Let's Do Math with Technology

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Abstract
Following the recommendation of the National Council of Teachers of Teachers of Mathematics regarding technology as being essential in the teaching and learning of mathematics a mathematics teacher education course and a secondary mathematics course, at a liberal arts college in the Southeast, were revised and the use of software programs and web sites were integrated into the curriculum. This study is a report of the integration of the software programs and web sites selected by the students for their usefulness and ease of use. Changes made to the curriculum of these courses due to the increased computer proficiency of the students are also reported.

Introduction

Principles and Standards for School Mathematics was published by the National Council of Teachers of Mathematics (NCTM) in 2000 to provide guidance to teachers, schools administrators, and other educational professional about the content and character of school mathematics. One of the principles stated in *Principles and Standards* is the technology principle. "Teaching is essential in teaching and learning mathematics: it influences the mathematics that is taught and enhances student learning." summarizes the ideology of NCTM regarding the technology principle.

The *Principles and Standards* together with the NCTM's *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) and *Professional Standards for Teaching Mathematics* (NCTM, 1991) motivated mathematics teachers and mathematics teacher educators to integrate technology in the mathematics classroom. At first this integration of technology was, in most cases, limited to graphing and statistical calculators. Graphing calculators were followed by mathematics software programs and statistical software programs followed statistical calculators. Examples of such software programs are M*athematica*, *Maple*, *Derive*, *Excel*, *MiniTab*, and *Fathom*. The use of the programming language LOGO in the classroom was widespread as far back as the late 70's with the advent of the personal computer. By the early 1990's a decline in the use of LOGO and the development of dynamic geometry programs lead to the use of programs such as the *Geometer's Sketchpad* and *Cabri Geometry* in the classroom.

These tools help achieve the goal of the technology principle of the NCTM, especially at the secondary level but there is a need for more at the elementary and middle grades level. The World Wide Web or Internet has an enormous potential for enhancing the teaching and learning of mathematics. This technology should also be used in the classroom.

In accordance with NCTM guidelines and increased access to the Internet in the schools in Fall 2002 a secondary mathematics education course and in Fall 2003 a K-8 mathematics education course, at a liberal arts college in the Southeast, were revised and the use of software programs and web sites were integrated into the curriculum. The secondary mathematics education course is offered in the Fall semester every other year. Students taking this course are either juniors or seniors majoring in secondary mathematics. Since Fall 2002 approximately 25 students have taken this course. The K-8 mathematics education course is offered every fall. Students taking this course are junior teacher education students. Every fall there are two sections with approximately 24 students in each section. Since Fall 2003 one hundred thirty education students have taken this course. This study is a report of the integration of the software programs and web sites selected by the students for their usefulness and ease of use. Changes

made to the curriculum of these courses during the last few years, due to the increased technology proficiency of the students, will also be reported.

What can software programs do?

A spreadsheet frees students from the laborious manipulation of numbers, so they can concentrate on the mathematical problem. Ways to use spreadsheets include as simulations, finding probabilities, inflation and depreciation problems, and interest problems. In geometry the spreadsheet program may be used to "discover" area and volume formulas, as well as to solve those types of problems. It can also be used to find the mean, median, mode, the standard deviation, and to draw statistical graphs. These types of problems may be solved using built-in formulas but also by having students write down the formulas by hand.

Another very useful program is a dynamic geometry program, which is a dynamic construction and exploration tool students can use to explore and understand mathematics. The user can construct, transform, measure, calculate and graph, among other things. A student can construct a geometric figure and then explore it by dragging the figure with the mouse. This stimulates problem solving skills and generalization by examination of many similar situations. This program can also be used to study symmetries, rotations, translations, and tessellations.

Why use Web Sites?

There are excellent web sites available on the World Wide Web with virtual manipultives. Virtual manipulatives are interactive, graphical representations and tools. Research in the area of virtual manipulatives indicates that students who use virtual manipulatives experience higher achievement or conceptual understanding in mathematics than those using associated traditional manipulatives or no manipulatives (Kieran & Hillel, 1990; Thompson, 1992). Izydirczak (2003) found that students using both virtual and traditional manipulatives showed a gain in conceptual understanding. Drickey (2001) found no statistically significant differences in achievement for students using traditional manipulatives, virtual manipultives, a combination of both traditional and virtual manipulatives, or no manipulatives.

These studies suggest that virtual manipulatives are more effective or at least as effective as traditional manipulatives. If that is the case and if excellent web sites area available on the Internet, these web sites should be used by mathematics teachers and mathematics teacher educators to enhance the teaching and learning of mathematics.

Software Programs

In Fall 2002, Excel, Geometer's Sketchpad and web sites, among other topics, were included in the secondary mathematics education course. Students that semester were not familiar with those programs and learning how to use them took much of the time allotted for that specific topic. Detailed written instructions were given to the students in order for them to learn how to use the software. Most of the assignments were related to how to use the program and not on how to do mathematics with the program.

The students in Fall 2004 were more familiar with those programs and were able to use them in approximately half of the time it took Fall 2002 students. Students were given the same detailed written instructions but assignments were related not only to how to use the program but also on how to do mathematics. In Fall 2006 students were already proficient in using those programs so the objective was how to do mathematics with the program and how to use it in the classroom to enhance the teaching and learning of mathematics.

Web Sites

In the three semesters the courses were offered many web sites were used. Students found most web sites to be useful. Among the favorite web sites of secondary math students were

Illuminations (http://illuminations.nctm.org), Cool Math (http://www.coolmath.com/), National Library of Virtual Manipulatives (http://nlvm.usu.edu), and Shodor Interactivate (http://shodor.org/interactivate) web sites. Teacher education students preferred Illuminations, National Library of Virtual Manipulatives, Shodor Interactivate, and E-examples (http://standards.nctm.org/document/eexamples/index.htm). Students stated they preferred these web sites because they were the most useful, easiest to use, and accessible without any kind of membership. Most of these web sites contained interactive figures. Students not only used these web sites while taking the course but also used them in their field experiences.

Illuminations

Illuminations was designed to improve the teaching and learning and to provide materials in support of the vision of the NCTM delineated in the *Principles and Standards for School Mathematics* (NCTM, 2000). It is part of Marco Polo Internet Content for the Classroom (http://www.marcopolo-education.org/home.aspx). The Illuminations web site has activities grouped by grade level. On each activity page there is an exploration section and links to the NCTM resources.

The Geometric Solids (http://www.illuminations.nctm.org/ActivityDetail.aspx?ID=70) activity helps the students learn various geometric solids and their properties and can be used from K-12. Students liked being able to rotate the figures and making them transparent as that made the solids appear more 3-dimensional.

The Histogram Tool (http://www.illuminations.nctm.org/ActivityDetail.aspx?ID=78) can be used from 3rd to 12th grade. Students liked this activity because it not only had sample data sets but also students could input their own data to be used in the activity. Given the emphasis given to Data Analysis recently, it was a very useful activity for the students.

Multiplying $(a+b)^2$ (http://illuminations.nctm.org/ActivityDetail.aspx?ID=127) This activity uses geometric shapes to show how to compute $(a + b)^2$. Students liked it because it gives a visual for the student to see why this result of this operation is not $a^2 + b^2$. This activity also includes a detailed exploration section that explains how the activity works and why our intuition for this problem is incorrect.

Cool Math

This website does just what the name suggests. It gives really cool math web sites including topics about algebra, geometry, trigonometry and calculus, math puzzles and math brain teasers, math humor, K-12 math teacher resources, and many more sites. This is a large site with over 5000 pages of content for students, teachers, and parents from ages 13 to 100.

Students liked the activity Determinants and Cramer's Rule for 2x2's (http://www.coolmath.com/algebra/Algebra2/07Systems3x3/05_det2x2cramer.htm) because it contains many examples and it allows the user to enter their own examples.

Students selected Adding and Subtraction Polynomials Lesson (http://www.coolmath.com/algebra/Algebra1/01Polynomials/05_polynomials-addition-subtraction.htm) because the text is very easily understood and it explains things in small steps.

Algebra Crunchers (http://www.coolmath.com/algebra/algebra-practice-problems.html). The Crunchers are divided into four main topics that are clearly defined. This was selected because of the variety of topics and exercises available.

National Library of Virtual Manipulatives

The National Library of Virtual Manipulatives (NLVM) is a project, supported by the National Science Foundation, that began in 1999 to develop a library of virtual manipulatives for mathematics instruction mainly in K-12. NLVM is organized by the NCTM content standards and by grade level. This is something that students found to be very useful for their field

experience as they could search directly for virtual manipulatives for a particular content standard at a certain grade range.

Because it was found very easy to at different grade levels The Sieve of Eratosthenes (http://nlvm.usu.edu/en/nav/frames_asid_158_g_1_t_1.html?open=instructions) was selected. Using this applet students reviewed multiples, common multiples and least common multiple as well as what are prime and composite numbers.

Students are able to visualize multiplication of two numbers as an area when using The Rectangle Multiplication applet (http://nlvm.usu.edu/en/nav/frames_asid_192_g_1_t_1.html). This applet also shows lattice multiplication and the common multiplication algorithm. Students liked that they could practice and compare the three different methods with this applet. Students also liked to be able to change the numbers to achieve different levels of difficulty.

The concept of subtraction of integers is usually taught using color chips. Students found Color Chips (http://nlvm.usu.edu/en/nav/frames_asid_162_g_2_t_1.html) to be helpful in illustrating subtraction of integers. This applet was found to be especially useful in examples where pairs of chips had to be used in order to be able to subtraction. Students also liked that this applet could be used with problems provided by the computer as well by the user.

Shodor Interactivate

Shodor Interactivate is a web site of Shodor Educational Foundation, Inc., a non-profit research and educational organization dedicated to the advancement of science and math education, specifically through the use of modeling and simulation technologies. The activities in Shodor Interactivate are arranged according to the Principles and Standards for School Mathematics delineated by NCTM.

Venn diagrams and the properties of sets are studied with The Venn Diagram Shape Sorter (http://www.shodor.org/interactivate/activities/venndia/index.html). Students can select using one set, two disjoint sets or two intersecting sets and the rules. The student may select the rule to use and then complete the Venn diagram or guess the rule.

Angles (http://www.shodor.org/interactivate/activities/angles/index.html) is an activity that allows students to practice their knowledge of acute, obtuse, and alternate angles. The student may select the number of alternatives but the problem and questions are created by the computer. Once the student has answered the problem, the student gets immediate feedback. In Why page of this activity there is a link to a related lesson plan.

Students create a game spinner with one to twelve sectors with The Probability Spinner (http://www.shodor.org/interactivate/activities/spinner3/index.html) to look at experimental and theoretical probabilities. With this applet students can see how the experimental probability approaches the experimental probability as the number of trials gets larger.

E-examples

The e-examples is the web site of the electronic examples of the *Principles and Standards for School Mathematics* (NCTM, 2000). The activities on this page are linked to the Standards and grouped by grade level. These activities have very clear instructions on how to use the applets making them very easy to use by teachers and students at all grade levels.

The Geoboard (http://standards.nctm.org/document/eexamples/chap4/4.2/index.htm), designed to be a Kindergarten through Grade 2 example to investigate the concept of triangles and properties of polygons, can be easily used in higher grades. One example students used it for was to show that irrational numbers are real numbers.

The Tangram (http://standards.nctm.org/document/eexamples/chap4/4.4/index.htm), designed to develop geometry understanding was another favorite. Students enjoyed performing the tasks, especially getting the 7 pieces to form a square and the challenge task of forming a

square, if possible, using 1, 2, 3, 4, 5, or 6 tangram pieces. The latter task indicates that this activity also can be extended to higher grades.

Students selected "Comparing the Properties of the Mean and the Median through the use of Technology" (http://standards.nctm.org/document/eexamples/chap4/4.4/index.htm). Students liked this activity because it was easy to see the effect of the outliers on the mean.

Conclusions

Regarding the use of software programs in the mathematics classroom there is no need to spend time teaching the students how to use the programs. When students take these courses, they are already proficient in how to use the programs. It is a matter of working with the creative and pedagogical side of the programs.

The five web sites discussed here are representative of what is available on the Internet for use by mathematics teachers and mathematics teacher educators. During these three years students have found that virtual manipulatives are more accessible than the traditional manipulatives they used when they were in school. The manipulatives used could also be used outside of class because they were always available on the Internet.

Using web sites in mathematics education courses has many advantages. Computers can be found in schools and most schools already have access to the Internet. If schools have it, why not use it? Another advantage is the availability of the Internet to the students. The Internet gives the student access to the mathematics taught and materials used in the classroom from their home or from the library. And let us not forget that students still enjoy using the computer and the Internet and they will be more motivated to do mathematics.

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