between the results of this study and previous studies. It would be interesting to see if similar results occur with a larger sample or a more diverse population.

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