Creative Thinking in Problem Solving

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Abstract

Solving problems in mathematics is in itself bedevilled by several problems. One is that students often cannot see what the problem is. Even when they can, a not unintelligent response can be 'why bother?'! Even when students get past the stage of viewing mathematics as mere symbol-shoving or even some form of black magic, the chance of indulging in creativity is rarely open to them, if only because 'shrewd guessing' and 'why?-asking' have been drummed out of them from an early age. While not everyone is creative, everyone can be taught to think creatively, because, even though problems are not always solved logically, every creative idea is logical in hindsight. My concern is how 21st century citizens will function in a society where we seem to have, for example, politicians who are scientifically illiterate but are making decisions about our technological future, or general medical practitioners who have trouble interpreting medical statistics but are making decisions about our primary health care. Are we really trying to educate *through* mathematics as well as *in* mathematics? This paper will try to tease out some of these issues in the context of some illustrations from pure and applied mathematics.

Introduction

The structure of the presentation is outlined in Figure 1.

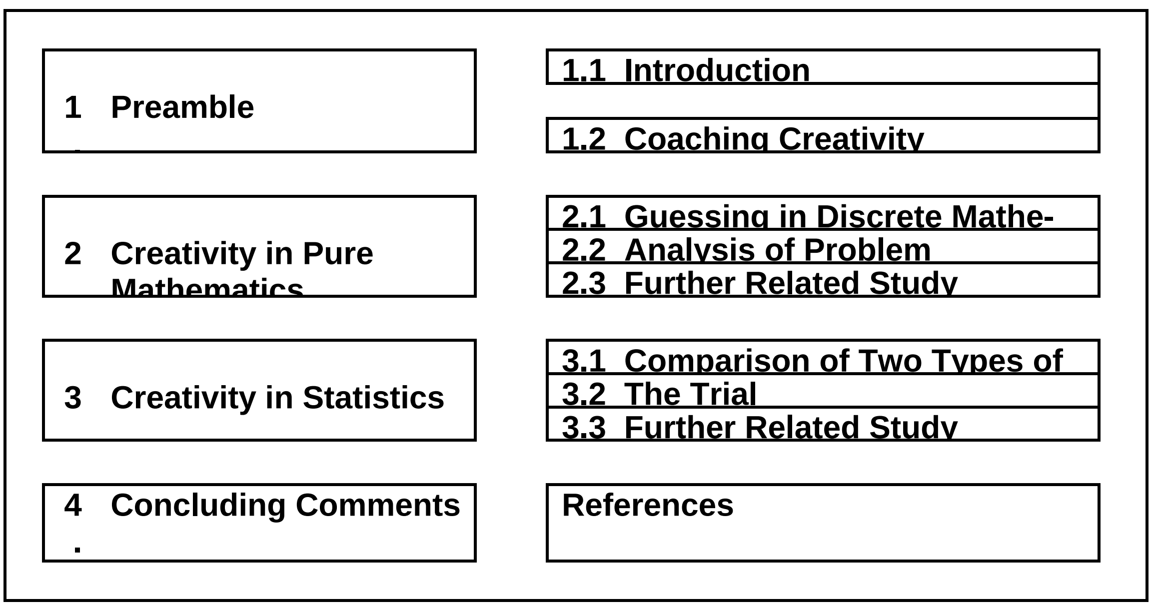

Figure 1: Flow chart of main ideas

By way of caveat there is nothing new in this paper: the models have been used previously, and the approach goes back to the fifties, pre-Piagetian developmental psychology! So why say anything now? The problems our students have in first year undergraduate mathematics do not seem different to us now than they were almost fifty years ago! Can we then say anything new? Perhaps! As Pascal said: "Don't tell me I have said nothing new. When one plays tennis it is the same ball which is used by both players, but one plays it better"! (Jaki, 1991: 276).

In the next section, we shall outline some general features of creativity. Then we shall consider the types of questions which provide opportunities for students (and teachers) to think creatively.

(It is important for students to get a glimpse of the thoughts which run through our minds when we first encounter a problem.)

Creativity, in any sense of the word, does not just happen. Can it be taught or even ‘coached’? In one sense, ‘no’. But like any “transfer of training” we can gear the learning experiences to facilitate it. These are the three phases of creativity (Alonso-Schökel, 1966: 212 - 239):

experience: imagination; emotions; senses (*cf.* Armstong, 1993);
intuition: effulgence; encouragement (Getzels and Csikszentmihalyi, 1975);
expression: notation as a tool of thought (Iverson, 1980).

Creativity in Pure Mathematics

Whatever the role of discrete mathematics in the liberal arts or science undergraduate curriculum, it has a very useful role in computer systems engineering, where it provides the conceptual framework for other courses such as data communication, digital techniques and logical design. However, a difficulty that soon emerges is that despite prior or concurrent courses in analysis, many students have little ability to solve problems that require more than manipulation of symbols according to algorithmic rules.

The following approach to guessing in discrete mathematics comes from Polya (1957), although the problem itself is not new (cf. Buck, 1943).

What is the maximum number of chunks of space one can obtain from five arbitrary planes? This guessing process, or insight and imagination, has often been successfully eliminated from a student’s repertoire (Forgasz & Leder, 1991).

In analysing the problem we see that it is not enough to guess, however shrewdly; the guess must also be tested (Shannon, 1991). What is a simpler problem? What is its simplest form? What is a similar problem? To test the situation under review we could use a piece of cheese and a knife as Polya intimates, or we could try a simpler case of the problem.

A common response to the question of what could be a simpler case of this problem, is one plane, which is certainly simpler although the simplest case is when there are no planes. Responses are then solicited for the maximum number of sections of space divided by 0, 1, 2, 3 planes in turn (the last row of Table 1). When we ask about 4 dividing planes, the most common answer is, not surprisingly, 16, but it is not easy to test so we try a similar problem.

Division of	by	Number of dividers					Sloane
a line	points	1	2	3	4	5	
a plane	lines	1	2	4	7		M1041
a space	planes	1	2	4	8		M1100

Table 1: Division of n-D space by (*n*-1)-D objects

Similar cases to this problem are to divide a plane by lines, and to divide a line by points. By encouraging simple drawings one can readily obtain from the class the numbers in the first two rows of the table.

At this stage the students should be only too happy to refine their previous guesses. Opportunities also arise to encapsulate their generalisations with suitable notation. For example, if n is the number of dividing points on a line, then u_n, the number of sections produced, is clearly given by $u_n = n + 1$. We now let v_n and w_n be the number of sections produced in the plane and in space, respectively. It is not difficult to see that $v_n = u_{n-1} + v_{n-1}, \quad n = 1,2,3$ and eventually that the answer to our original problem is $w_5 = v_4 + w_4 = 26$

Further related study is to see if you can

relate u_n, v_n, w_n to number trees? (Atanassov *et al*, 2002);

extend the table to include hyperplanes? (Ho & Zimmerman, 2006);
generate starting sums in this table? (Gould, 1964);
connect with Fibonacci and polygonal numbers in general? (Lind, 1965).

Creativity in Statistics

Hypoglycaemic reactions in patients with diabetes are always serious and can be fatal. A sudden lowering of blood-sugar levels can be due to too much insulin, too much exercise, or not enough carbohydrate. Symptoms can include dizziness, sweating, loss of balance, nausea and irritability. Very brittle patients with a long history of diabetes can become asymptomatic. Soon after the introduction of (semi-synthetic) "human insulin" to replace porcine insulin there was debate about reports that some insulin-treated diabetic patients lose awareness of hypoglycaemic symptoms on changing from porcine to human insulin. In a double-blind, randomized cross-over study, differences were sought between porcine and human insulin in the frequency and characteristics of hypoglycaemic episodes among patients who reported a reduction of awareness of hypoglycaemia after changing treatment (Colagiuri, *et al*., 1992). Trials of this nature provide so-called "Level One Evidence" which is often required "as proof" by government departments of public health.

In this trial, fifty patients were studied. They had been referred by their physicians because of complaints of lack of awareness of the symptoms of the onset of hypoglycaemia while on human insulin. They had had diabetes for a mean of 20 (SD 12) years and 70% had good or acceptable glycaemic control. Each patient was treated in a double-blind manner for four 1-month periods, two with human and two with porcine insulin, in random order. Figure 2 shows the six possible combinations for any one patient.

H	**H**	**H**	**P**	**P**	**P**
H	**P**	**P**	**H**	**H**	**P**
P	**H**	**P**	**P**	**H**	**H**
P	**P**	**H**	**H**	**P**	**H**

Figure 2: Combinations of human (H) and porcine (P) insulins over 4 months

Only 2 patients correctly identified the sequence of insulin treatments used, yet 8 or 9 would have been expected t do so by chance alone. The mean percentage of hypoglycaemic episodes associated with reduced or absent awareness was 64% (SD 30%) for human insulin and 69% (SD 31%) for porcine insulin. No statistically significant differences could be found between the insulin species with respect to glycaemic control or the frequency, timing, severity, or awareness of hypoglycaemia. Would you be convinced? (Reduced hypoglycaemic awareness over time is common with both human and porcine insulins.)

The preceding example raises the issue of statistical numeracy among primary health care professionals who often have to interpret findings on their own. Thus the following short multiple-choice test is included for you to try sometime. The questions are included because chance and probability which intrude increasingly into our daily lives are so misunderstood (*cf*. Barragués *et al*, 2006; Kapadia and Borovcnik, 1991). This quiz was originally given to me by Professor Peter Petocz of Macquarie University, Sydney.

The point at issue is that statistical thinking is at the heart of the curriculum for the well-educated citizen of the 21st century. My thinking on this is well captured – much better than I could ever express it in Franklin's (2001) erudite exposition on the pre-history of conjecture.

1 If we accept a null hypothesis using a statistical test, is the null hypothesis actually true?

(A) Yes (B) No (C) We cannot tell

2 When we reject a null hypothesis using a statistical test, is the null hypothesis actually false?

(A) Yes (B) No (C) We cannot tell

3 A statistical test was carried out using a 1% level of significance and the null hypothesis was rejected. Would the null hypothesis have been rejected using a 5% level of significance?

(A) Yes (B) No (C) We cannot tell

4 A statistical test was carried out using a 5% level of significance and the null hypothesis was rejected. Would the null hypothesis have been rejected using a 1% level of significance?

(A) Yes (B) No (C) We cannot tell

5 If a null hypothesis is actually true, is 1% the probability of accepting H_0 when we perform the test at 1% significance?

(A) Yes (B) No (C) We cannot tell

6 If a null hypothesis is actually false, is 1% the probability of rejecting H_0 when we perform the test at 1% significance?

(A) Yes (B) No (C) We cannot tell

Concluding Comments

The comments outlined here are intended to sensitize readers to issues and to provoke discussion: for more depth the interested reader is referred to some of the "Problem Books in Mathematics" such as Engel (1998). Nor are the comments here necessarily linked to any particular philosophy of curriculum development (*cf*. Andrews, 1996; Schoenfeld, 2004), though I am biased in favour of 'guided discovery' since my days at the Australian Council for Educational Research in the 1960s. They are based on fifty years of teaching during which I have seen many well-intentioned curriculum projects come and go.

Yet realistically where are we? Two recent Australian reports are less than optimistic. Maiden (2007) reports a key finding of a national snapshot of literacy and numeracy skills which seems to indicate that about 20 percent of students in Year 7 in Australian schools lack the basic skills required to progress through primary school. At the other end of the scale, a national strategic review of mathematical sciences research in Australia (Rubinstein, 2006) has revealed the parlous state of the ever-shrinking university mathematics departments with perhaps only the Melbourne, Australian National, New South Wales and Sydney with the critical mass needed to teach users of the mathematical sciences as well as those doing advanced study in pure and applied mathematics and statistics, though the recent (May 2007) Federal budget is promising a significant increase in funding for undergraduate mathematics and statistics students.

I am still passionate about the place of mathematics in a general comprehensive education for all citizens in the 21st century. Without mathematics civilization would still be in the dark ages. Without mathematics science, engineering and technology would not have made their spectacular advance in the 19th and 20th centuries. Without mathematics the citizens of the 21st century will not have the conceptual framework to be able to evaluate critically the ethical and legal issues associated with advances in the social and health sciences. In the words of Galileo: "That vast book which stands forever open before our eyes I mean the universe cannot be read until we have learned the language. It is written in mathematical language, and its characteristics are triangles, circles and other geometric figures, without which it is humanly impossible to comprehend a single word; without these one is wandering about a dark labyrinth" (Bartlett, 1968: 211b).

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